Physics 110

Exam #2

November 1, 2019

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 9 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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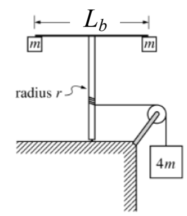
1. Suppose that you are conducting an experiment on rotational motion and that you have a uniform bar of mass $M_b = 1kg$ and length $L_b = 30cm$ has two point masses (each of mass $m = 100g$) attached to each end as shown as experiment A in the figure on the right. The bar is attached to a rod of radius $r = 1cm$ around which a light string is wound. The string passes over a massless pulley and a hanging block of mass $M_h = 4m$ is attached. The system is released from rest.



Experiment A to investigate rotational motion.

a. From an examination of the forces and torques that act on the system, what is the translational acceleration of the hanging mass $M_h = 4m$ after it has fallen through a distance $D = 50cm$?

$$I_{system} = I_{bar} + 2I_{Pm} = \tfrac{1}{12}M_bL_b^2 + 2m\left(\tfrac{L_b}{2}\right)^2 = \tfrac{1}{12}M_bL_b^2 + \tfrac{1}{2}mL_b^2$$

$$\sum F_{apparatus,x}: \quad F_T - F_{\text{stand},x} = M_{apparatus}a_x = 0$$
$$\sum F_{apparatus,y}: \quad F_N - M_bg - 2mg = M_{apparatus}a_y = 0$$

$$\sum \tau: \quad rF_T = I_{system}\alpha = I_{system}\left(\frac{a}{r}\right) \rightarrow F_T = \frac{\left(\tfrac{1}{12}M_bL_b^2 + \tfrac{1}{2}mL_b^2\right)a}{r^2}$$
$$\sum F_{M_H}: \quad F_T - M_hg = -M_ha \rightarrow F_T = M_hg - M_ha = 4mg - 4ma$$

$$F_T = 4mg - 4ma = \frac{\left(\tfrac{1}{12}M_bL_b^2 + \tfrac{1}{2}mL_b^2\right)a}{r^2}$$
$$a = \frac{4mg}{\left[\left(\frac{m}{2} + \frac{M_b}{12}\right)\frac{L_b^2}{r^2} + 4m\right]} = \frac{4 \times 0.1kg \times 9.8\frac{m}{s^2}}{\left[\left(\frac{0.1kg}{2} + \frac{1kg}{12}\right)\left(\frac{0.3m}{0.01m}\right)^2 + 4(0.1kg)\right]} = 0.03\frac{m}{s^2}$$

b. Using energy methods, after the hanging mass M_h has fallen through a distance D, what is its translational speed?

$$\Delta E_{system} = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = 0$$

$$0 = \left(\tfrac{1}{2} M_h v_f^2 - 0\right) + \left(\tfrac{1}{2} I\omega_f^2 - 0\right) + \left(0 - M_h gD\right)$$

$$0 = \tfrac{1}{2} M_h v_f^2 + \tfrac{1}{2} I\omega_f^2 - M_h gD = \tfrac{1}{2}(4m)v_f^2 + \tfrac{1}{2}\left(\tfrac{1}{12} M_b L_b^2 + \tfrac{1}{2} m L_b^2\right)\left(\tfrac{v_f^2}{r^2}\right) - (4m)gD$$

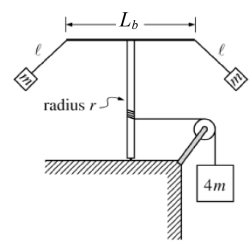
$$v_f = \sqrt{\frac{4mgD}{2m + \left[\frac{m}{4} + \frac{M_b}{24}\right]\left(\frac{L_b}{r}\right)^2}} = \sqrt{\frac{4 \times 0.1kg \times 9.8\frac{m}{s^2} \times 0.5m}{2(0.1kg) + \left[\frac{0.1kg}{4} + \frac{1kg}{24}\right]\left(\frac{0.3m}{0.01m}\right)^2}} = 0.18\tfrac{m}{s}$$

c. When the hanging mass $M_h = 4m$ has fallen through a distance D, the kinetic energy at that instant, in the system is

1. greater than $4mgD$.
2. equal to $4mgD$.

   $\Delta E = \Delta K_T + \Delta K_R + \Delta U_g = 0 \rightarrow \Delta K_T + \Delta K_R = -\Delta U_g = 4mgD$
3. less than $4mgD$.
4. unable to be determined from the information given.

d. Suppose that instead of the situation in parts a, b and c we have the following experiment to investigate rotational motion. In experiment B on the right, the uniform bar still has mass $M_b = 1kg$ and length $L_b = 30cm$ but now each point mass ($m = 100g$) is attached to the end of the bar by a light string of length l. Now, when the hanging mass $M_h = 4m$ has fallen through a distance D, the kinetic energy at that instant, in the system is



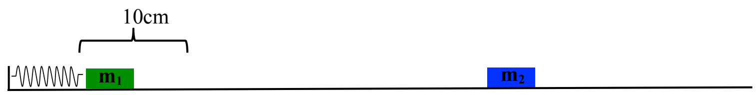
Experiment B

Experiment B used to investigate rotational motion.

1. greater than $4mgD$.
2. equal to $4mgD$.
3. less than $4mgD$.

   $\Delta E = \Delta K_T + \Delta K_R + \Delta U_g = 0 \rightarrow \Delta K_T + \Delta K_R = -\Delta U_{g,M_H} - \Delta U_{g,m} = 4mgD - \Delta U_{g,m}$
4. unable to be determined from the information given.

2. A spring of unknown stiffness is compressed by an amount $x = 10cm$ from its equilibrium position at which point mass $m_1 = 3kg$ is placed. The system is released from rest and when the spring reaches its equilibrium position the mass loses contact with the spring. Assume that the horizontal surface is frictionless.



a. Point mass m_1 makes a head–on collision with an initially stationary point mass $m_2 = 4kg$. After the collision the two masses move off together with a speed of $V = 2\frac{m}{s}$. What is the stiffness of the spring?

$$m_1 v_1 = (m_1 + m_2)V \rightarrow v_1 = \left(\frac{m_1 + m_2}{m_1}\right)V = \left(\frac{3kg + 4kg}{3kg}\right) \times 2\tfrac{m}{s} = 4.7\tfrac{m}{s}$$

$$\Delta E_{system} = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = 0$$

$$0 = \left(\tfrac{1}{2} m_1 v_f^2 - 0\right) + \left(0 - \tfrac{1}{2} k x_i^2\right) \rightarrow k = \frac{m_1 v_f^2}{x_i^2} = \frac{3kg \times \left(4.7\tfrac{m}{s}\right)^2}{(0.10m)^2} = 6627\tfrac{N}{m}$$

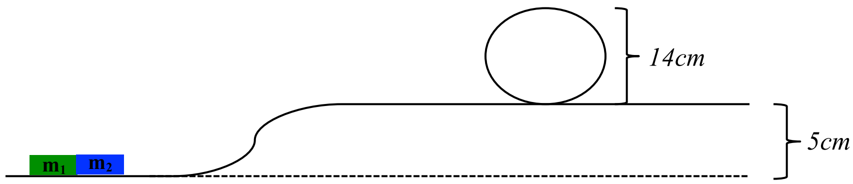
b. The percent of the initial kinetic energy lost to the collision between the two point masses m_1 and m_2 is most likely given by

1. $\% = \left(\frac{m_1}{m_2} - 1\right) \times 100$.
2. $\% = \left(\frac{m_2}{m_1} - 1\right) \times 100$.
3. $\% = \left(\frac{m_1 + m_2}{m_2} - 1\right) \times 100$.
4. $\% = \left(\frac{m_1}{m_1 + m_2} - 1\right) \times 100$.

$$\frac{K_f - K_i}{K_i} = \frac{K_f}{K_i} - 1 = \frac{\frac{1}{2}(m_1 + m_2)\left[\frac{m_1^2}{(m_1+m_2)^2} v_1^2\right]}{\frac{1}{2} m_1 v_1^2} - 1 = \frac{m_1}{m_1 + m_2} - 1$$

5. None of the above will give the correct expression for the energy lost to the collision.

c. Suppose that after the collision the system of point masses m_1 and m_2 slide up a $5cm$ hill tall and then around the loop-the-loop with diameter $14cm$. What is the magnitude of the reaction force of the tract on the masses m_1 and m_2 at the top of the loop-the-loop? Assume that all of the surfaces are frictionless.



$$\Delta E_{system} = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = 0$$
$$0 = \left(\tfrac{1}{2}(m_1+m_2)v_{top}^2 - \tfrac{1}{2}(m_1+m_2)v_{bottom}^2\right) + \left((m_1+m_2)gy_{top} - 0\right)$$
$$v_{top} = \sqrt{v_{bottom}^2 - 2gy_{top}} = \sqrt{\left(2\tfrac{m}{s}\right)^2 - 2\times 9.8\tfrac{m}{s^2}\times 0.19m} = 0.52\tfrac{m}{s}$$

$$\sum F_{y,top} : -F_N - (m_1+m_2)g = -(m_1+m_2)\frac{v_{top}^2}{R} \rightarrow F_N = (m_1+m_2)\left(g - \frac{v_{top}^2}{R}\right)$$
$$F_N = (m_1+m_2)\left(g - \frac{v_{top}^2}{R}\right) = (3kg+4kg)\left(9.8\tfrac{m}{s^2} - \frac{\left(0.52\tfrac{m}{s}\right)^2}{0.07m}\right) = 41.2N$$

d. Suppose that instead of rising up the $5cm$ tall hill and then around the loop-the-loop, the masses m_1 and m_2 instead fell down a hill of the same height and then around the loop-the-loop track of the same diameter. In this case the magnitude of the reaction force of the track on masses m_1 and m_2 would be

1. less than the magnitude found in part c.
2. equal to the magnitude found in part c.
3. greater than the magnitude found in part c.
4. dependent on the stiffness of the spring
5. unable to be determined from the information given.

3. Hip problems, like lower backaches and pain, are often associated with an underlying medical condition such as rheumatoid arthritis, osteoarthritis, tendonitis, pelvic floor issues and being overweight. As with the back, forces on the hip from the legs can be several times that of a person's body weight. To see how large these forces can be consider the model of the human leg as shown below which illustrates several forces at play as you, say stand on one foot, or as you walk slowly. The figure on the left, below, illustrates the forces on the leg and hip while the figure on the right, below, is a cartoon diagram illustrating the various forces and distances from the pivot involved.

   F_N is the reaction force from the floor with supports the body's weight F_{WB} and F_{WL} is the weight of the leg, assumed uniform and given as $0.16F_{WB}$. F_M is the force due to the muscles in the hip, called the abductor muscles. The hip abductor muscles are responsible for moving the leg away from the body and help rotate the leg at the hip joint. The hip abductors are necessary for stability when walking or standing on one leg. Lastly, F_R is the reaction force on the leg from the hip itself and this is what we'd like to calculate. Let $\theta = 70^0$.

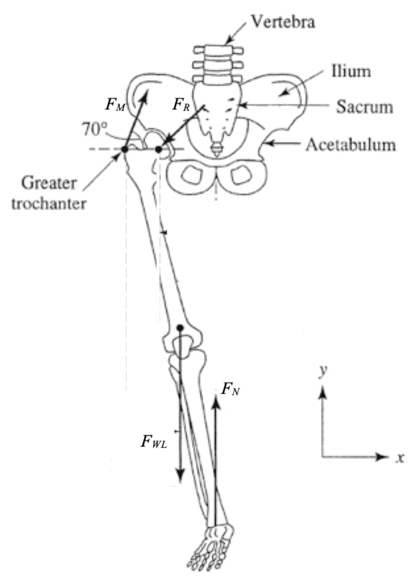
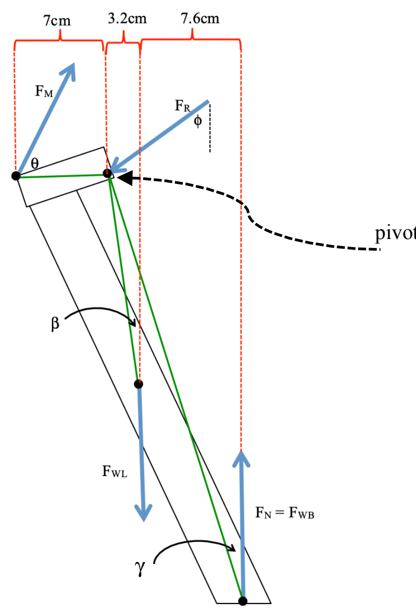


Figure adapted from Physics of the Human Body by Herman Irving



a. In the diagram on the right above on the previous page, the green lines represent the radial distances from the pivot to the each applied force and the angles between the radial distances and the applied forces are given. Using this information, what are the expressions for the sum of the forces in the horizontal and vertical directions and the sum of the torques about the pivot? Let $\theta = 70^0$ and take clockwise as the positive direction for the torque.

$$\sum F_x : F_M \cos\theta - F_R \sin\phi = ma_x = 0$$
$$\rightarrow F_R \sin\phi = F_{Rx} = F_M \cos\theta$$
$$\sum F_y : F_M \sin\theta - F_R \cos\phi - F_{WL} + F_N = ma_y = 0 \rightarrow F_R \cos\phi = F_{Ry} = F_M \sin\theta - 0.16 F_{WB} + F_{WB}$$
$$\rightarrow F_R \cos\phi = F_{Ry} = F_M \sin\theta - 0.16 F_{WB} + F_{WB} = F_M \sin\theta + 0.84 F_{WB}$$

$$\sum \tau : \ + r_{F_M} F_M \sin\theta - r_{F_N} F_N \sin\gamma + r_{F_{WL}} F_{WL} \sin\beta = I_{system}\alpha = 0$$
$$\rightarrow 0 = \left(0.07m \times \sin 70\right) F_M - \left(0.108m\right) F_{WB} + \left(0.032m \times 0.16\right) F_{WB}$$
$$F_M = 1.56 F_{WB}$$

$$\therefore F_R \sin\phi = F_{Rx} = F_M \cos\theta = 1.56 F_{WB} \cos 70 = 0.534 F_{WB}$$
$$\therefore F_R \cos\phi = F_{Ry} = F_M \sin\theta + 0.84 F_{WB} = 1.56 F_{WB} \sin 70 + 0.84 F_{WB} = 2.31 F_{WB}$$

From the geometry of the system we can get:

$$\sin\gamma = \frac{0.108m}{r_{F_N}} \rightarrow 0.108m = r_{F_N} \sin\gamma$$

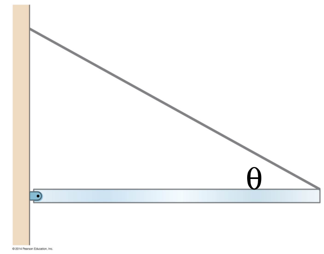
$$\sin\beta = \frac{0.032m}{r_{F_{WL}}} \rightarrow 0.032m = r_{F_{WL}} \sin\beta$$

b. What is the magnitude and direction of F_R in terms of the weight of the body, F_{WB}?

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \ @ \phi = \tan^{-1}\left(\tfrac{F_{Ry}}{F_{Rx}}\right)$$
$$F_R = \sqrt{\left(0.534 F_{WB}\right)^2 + \left(2.31 F_{WB}\right)^2} \ @ \phi = \tan^{-1}\left(\tfrac{0.534 F_{WB}}{2.31 F_{WB}}\right)$$
$$F_R = 2.37 F_{WB} \ @ \phi = 13^0$$

If we define Δ to be the angle with respect to the horizontal, then $\Delta = 90 - \phi = 77^0$ and since this is greater than θ, the system is stable against rotation and you won't fall over sideways.

c. Suppose that you have the following system in which a uniform beam is hinged at one end and held in position by a cable as shown on the right. The tension in the cable



1. must be at least half of the weight of the beam, no matter what angle the cable makes with the horizontal.

$$\tau_{hinge,left} = \frac{L}{2} M_{beam} g - LF_T \sin\theta \rightarrow F_T = \frac{M_{beam} g}{2 \sin\theta} = \begin{cases} \theta = 0^0;\ F_T \rightarrow \infty \\ \theta = 90^0;\ F_T \rightarrow \dfrac{M_{beam} g}{2} \end{cases}.$$

2. could be less than the beam's weight for some angles the cable makes with the horizontal.
3. will be half of the beam's weight for all angles the cable makes with the horizontal.
4. will be equal to the beam's weight for all angles the cable makes with the horizontal.
5. cannot be determined for this situation.

d. Two children, Alex and Samantha, are sitting on a merry-go-round. Alex is at a point halfway between the center of the merry-go-round and the outer edge, while Sam is sitting on the outer edge. The merry-go-round makes one revolution every $2s$. Alex's linear velocity is

1. one quarter of Samantha's linear velocity.
2. half of Samantha's linear velocity.

$$v = r\omega \rightarrow \omega = \frac{v_A}{r_A} = \frac{v_S}{r_S} \rightarrow v_A = \frac{r_A}{r_S} v_S = \frac{0.5R}{R} v_S = \frac{v_S}{2}.$$

3. the same as Samantha's linear velocity.
4. twice that of Samantha's linear velocity.
5. four times that of Samantha's linear velocity.

Physics 110 Formulas

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Uniform Circular Motion

$$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x{}^2 + v^2{}_y}$

direction of a vector: $\phi = \tan^{-1}\left(\dfrac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\,{}^{m}\!/\!_{s^2}$ $\quad G = 6.67\times10^{-11}\,{}^{Nm^2}\!/\!_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}\!/\!_{mole}$ $\quad k_B = 1.38\times10^{-23}\,{}^{J}\!/\!_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}\!/\!_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}\!/\!_{s}$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\cdot dt$$

$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

$$\vec{F}_s = -k\vec{x}$$

$$|\vec{F}_{fr}| = \mu|\vec{F}_N|$$

Work/Energy

$$K_T = \tfrac{1}{2}mv^2$$

$$K_R = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = F\Delta x \,\mathrm{Cos}\,\theta = \Delta K_T$$

$$W_R = \tau\theta = \Delta K_R$$

$$W_{net} = W_R + W_T = \Delta K_R + \Delta K_T$$

$$\Delta K_R + \Delta K_T + \Delta U_g + \Delta U_S = \Delta E_{system} = 0$$

$$\Delta K_R + \Delta K_T + \Delta U_g + \Delta U_S = \Delta E_{system} = W_{fr} = -F_{fr}\Delta x$$

Heat

$$T_C = \tfrac{5}{9}[T_F - 32]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}(1 + \alpha\Delta T)$$

$$A_{new} = A_{old}(1 + 2\alpha\Delta T)$$

$$V_{new} = V_{old}(1 + \beta\Delta T):\ \beta = 3\alpha$$

$$PV = Nk_BT$$

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta:\ v = r\omega:\ a_t = r\alpha$$

$$a_r = r\omega^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0};\quad I_o = 1\times10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L};\ f_n = nf_1 = n\frac{v}{4L}$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho gd$$

$$F_B = \rho gV$$

$$A_1v_1 = A_2v_2$$

$$\rho_1A_1v_1 = \rho_2A_2v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho gh_2$$

Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

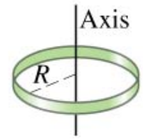
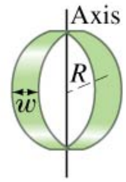
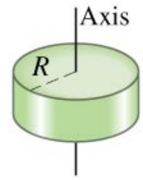
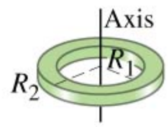
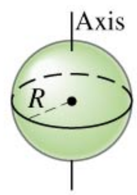
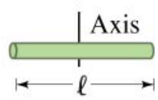
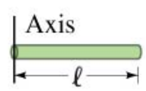
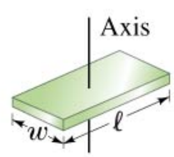
$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2 \rho v A^2$$

	Object	Location of axis		Moment of inertia
(a)	**Thin hoop,** radius R	Through center		MR^2
(b)	**Thin hoop,** radius R width w	Through central diameter		$\frac{1}{2}MR^2 + \frac{1}{12}Mw^2$
(c)	**Solid cylinder,** radius R	Through center		$\frac{1}{2}MR^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius R	Through center		$\frac{2}{5}MR^2$
(f)	**Long uniform rod,** length ℓ	Through center		$\frac{1}{12}M\ell^2$
(g)	**Long uniform rod,** length ℓ	Through end		$\frac{1}{3}M\ell^2$
(h)	**Rectangular thin plate,** length ℓ, width w	Through center		$\frac{1}{12}M(\ell^2 + w^2)$