Physics 110

Exam #2

October 30, 2020

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg) \times (2\frac{m}{s}) = 10\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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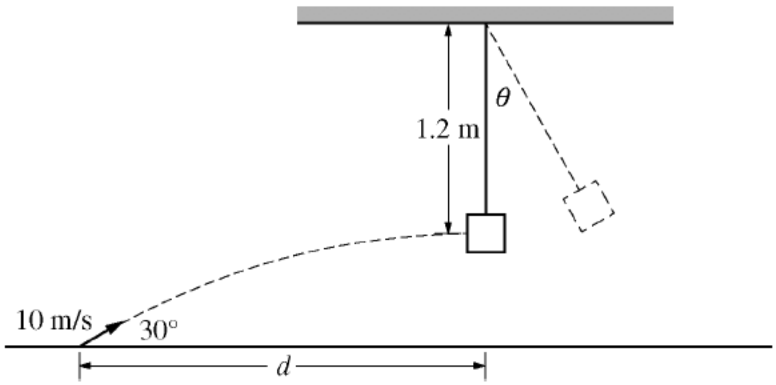
1. A small dart of mass $m_d = 100g$ is launched at an angle of 30^0, measured with respect to the horizontal, with initial speed $10\frac{m}{s}$. When the dart reaches its highest vertical point, it is moving horizontally, and it collides with and sticks to a wooden block of mass $m_b = 500g$ that is suspended at the end of a very light rope of length $l = 1.2m$. The block and dart then swing upward through an angle θ, measured with respect to the vertical, as shown below.



a. What is the speed of the dart/block system immediately after the collision?

$$v_{fx} = v_{ix} = v_i \cos\theta = 10\frac{m}{s} \times \cos 30 = 8.66\frac{m}{s}$$

$$\Delta p = 0 \to p_f - p_i = 0 \to p_i = p_f \to m_d v_{id} = (m_d + m_b)V$$

$$V = \left(\frac{m_d}{m_d+m_b}\right) v_i = \left(\frac{0.1kg}{0.1kg+0.5kg}\right) \times 8.66\frac{m}{s} = 1.44\frac{m}{s}$$

b. Through what angle θ did the dart/block system swing?

$$\Delta E = 0 = \Delta K + \Delta U_g + \Delta U_s$$

$$0 = \left(0 - \frac{1}{2}(m_d + m_b)V^2\right) + \left((m_d + m_b)gy_f - (m_d + m_b)gy_i\right)$$

$$y_f = \frac{V^2}{2g} = \frac{\left(1.44\frac{m}{s}\right)^2}{2\times 9.8\frac{m}{s^2}} = 0.11m$$

$$\cos\theta = \frac{L - y_f}{L} = \frac{1.2m - 0.11m}{1.2m} = 0.9083 \to \theta = \cos^{-1} 0.9803 = 27.4^0$$

c. Derive an expression (and then evaluate it with the numbers you have) for the fraction of energy that was lost to the collision?

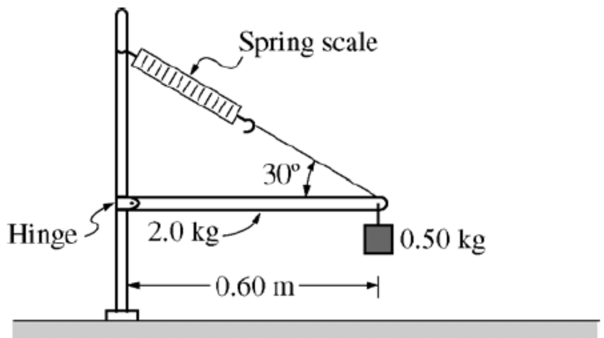
$$f = \frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = \frac{\frac{1}{2}(m_d + m_b)V^2 - \frac{1}{2}m_d v_i^2}{\frac{1}{2}m_d v_i^2} = \frac{(m_d + m_b)V^2}{m_d v_i^2} - 1 = \frac{(0.1kg + 0.5kg)\left(1.44\frac{m}{s}\right)^2}{0.1kg\left(8.66\frac{m}{s}\right)^2} - 1$$

$$f = -0.834$$

d. Suppose a second experiment is done with a new dart with mass $m_{new,dart} = 2m_d$. The dart is launched at the same initial speed and angle and collides with a identical block. Compared to angle θ that you calculated in part b, which of the following gives the correct statement about the new angle θ_{new} through which the system now swings?

1. $\theta_{new} < \theta$ because the speed of the dart/block system is lower after the collision.
2. $\theta_{new} < \theta$ because the speed of the dart/block system is higher after the collision
3. $\theta_{new} = \theta$ because the mass has no effect on the angle.
4. $\theta_{new} > \theta$ because the speed of the dart/block system is lower after the collision.
5. $\theta_{new} > \theta$ because the speed of the dart/block system is higher after the collision

2. A horizontal uniform rod has a length $l = 0.60m$ and mass $m_r = 2kg$ assumed ot act at the rod's center. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of $30^0$with respect to the rod. A spring scale of negligible mass is used to measure the tension in the cord. At the right end of the rod a block of mass $m_b = 0.50kg$ is attached as shown below.



a. What would the spring scale read for the tension in the cord and if the spring in the scale had a stiffness of $k = 1000\frac{N}{m}$, how far out would the spring be pulled from equilibrium?

$$\sum \tau: -\frac{l}{2}m_r g \sin 90 - l m_b g \sin 90 + l F_T \sin\theta = I\alpha = 0$$

$$F_T = \frac{m_r g}{2} + m_b g = \left(\frac{2kg}{2} + 0.5kg\right)\frac{9.8\frac{m}{s^2}}{\sin 30} = 29.4N$$

$$F_T = kx \rightarrow x = \frac{F_T}{k} = \frac{29.4N}{1000\frac{N}{m}} = 0.029m = 29mm$$

b. What is the magnitude and direction of the reaction force of the hinge?

$$\sum F_x:\ F_R\cos\phi - F_T\cos\theta = ma_x = 0$$

$$\sum F_y:\ F_R\sin\phi - F_T\sin\theta - m_r g - m_b g = ma_y = 0$$

$$\rightarrow F_R\cos\phi = F_T\cos\theta = 29.4N \times \cos 30 = 25.5N$$

$$\rightarrow F_R\sin\phi = -F_T\sin\theta + m_r g + m_b g$$
$$= -29.4N \times \sin 30 + (2kg + 0.5kg) \times 9.8\frac{m}{s^2} = 9.8N$$

$$\frac{F_R\sin\phi}{F_R\cos\phi} = \tan\phi = \frac{9.8N}{25.5N} = 0.384 \rightarrow \phi = \tan^{-1} 0.384 = 21^0$$
$$F_R\sin\phi = 9.8N \rightarrow F_R = \frac{9.8N}{\sin 21} = 27.4N$$

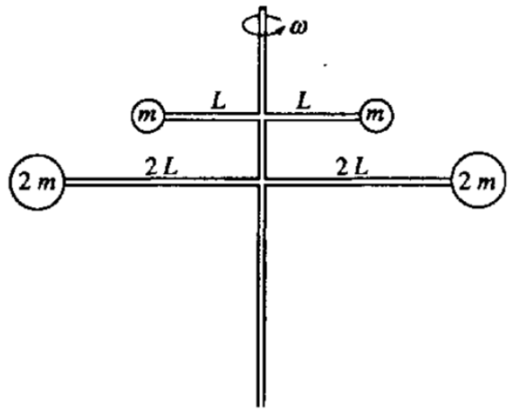
c. If the cord is cut, what is the initial angular acceleration of the rod? Hint: The moment of inertia of a rod spun about one end is $I = \frac{1}{3}ML^2$.

$$-\frac{l}{2}m_r g - l m_b g = I\alpha = \left(\frac{1}{3}m_r l^2 + m_b l^2\right)\alpha$$

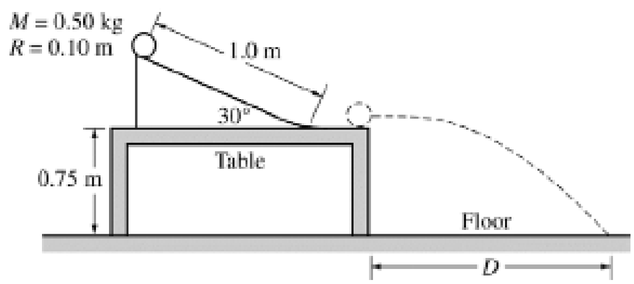
$$\alpha = \frac{-\frac{l}{2}m_r g - l m_b g}{\frac{1}{3}m_r l^2 + m_b l^2} = -\left(\frac{\frac{2kg}{2}+0.5kg}{\frac{1}{3}(2kg)+0.5kg}\right)\frac{9.8\frac{m}{s^2}}{0.6m} = -20.9\frac{rad}{s^2} \text{ or } 20.9\frac{rad}{s^2}\ clockwise$$

d. The rigid body shown in the diagram above consists of a vertical support post and two horizontal crossbars with spheres attached. The masses of the spheres and the lengths of the crossbars are indicated in the diagram. The body rotates about a vertical axis along the support post with constant angular speed ω. If the masses of the support post and the crossbars are negligible, what is the ratio of the angular momentum of the two upper spheres to that of the two lower spheres?

1. $\frac{L_{upper}}{L_{lower}} = 8.$
2. $\frac{L_{upper}}{L_{lower}} = 4.$
3. $\frac{L_{upper}}{L_{lower}} = 2.$
4. $\frac{L_{upper}}{L_{lower}} = \frac{1}{2}.$
5. $\frac{L_{upper}}{L_{lower}} = \frac{1}{4}.$
6. $\frac{L_{upper}}{L_{lower}} = \frac{1}{8}.$



3. A uniform solid cylinder of mass $M = 0.50kg$, radius $R = 0.1m$, and moment of inertia $I_C = \frac{1}{2}MR^2$ is released from rest and rolls without slipping down a long ramp inclined at an angle of 30^0measured with respect to the table. At the end of the ramp, the cylinder is launched from a horizontal table of height $\Delta y = 0.75m$ and lands on the floor a distance D away from the edge of the table as shown below. There is a smooth transition from the ramp to the table and the motion occurs with no frictional energy losses.



a. Using energy ideas, what is the total kinetic energy of the cylinder at it reaches the horizontal table and what is the rotational speed of the cylinder about an axis through its center at this point?

$$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s$$
$$0 = (K_{fT} - 0) + (K_{fR} - 0) + (Mgy_f - Mgy_i)$$
$$0 = K_{fT} + K_{fR} + (MgR - Mg(d\sin\theta + R))$$
$$K_{fT} + K_{fR} = K_{total} = Mgd\sin\theta = 0.5kg \times 9.8\tfrac{m}{s^2} \times 1.0m \times \sin 30 = 2.45J$$

$$K_{total} = \tfrac{1}{2}Mv_f^2 + \tfrac{1}{2}I\omega_f^2 = \tfrac{1}{2}M(R\omega_f)^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega_f^2 = \tfrac{3}{4}MR^2\omega_f^2$$

$$\omega_f = \sqrt{\frac{4K_{total}}{3MR^2}} = \sqrt{\frac{4\times 2.45J}{3\times 0.5kg\times(0.1m)^2}} = 25.6\frac{rad}{s}$$

b. Using energy ideas, what is the impact speed, v_f, of the cylinder with the floor? Hint: The cylinder continues to rotate when it comes off of the table.

$$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s$$

$$0 = \left(\frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2\right) + \left(\frac{1}{2}Mv_f^2 - \frac{1}{2}Mv_i^2\right) + \left(Mgy_f - Mgy_i\right)$$

$$0 = \left(\frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2 - \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2\right) + \left(\frac{1}{2}Mv_f^2 - \frac{1}{2}M(R\omega_i)^2\right) + \left(Mg(R) - Mg(R + y_i)\right)$$

$$\frac{3}{4}Mv_f^2 = Mgy_i + \frac{3}{4}MR^2\omega_i^2 \rightarrow v_f = \sqrt{\frac{4}{3}gy_i + R^2\omega_i^2}$$

$$v_f = \sqrt{\left(\frac{4}{3} \times 9.8\frac{m}{s^2} \times 0.75\right) + \left(0.1m \times 25.6\frac{rad}{s}\right)^2} = 4\frac{m}{s}$$

c. Suppose a sphere of the same mass and radius is rolled down the same ramp. If the sphere has moment of inertia $I_s = \frac{2}{5}MR^2$, which of the following gives the correct relationship between the total kinetic energy of the sphere (K_S) to the cylinder (K_C) when they both reach the floor?

1. $K_S < K_C$ since the moment of inertia of the sphere is less.
2. $K_S < K_C$ since the moment of inertia of the sphere is greater.
3. $K_S = K_C$ since they both fall through the same height Δy.
4. $K_S > K_C$ since the moment of inertia of the sphere is less.
5. $K_S > K_C$ since the moment of inertia of the sphere is greater.

d. What is the angular acceleration of the cylinder about its center as it rolls without slipping down the incline? Hints: Write the forces parallel to the ramp and what force causes a torque about the center of the cylinder in the system?

$$\sum F_x:\ -F_{fr} + Mg\sin\theta = Ma = MR\alpha \rightarrow F_{fr} = Mg\sin\theta - MR\alpha$$

$$\sum \tau:\ \ RF_{fr} = -MR^2\alpha + RgM\sin\theta = I\alpha = \frac{1}{2}MR^2\alpha$$

$$\frac{3}{2}MR^2\alpha = RMg\sin\theta$$

$$\alpha = \frac{2g}{3R}\sin\theta = \frac{2\times 9.8\frac{m}{s^2}}{3\times 0.1m}\sin 30 = 32.7\frac{rad}{s}$$

Physics 110 Formulas

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \mathrm{D}x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \mathrm{D}y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2pr}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles *Triangles* *Spheres*

$C = 2pr \quad A = \frac{1}{2}bh \quad A = 4pr^2$

$A = pr^2 \quad V = \frac{4}{3}pr^3$

Quadratic equation : $ax^2 + bx + c = 0,$

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x{}^2 + v^2{}_y}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8 \,{}^{m}\!/\!_{s^2} \quad G = 6.67´\ 10^{-11}\, {}^{Nm^2}\!/\!_{kg^2}$

$N_A = 6.02´\ 10^{23}\, {}^{atoms}\!/\!_{mole} \quad k_B = 1.38´\ 10^{-23}\, {}^{J}\!/\!_{K}$

$s = 5.67´\ 10^{-8}\, {}^{W}\!/\!_{m^2K^4} \quad v_{sound} = 343\, {}^{m}\!/\!_{s}$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F} \cdot dt$

$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$

$\vec{F}_s = -k\vec{x}$

$|\vec{F}_{fr}| = \mu |\vec{F}_N|$

Work/Energy

$K_T = \frac{1}{2}mv^2$

$K_R = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = Fd\,\mathrm{Cos}\,\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + a\mathrm{D}T)$

$A_{new} = A_{old}(1 + 2a\mathrm{D}T)$

$V_{new} = V_{old}(1 + b\mathrm{D}T): \ b = 3a$

$PV = Nk_B T$

$\frac{3}{2}k_B T = \frac{1}{2}mv^2$

$\mathrm{D}Q = mc\mathrm{D}T$

$P_C = \frac{\mathrm{D}Q}{\mathrm{D}t} = \frac{kA}{L}\mathrm{D}T$

$P_R = \frac{\mathrm{D}Q}{\mathrm{D}T} = esAT^4$

$\mathrm{D}U = \mathrm{D}Q - \mathrm{D}W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta: \ v = r\omega: \ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = fl = (331 + 0.6T)\,\frac{m}{s}$

$b = 10\log\frac{I}{I_0}; \quad I_o = 1´\ 10^{-12}\,\frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1 v_1 = A_2 v_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$w = 2pf = \frac{2p}{T}$

$T_S = 2p\sqrt{\frac{m}{k}}$

$T_P = 2p\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2pt}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2pt}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2pt}{T}\right)$

$v = fl = \sqrt{\frac{F_T}{m}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2p^2 f^2 rvA^2$