Physics 110

Exam #2

October 19, 2026

Name_____

Please read and follow these instructions carefully:

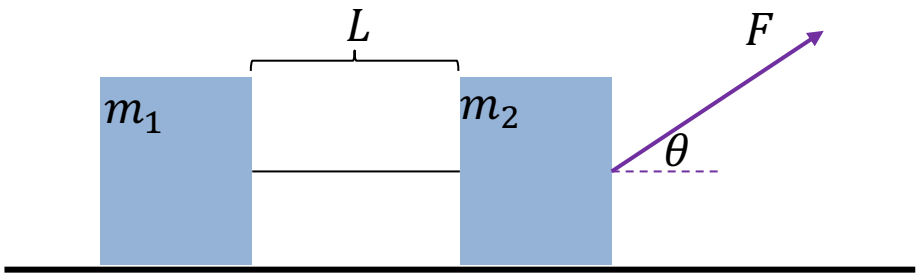
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. Erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg) \times \left(2\,\frac{m}{s}\right) = 10\,\frac{kg \cdot m}{s}$
- Give standard SI units with your results.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Formulas or constants not explicitly given on the formula sheet need to be derived before they can be used or full credit will not be given even if the formula or answer is correct.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 4 points.

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. Two blocks of masses $m_1 = 2kg$ and $m_2 = 5kg$ are being pulled by a force with magnitude $F = 50N$ oriented at an angle $\theta = 50^0$ measured with respect to the horizontal. The two masses are connected together by a massless rope of length $L = 50cm$. Friction exists between each block and the surface with coefficient of friction $\mu = 0.25$.



a. What are the reaction forces exerted on each block by the table?

Taking vertically up as the positive y-direction we have:

For m_1 in the y-direction:

$$F_{N1} - F_{W1} = F_{N1} - m_1 g = m_1 a_y = 0 \rightarrow F_{N1} = m_1 g = 2kg \cdot 9.8\frac{m}{s^2} = 19.6N$$

For m_2 in the y-direction:

$$F_{N2} - F_{W2} + F_y = F_{N1} - m_2 g + F\sin\theta = m_2 a_y = 0 \rightarrow F_{N2} = m_2 g - F\sin\theta$$

$$F_{N2} = 5kg \cdot 9.8\frac{m}{s^2} - 50N\sin 50 = 10.7N$$

b. What is the acceleration of the two-block system down the horizontal surface?

Taking to the right as the positive x-direction we have:

$$F_x - F_{fr1} - F_{fr2} = F\cos\theta - \mu F_{N1} - \mu F_{N2} = (m_1 + m_2)a$$

$$\rightarrow a = \frac{F\cos\theta - \mu(F_{N1} + F_{N2})}{m_1 + m_2}$$

$$a = \frac{50N\cos 50 - 0.25(19.6N + 10.7N)}{2kg + 5kg} = 3.51\frac{m}{s^2}$$

c. What are the magnitudes and directions of the tension force in the ends of the rope that are physically connected to each block? Do these forces obey Newton's 3rd law? Explain your answer.

Taking to the right as the positive x-direction we have:

The tension force for m_1:

$F_{T1} - F_{fr1} = m_1 a \rightarrow F_{T1} = m_1 a + F_{fr1} = m_1 a + \mu m_1 g$

$F_{T1} = 2kg \cdot \left(3.51\frac{m}{s^2} + 0.25 \cdot 9.8\frac{m}{s^2}\right) = 11.92N$ pointing to the right.

The tension force for m_2:

$-F_{T2} - F_{fr2} + F\sin\theta = m_2 a \rightarrow F_{T2} = F\sin\theta - m_2 a + F_{fr2}$

$F_{T2} = F\sin\theta - m_2 a + \mu m_1 g = 50N\sin 50 - 5kg \cdot 3.51\frac{m}{s^2} + 0.25 \cdot 5kg \cdot 9.8\frac{m}{s^2}$

$F_{T2} = 11.92N$ to the left.

Since these forces are equal in magnitude and oppositely directed, they obey Newton's 3rd law.

d. How much work was done on each block if the blocks are pulled from rest along the horizontal surface for a time $t = 4s$?

In this time, the block moves a horizontal distance:

$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 = \frac{1}{2}a_x t^2 = \frac{1}{2} \cdot 3.51\frac{m}{s^2} \cdot (4s)^2 = 28.1m$

The work done on m_1:

$W_1 = F_{net,1} \cdot \Delta x\cos\theta = m_1 a\Delta x\cos\theta = 2kg \cdot 3.51\frac{m}{s^2} \cdot 28.1m \cdot \cos 0 = 197.12J$

The work done on m_2:

$W_2 = F_{net,2} \cdot \Delta x\cos\theta = m_2 a\Delta x\cos\theta = 5kg \cdot 3.51\frac{m}{s^2} \cdot 28.1m \cdot \cos 0 = 493.16J$

e. Using the work-kinetic energy theorem, what is the speed of the two-block system after being pulled along the horizontal surface for a time $t = 4s$?

$$W_{net} = W_1 + W_2 = \Delta K = \Delta K_1 + \Delta K_2$$

$$W_{net} = W_1 + W_2 = \left(\frac{1}{2}m_1 v_{1f}^2 - \frac{1}{2}m_1 v_{1i}^2\right) + \left(\frac{1}{2}m_2 v_{2f}^2 - \frac{1}{2}m_2 v_{2i}^2\right) = \frac{1}{2}(m_1 + m_2)v_f^2$$

$$v_f = \sqrt{\frac{2W_{net}}{m_1 + m_2}} = \sqrt{\frac{2(W_1 + W_2)}{m_1 + m_2}} = \sqrt{\frac{2(197.12J + 493.16J)}{2kg + 5kg}} = 14\frac{m}{s}$$

f. Suppose that at a time $t = 4s$ you let go of the rope connected to the block of mass m_2. How far from where you let go of the rope will the blocks of masses m_1 and m_2 move before coming to rest?

When you let go of the rope, F vanishes and the blocks slow to rest due to friction.

$$W_{net} = W_1 + W_2 = W_{fr1} + W_{fr2} = \Delta K = \Delta K_1 + \Delta K_2$$

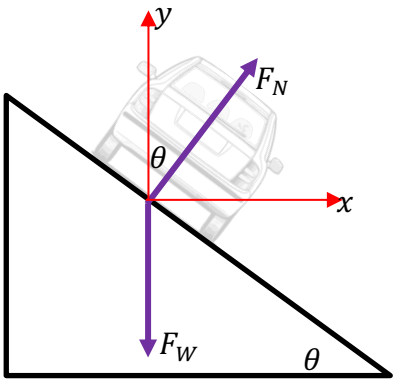
$$W_{net} = W_{fr1} + W_{fr2} = \frac{1}{2}(m_1 + m_2)v_f^2 - \frac{1}{2}(m_1 + m_2)v_i^2 = -\frac{1}{2}(m_1 + m_2)v_i^2$$

$$W_{net} = -\mu F_{N1}\Delta x - \mu F_{N2}\Delta x = -\frac{1}{2}(m_1 + m_2)v_i^2 \rightarrow \Delta x = \frac{(m_1 + m_2)v_i^2}{2\mu(F_{N1} + F_{N2})}$$

$$\Delta x = \frac{(m_1 + m_2)v_i^2}{2\mu g(m_1 + m_2)} = \frac{v_i^2}{2\mu g} = \frac{\left(14\frac{m}{s}\right)^2}{2 \cdot 0.25 \cdot 9.8\frac{m}{s^2}} = 40m$$

2. A car of mass m is negotiating a corner in a road that has a radius of curvature R and is banked at an angle of θ measured with respect to the horizontal at a constant speed v as shown below.

a. From a carefully labeled free-body diagram, derive an expression for the magnitude of the reaction force of the roadway on the car? Your expression should depend on the bank angle θ. Does your expression make sense? That is, explain the limiting cases for θ, namely what do you get if $\theta = 0^0$ and $\theta = 90^0$. What do they mean and are these reasonable?



$$F_{net,y} = F_{Ny} - F_W = F_N \cos\theta - mg = ma_y = 0$$

$$F_N = \frac{mg}{\cos\theta}$$

If $\theta = 0^0$, then the car is sitting on a horizontal surface and the normal force from the road is $F_N = mg$ as expected.

As $\theta \to 90^0$, the normal force from the road is $F_N \to \infty$. This too is expected. As the car becomes vertical the weight wants to pull it down to the ground. To keep it in the horizontal circle, the car would need to push on the almost vertical wall with a very large force. Thus, the wall pushes back with a very large reaction force by Newton's 3rd law.

b. From your carefully labeled free-body diagram, derive an expression for the speed v at which the car can take the corner without having to rely on friction? This is called the design speed. Further, suppose that the car has a mass $m = 1200kg$, the road has a radius of curvature $R = 87m$ $(\sim 270feet)$ and is banked at an angle of $\theta = 20^0$ measured with respect to the horizontal. What is the speed that the roadway is designed for? To earn full credit, you need to derive an expression for the speed v. Simply stating a formula or evaluating a formula with no derivation will earn minimal credit.

$$F_{net,x} = F_{Nx} = F_N \sin\theta = ma_c = m\frac{v^2}{R}$$

$$F_N \sin\theta = \left(\frac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta = m\frac{v^2}{R}$$

$$v = \sqrt{Rg\tan\theta} = \sqrt{87m \cdot 9.8\frac{m}{s^2} \cdot \tan 20} = 17.6\frac{m}{s}\left(\sim 40\frac{mi}{hr}\right)$$

c. Suppose that you want to take the curve at a speed higher than the design speed, say perhaps $v = 55\frac{mi}{hr} (= 24\frac{m}{s})$. In this case you would have to rely on friction to help you take the corner. What coefficient of friction μ would be needed to safely take the corner at this speed using the numbers from part b?

To take the corner at a high rate of speed, the car would tend to slide up the track to a larger radius. To keep the radius fixed, we need friction to point down the track to help us take the corner.

$$F_{net,y} = F_{Ny} - F_W - F_{Fry} = F_N \cos\theta - mg - \mu F_N \sin\theta = ma_y = 0$$

$$\rightarrow F_N = \frac{mg}{\cos\theta - \mu \sin\theta}$$

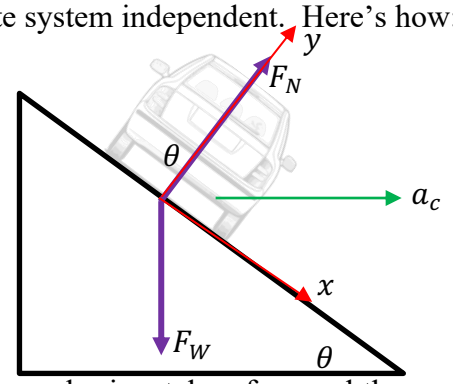
$$F_{net,x} = F_{Nx} + F_{frx} = F_N \sin\theta + \mu F_N \cos\theta = ma_c = m\frac{v^2}{R}$$

$$F_N(\sin\theta + \mu\cos\theta) = mg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right) = m\frac{v^2}{R} \rightarrow \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{Rg}$$

$$\frac{v^2}{Rg} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta} \rightarrow \mu = \frac{\frac{v^2}{Rg} - \tan\theta}{1 + \frac{v^2}{Rg}\tan\theta} = \frac{\frac{\left(24\frac{m}{s}\right)^2}{87m \cdot 9.8\frac{m}{s^2}} - \tan 20}{1 + \frac{\left(24\frac{m}{s}\right)^2}{87m \cdot 9.8\frac{m}{s^2}}\tan 20} = 0.25$$

As an aside, many of you wanted to tilt your coordinate axes so that the y-axis is perpendicular to the incline and the x-axis is parallel to the incline. You can certainly do it this way. The expressions/answers are coordinate system independent. Here's how:

a. In the y-direction we have:



$$F_N - F_W \cos\theta = ma_{c,y} = \frac{mv^2}{R}\sin\theta$$

$$\rightarrow F_N = mg\cos\theta + \frac{mv^2}{R}\sin\theta$$

If $\theta = 0^0$ then $F_N = mg$ and the car is sitting on a horizontal surface and the normal force should equal the wight. This seems reasonable.

If $\theta = 90^0$ then $F_N = \frac{mv^2}{R}$ and the car turning in horizontal circle parallel to the ground and the wall has to react to that and push back. This is the rotor problem from class. This too seems reasonable.

b. In the x-direction we have:

$F_W \sin\theta = ma_{c,x} = \frac{mv^2}{R}\cos\theta \rightarrow v = \sqrt{Rg\tan\theta}$ which is exactly what we got when we didn't tilt the axes. The choice of coordinate system does not matter.

c. The coefficient of friction:

In the x-direction we have:

$$F_W \sin\theta + F_{fr} = mg\sin\theta + \mu F_N = \frac{mv^2}{R}\cos\theta$$

In the y-direction we have the same expression as in part a since friction points down the ramp and thus does not point perpendicular to the ramp in this coordinate system.

$$\rightarrow mg\sin\theta + \mu F_N = mg\sin\theta + \mu\left(mg\cos\theta + \frac{mv^2}{R}\sin\theta\right) = \frac{mv^2}{R}\cos\theta$$

$$mg\frac{\sin\theta}{\cos\theta} + \mu\left(mg\frac{\cos\theta}{\cos\theta} + \frac{mv^2}{R}\frac{\sin\theta}{\cos\theta}\right) = \frac{mv^2}{R}$$

$$mg\tan\theta + \mu\left(mg + \frac{mv^2}{R}\tan\theta\right) = \frac{mv^2}{R} \rightarrow \mu = \frac{\frac{v^2}{Rg} - \tan\theta}{1 + \frac{mv^2}{R}\tan\theta}$$

Which is the exact same expression as we got in part c. Thus, the choice of coordinate system does not matter.

d. A heavy semi-truck and a lightweight car approach the same banked curve with identical tire conditions, that is μ is the same for both the semi-truck and the car. Does one vehicle have a higher maximum safe speed before skidding, or can they both navigate the curve at the same maximum speed? Explain your answer in complete sentences using as much physics as possible.

In either case, whether we use friction or not to negotiate the curve, the formula for the speed of the vehicle is independent of the mass of the vehicle. Therefore, both can take the curve at the same speed.

e. Suppose the curve in the roadway approximated a semi-circle. How long would it take for a vehicle to negotiate the curve at the speed in part c?

$$v = \frac{\pi R}{T} \rightarrow T = \frac{\pi R}{v} = \frac{\pi \cdot 87m}{24\frac{m}{s}} = 11.4s$$

f. As the vehicle negotiates the semi-circle around the corner at the design speed, how much work is done on the vehicle by all of the forces that act? Does your answer make sense? Explain.

Let $\Delta S = \pi R$ be the distance traveled around the semi-circle as the vehicle corners.

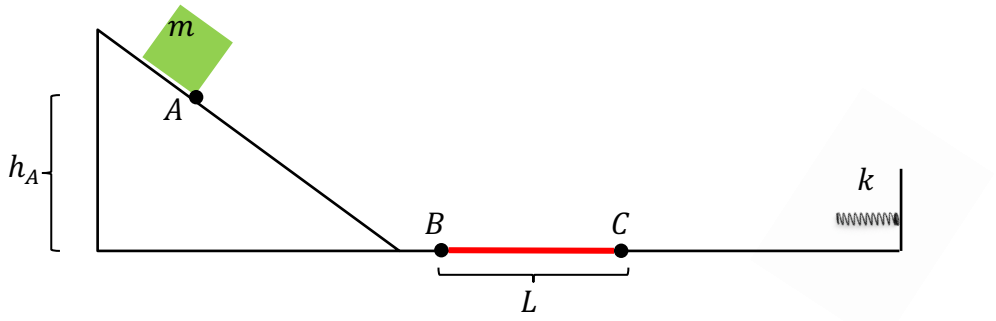
$$W_{F_N} = F_N \Delta S \cos\phi = F_N \cdot \pi R \cdot \cos 90 = 0$$

$$W_{F_W} = F_W \Delta S \cos\phi = mg \cdot \pi R \cdot \cos 90 = 0$$

$$W_{net} = W_{F_N} + W_{F_W} = \Delta K = 0$$

If the vehicle is taking the turn at a constant speed, then there is no change in the vehicle's kinetic energy and thus the work done on the vehicle must be zero. This is exactly what we get if we calculate the work done on the vehicle by the normal force and the weight as the car takes the corner.

3. A block of mass $m = 3kg$ is released from rest at point A which is a height $h_A = 4m$ above the ground. The block slides down the ramp, assumed to be frictionless, to point B on the horizontal surface. The distance between points B and C is $L = 50cm$ long and over this region there is a rough patch where friction exists with a coefficient of friction $\mu = 0.8$. When the block exits point C it slides across a frictionless surface until it encounters a horizontal spring with stiffness $k = 600\frac{N}{m}$ that is attached to an immovable wall.



a. Using energy ideas, what is the speed of the block at point B?

$$\Delta E = 0 = \Delta K + \Delta U_g + \Delta U_S = \left(\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2\right) + (mgy_B - mgy_A)$$

$$0 = \frac{1}{2}mv_B^2 - mgy_A \rightarrow v_B = \sqrt{2gy_A} = \sqrt{2 \cdot 9.8\frac{m}{s^2} \cdot 4m} = 8.85\frac{m}{s}$$

b. How far is the spring compressed bringing the bock momentarily to rest? Assume that the spring is initially at its equilibrium position when the block strikes it.

From point C to the spring by calculating the speed after the rough patch from point B

$$\Delta E = 0 = \Delta K + \Delta U_g + \Delta U_S = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_C^2\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right)$$

$$0 = -\tfrac{1}{2}mv_C^2 + \tfrac{1}{2}kx_f^2 \rightarrow x_f = \sqrt{\frac{m}{k}}v_c = \sqrt{\frac{3kg}{600\frac{N}{m}}} \cdot 8.4\frac{m}{s} = 0.594m = 59.4cm$$

$$W_{fr} = F_{fr}\Delta x \cos\phi = \mu mgL\cos 180 = -\mu mgL = \Delta K = \tfrac{1}{2}mv_C^2 - \tfrac{1}{2}mv_B^2$$

$$v_C = \sqrt{v_B^2 - 2\mu gL} = \sqrt{\left(8.85\frac{m}{s}\right)^2 - 2 \cdot 0.8 \cdot 9.8\frac{m}{s^2} \cdot 0.5m} = 8.4\frac{m}{s}$$

We could have done this from point A to the spring too.

$$\Delta E = W_{fr} = \Delta K + \Delta U_g + \Delta U_S \rightarrow F_{fr}\Delta x \cos\phi = (mgy_C - mgy_A) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right)$$

$$-\mu mgL = -mgy_A + \tfrac{1}{2}kx_f^2 \rightarrow x_f = \sqrt{\frac{2mg(y_A - \mu L)}{k}}$$

$$x_f = \sqrt{\frac{2 \cdot 3kg \cdot 9.8\frac{m}{s^2}(4m - 0.8 \cdot 0.5m)}{600\frac{N}{m}}} = 0.59m = 59cm$$

c. The spring will launch the block from rest back towards the ramp. On its first pass back across the rough patch what is the vertical displacement of the block back up the ramp? Use energy ideas.

From the spring to the patch:

$$\Delta E = 0 = \Delta K + \Delta U_g + \Delta U_S = \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right) + \left(\tfrac{1}{2}mv_C^2 - \tfrac{1}{2}mv_i^2\right)$$

$$0 = -\tfrac{1}{2}kx_i^2 + \tfrac{1}{2}mv_C^2 \rightarrow v_C = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{600\frac{N}{m}}{3kg}} \cdot 0.594m = 8.4\frac{m}{s}$$

Across the patch:

$$W_{fr} = F_{fr}\Delta x \cos\phi = \mu mgL\cos 180 = -\mu mgL = \Delta K = \tfrac{1}{2}mv_B^2 - \tfrac{1}{2}mv_C^2$$

$$v_B = \sqrt{v_C^2 - 2\mu gL} = \sqrt{\left(8.4\frac{m}{s}\right)^2 - 2 \cdot 0.8 \cdot 9.8\frac{m}{s^2} \cdot 0.5m} = 7.9\frac{m}{s}$$

Up the ramp to maximum height:

$$\Delta E = 0 = \Delta K + \Delta U_g + \Delta U_S = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_B^2\right) + \left(mgy_f - mgy_B\right)$$

$$0 = mgy_f - \tfrac{1}{2}mv_B^2 \rightarrow y_f = \frac{v_B^2}{2g} = \frac{\left(7.9\frac{m}{s}\right)^2}{2 \cdot 9.8\frac{m}{s^2}} = 3.2m$$

We could have also done this from the spring to the final height.

$$\Delta E = W_{fr} = \Delta K + \Delta U_g + \Delta U_S \rightarrow F_{fr}\Delta x \cos\phi = \left(mgy_f - mgy_C\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right)$$

$$y_f = \frac{+\frac{1}{2}kx_i^2 - \mu mgL}{mg} = \frac{kx_i^2}{2mg} - \mu L = \frac{600\frac{N}{m} \cdot (0.594m)^2}{2 \cdot 3kg \cdot 9.8\frac{m}{s^2}} - 0.8 \cdot 0.5m = 3.2m$$

d. How many times does the block cross the rough patch of ground between points B and C?

Every time the block crosses the patch it loses energy due to friction. The energy lost:

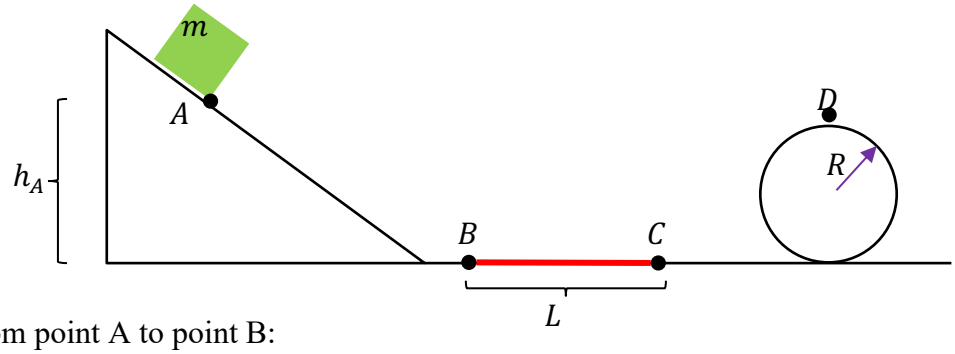
$$W_{fr} = F_{fr}\Delta x \cos\phi = \mu mgL \cos 180 = -\mu mgL = -0.8 \cdot 3kg \cdot 9.8\frac{m}{s^2} \cdot 0.5m$$

$$W_{fr} = -11.76J$$

The number of passes is the total initial energy divided by the energy loss per pass.

$$\# = \frac{E_{total}}{|W_{fr}|} = \frac{mgy_A}{11.76J} = \frac{3kg \cdot 9.8\frac{m}{s^2} \cdot 4}{11.76J} = \frac{117.6J}{11.76J} = 10$$

e. Suppose we remove the spring and replace it with a loop-the-loop track with radius $R = 1.5m$. The block is again released from rest at point A. Using energy ideas what will be the speed of the block at point D located at the top of the loop?



From point A to point B:

$$\Delta E = 0 = \Delta K + \Delta U_g + \Delta U_S = \left(\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2\right) + (mgy_B - mgy_A)$$

$$0 = \frac{1}{2}mv_B^2 - mgy_A \rightarrow v_B = \sqrt{2gy_A} = \sqrt{2 \cdot 9.8\frac{m}{s^2} \cdot 4m} = 8.85\frac{m}{s}$$

From point B to point C:

$$W_{fr} = F_{fr}\Delta x \cos\phi = \mu mgL \cos 180 = -\mu mgL = \Delta K = \frac{1}{2}mv_C^2 - \frac{1}{2}mv_B^2$$

$$v_C = \sqrt{v_B^2 - 2\mu gL} = \sqrt{\left(8.85\frac{m}{s}\right)^2 - 2 \cdot 0.8 \cdot 9.8\frac{m}{s^2} \cdot 0.5m} = 8.4\frac{m}{s}$$

From point C to point D:

$$\Delta E = 0 = \Delta K + \Delta U_g + \Delta U_S = \left(\frac{1}{2}mv_D^2 - \frac{1}{2}mv_C^2\right) + (mgy_D - mgy_C)$$

$$0 = \frac{1}{2}mv_D^2 - \frac{1}{2}mv_C^2 + mgy_D$$

$$\rightarrow v_D = \sqrt{v_C^2 + 2gy_D} = \sqrt{\left(8.4\frac{m}{s}\right)^2 - 2 \cdot 9.8\frac{m}{s^2} \cdot (2 \cdot 1.5m)} = 3.4\frac{m}{s}$$

We could have also done this from point A to point D.

$$\Delta E = W_{fr} = \Delta K + \Delta U_g + \Delta U_S \rightarrow -\mu mgL = \left(\frac{1}{2}mv_D^2 - \frac{1}{2}mv_A^2\right) + (mgy_D - mgy_A)$$

$$v_D = \sqrt{2g(y_A - y_D - \mu L)} = \sqrt{2 \cdot 9.8\frac{m}{s^2} \cdot (4m - (2 \cdot 1.5m) - 0.8 \cdot 0.5m)} = 3.4\frac{m}{s}$$

f. If the block slides around the inside of the loop, what is the magnitude of the reaction force of the track on the block when the block is at the top of the loop?

Taking up as the positive y-direction we have:

$$-F_N - F_W = -ma_c = -m\frac{v_D^2}{R} \rightarrow F_N = m\frac{v_D^2}{R} - mg$$

$$F_N = \frac{3kg \cdot \left(3.4\frac{m}{s}\right)^2}{1.5m} - 3kg \cdot 9.8\frac{m}{s^2} = -5.9N$$

Since this is negative and magnitudes cannot be negative, this means that the block actually never makes it around the loop. The minimum speed needed is $v = \sqrt{Rg} = \sqrt{1.5 \cdot 9.8\frac{m}{s^2}} = 3.8\frac{m}{s}$. Since the speed needed to just make it around is $3.8\frac{m}{s}$ hand the speed the block has if it could make it to point D is $3.8\frac{m}{s}$, the block doesn't ever get to point D by going around the circle. This is an artifact of the numbers used.