# Physics 110

# Exam #2

# May 22, 2020

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\frac{m}{s}) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 7 points

| | |
|---|---|
| Problem #1 | /24 |
| Problem #2 | /24 |
| Problem #3 | /24 |
| Total | /72 |

*I affirm that I have carried out my academic endeavors with full academic honesty.*

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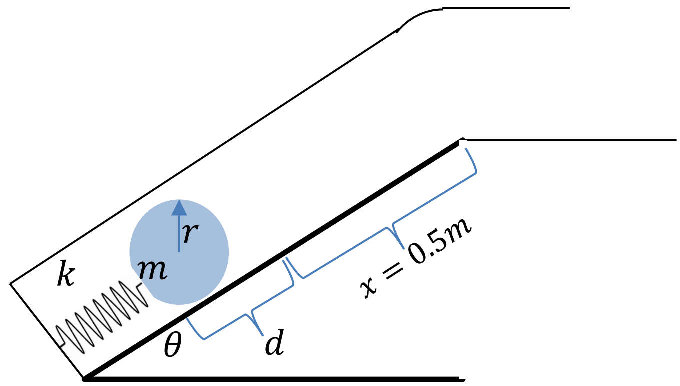
1. A marble of mass $m = 200g$ and radius $r = 1.3cm$ is pressed against a spring of stiffness $k = 100\frac{N}{m}$ by an amount $d = 20cm$ measured from the equilibrium position of the spring.

   a. If the marble is released from rest when the spring returns to its equilibrium position, what is the translational speed of the marble at the top of the ramp? Let the marble travels a distance $x = 0.5m$ from the point of losing contact with the spring to the top of the ramp? The marble is constrained to stay in the tube until the marble is on the horizontal surface. Assume the ramp/tube are frictionless and the marble rolls without slipping along the $\theta = 30^0$ incline. You may need $I = \frac{2}{5}mr^2$.




   Conservation of energy gives:

   $\Delta E = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = 0$

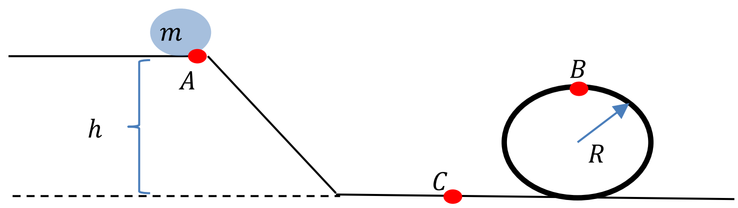
   $\Delta E = \left(\frac{1}{2}mv_f^2 - 0\right) + \left(\frac{1}{2}I\omega_f^2 - 0\right) + \left(mgy_f - mgy_i\right) + \left(0 - \frac{1}{2}kd^2\right) = 0$

   $\Delta E = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_f}{r}\right)^2 + mg\left((d + x)\sin\theta\right) - \frac{1}{2}kd^2 = 0$

   $v = \sqrt{\frac{10}{14}\left(\frac{k}{m}d^2 - 2g(d + x)\sin\theta\right)}$

   $v = \sqrt{\frac{10}{14}\left(\frac{100\frac{N}{m}}{0.2kg}(0.2m)^2 - 2 \times 9.8\frac{m}{s^2}(0.2m + 0.5m)\sin 30\right)} = 3.1\frac{m}{s}$

b. When the marble exits the tube at the top of the ramp, it rolls along a horizontal portion of the track until it reaches point A. At point A the marble rolls down the track and around the loop-the-loop. What is the radius of the loop-the-loop if the marble just barely makes it around the loop at point B. Assume that the vertical distance from point A to point C is $h = 2.5m$?




Conservation of energy gives:

$$\Delta E = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = 0$$

$$\Delta E = \left(\tfrac{1}{2}mv_B^2 - \tfrac{1}{2}mv_A^2\right) + \left(\tfrac{1}{2}I\omega_B^2 - \tfrac{1}{2}I\omega_A^2\right) + (mgy_B - mgy_A) = 0$$

$$\Delta E = \tfrac{1}{2}m(v_B^2 - v_A^2) + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\left(\frac{v_B^2}{r^2} - \frac{v_A^2}{r^2}\right) + mg(2R - h) = 0$$

The marble barely makes it around the loop at point B:

$$-F_N - F_W = -F_N - mg = ma_y = -m\frac{v_B^2}{r} \rightarrow v_B^2 = Rg$$

Returning to conservation of energy:

$$\Delta E = \tfrac{1}{2}m(Rg - v_A^2) + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\left(\frac{Rg}{r^2} - \frac{v_A^2}{r^2}\right) + mg(2R - h) = 0$$

$$\Delta E = \tfrac{1}{2}(Rg - v_A^2) + \tfrac{1}{5}(Rg - v_A^2) + g(2R - h) = 0$$

$$\Delta E = \tfrac{7}{10}(Rg - v_A^2) + g(2R - h) = 0$$

$$\rightarrow \tfrac{27}{10}Rg = \tfrac{7}{10}v_A^2 + gh$$

$$R = \frac{7}{27}\frac{v_A^2}{g} + \frac{10}{27}h = \frac{7}{27}\left(\frac{\left(3.1\frac{m}{s}\right)^2}{9.8\frac{m}{s^2}}\right) + \frac{10}{27}(2.5m) = 1.17m$$

c. What is the magnitude of the angular momentum of the marble at point C?

Conservation of energy between points A and C give the rotational speed of the ball.

$$\Delta E = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = 0$$

$$\Delta E = \left(\tfrac{1}{2}mv_C^2 - \tfrac{1}{2}mv_A^2\right) + \left(\tfrac{1}{2}I\omega_C^2 - \tfrac{1}{2}I\omega_A^2\right) + (mgy_C - mgy_A) = 0$$

$$\Delta E = \tfrac{1}{2}m(v_C^2 - v_A^2) + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)(\omega_C^2 - \omega_{BA}^2) - mgh = 0$$

$$\Delta E = \tfrac{1}{2}mr^2(\omega_C^2 - \omega_A^2) + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)(\omega_C^2 - \omega_A^2) - mgh = 0$$

$$\Delta E = \tfrac{1}{2}mr^2(\omega_C^2 - \omega_A^2) + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)(\omega_C^2 - \omega_A^2) - mgh = 0$$

$$\Delta E = \tfrac{7}{10}mr^2(\omega_C^2 - \omega_A^2) - mgh = 0$$

$$w_C = \sqrt{\omega_A^2 + \frac{10}{7}\frac{gh}{r^2}} = \sqrt{\frac{v_A^2 + \frac{10}{7}gh}{r^2}} = \sqrt{\frac{\left(3.1\frac{m}{2}\right)^2 + \frac{10}{7}\times 9.8\frac{m}{s^2}\times 2.5m}{(0.013)^2}} = 514\frac{rad}{s}$$

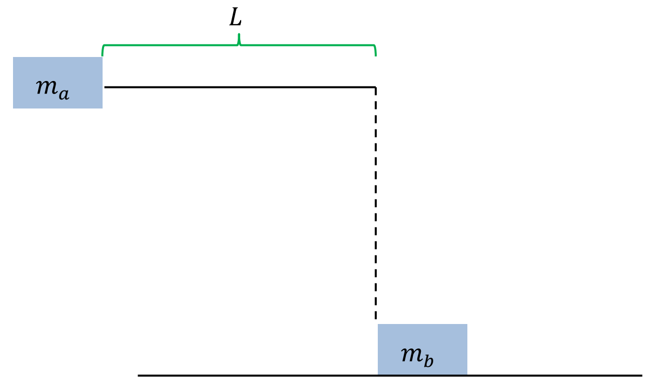
The angular momentum:

$$L = I\omega = \tfrac{2}{5}mr^2\omega_C = \frac{2}{5}\times 0.2kg \times (0.013)^2 \times 514\frac{rad}{s} = 0.007\frac{kgm^2}{s}$$

d. Suppose instead that you have the following setup. Two objects (a ball and a block) are released from rest at the top of identical ramps. The ball rolls down the ramp without slipping and the block slides down the ramp without friction. Which object reaches the bottom first?

1. The ball because it gains rotational kinetic energy but the block does not.
2. The ball because it gains mechanical energy due to a torque exerted on it, but the block does not.
3. The block because it does not lose mechanical energy due to friction but the ball does.
4. The block because it does not gain rotational kinetic energy but the ball does.
5. None of the above answers are correct.

2. A block of mass $m_a = m$ is attached to the end of a very light rod of length $L = 2.0m$ and held horizontally at rest. The block of mass $m_a$ is then released from rest and allowed to fall. When the rod is vertical the mass $m_a$ collides with a second block of identical mass $m_b = m_a = m$ initially at rest on a horizontal frictionless surface.




a. What is the translational speed of the block of mass $m_a$ just before the collision? You may need that the moment of inertia of a point mass is $I = mr^2$.

Conservation of energy gives:

$$\Delta E = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = 0$$

$$\Delta E = \left(\frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2\right) + \left(m_a g y_f - m_a g y_i\right) = 0$$

$$\Delta E = \left(\frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2\right) + \left(m_a g y_f - m_a g y_i\right) = 0$$

$$\omega_f = \sqrt{\frac{2m_a gL}{I}} = \sqrt{\frac{2m_a gL}{m_a L^2}} = \sqrt{\frac{2g}{L}} = \sqrt{\frac{2 \times 9.8\frac{m}{s^2}}{2.0m}} = 3.1\frac{rad}{s}$$

The translational speed:

$$v = r\omega = 2.0m \times 3.1\frac{rad}{s} = 6.26\frac{m}{s}$$

b. Assuming the collision is inelastic and that of the 50% initial kinetic energy is lost in the collision, what is the speed of each block after the collision? Assume that the block of mass $m_a$ moves to the right after the collision.

Conservation of momentum for the collision gives:

$$\Delta p_x = 0 \rightarrow p_{ix} = p_{fx} \rightarrow m_a v = m_a v_{fa} + m_b v_{fb} \rightarrow v = v_{fa} + v_{fb}$$

The collision is inelastic and energy is lost during the collision. The fraction of the energy lost is:

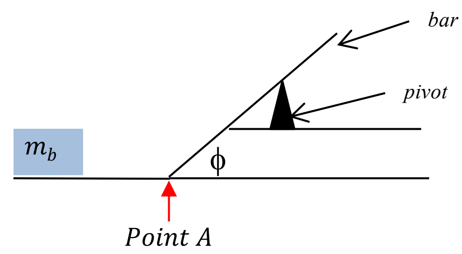
$$f = \frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = \frac{\frac{1}{2}m_a v_{fa}^2 + \frac{1}{2}m_b v_{fb}^2}{\frac{1}{2}m_a v^2} = \frac{v_{fa}^2 + v_{fb}^2}{v^2} = \frac{v_{fa}^2 + (v - v_a)^2}{v^2} = \frac{v_{fa}^2 + v^2 + v_{fa}^2 - 2vv_a}{v^2}$$

$$fv^2 = 2v_f^2 - 2vv_{fa} + v^2 \rightarrow 2v_f^2 - 2vv_{fa} + (1-f)v^2 = 0$$

$$v_{fa} = \frac{2\mathrm{v} \pm \sqrt{(-2v)^2 - 4 \times 2 \times (1-f)v^2}}{2 \times 2} = \frac{2\mathrm{v} \pm \sqrt{4v^2 - 8 \times (1-0.5)v^2}}{4} = \frac{2\mathrm{v} \pm \sqrt{4v^2 - 4v^2}}{4} = \frac{v}{2} = 3.13\frac{m}{s}$$

$$v_{fb} = v - v_a = v - \frac{v}{2} = \frac{v}{2} = 3.13\frac{m}{s}$$

c. After the collision, the block of mass $m_b = 3kg$ slides across the frictionless horizontal surface until it reaches point A. To the right of point A, there is a bar of mass $m_{bar} = 1kg$, length $L = 1m$ and between the bar and the block there is friction with coefficient of friction $\mu = 0.1$. When the block of mass $m_b$ slides onto the bar, does the bar rotate about the pivot? If the bar rotates about the pivot, what is the initial torque about the pivot? If the bar does not rotate about the pivot, what is the reaction force on the bar at point A? Assume that $\phi = 20^0$ and that the pivot is located at a distance of $70\%$ of the way from point A to the end of the bar.




To determine whether the bar will rotate or not, we need to know where the mass comes to rest. Conservation of energy gives:

$\Delta E = W_{fr} = \Delta K_R + \Delta K_T + \Delta U_g + \Delta U_s$

$\Delta E = W_{fr} = -\mu F_N \Delta x = -\mu m_b g \cos\phi\, x$

$= \left(\frac{1}{2} m_b v_{bf}^2 - \frac{1}{2} m_b v_{bi}^2\right) + \left(m_b g y_{bf} - m_b g y_{bi}\right)$

$-\mu m_b g \cos\phi\, x = -\frac{1}{2} m_b v_{bi}^2 + m_b g x \sin\phi$

$$x = \frac{v_{bi}^2}{2g(\sin\phi + \mu\cos\phi)} = \frac{\left(3.13\frac{m}{s}\right)^2}{2\times 9.8\frac{m}{s^2}(\sin 20 + 0.1\cos 20)} = 1.12m$$

Since this is greater than $0.7m$ and the end of the bar, the bar does not rotate.

The reaction force at point A taking the torques about the pivot:

$\tau = 0 = \tau_{bar} + \tau_{F_N} = +r_{bar} F_{w,bar} \sin(90-\phi) - r_{F_N} F_N \sin(90-\phi)$

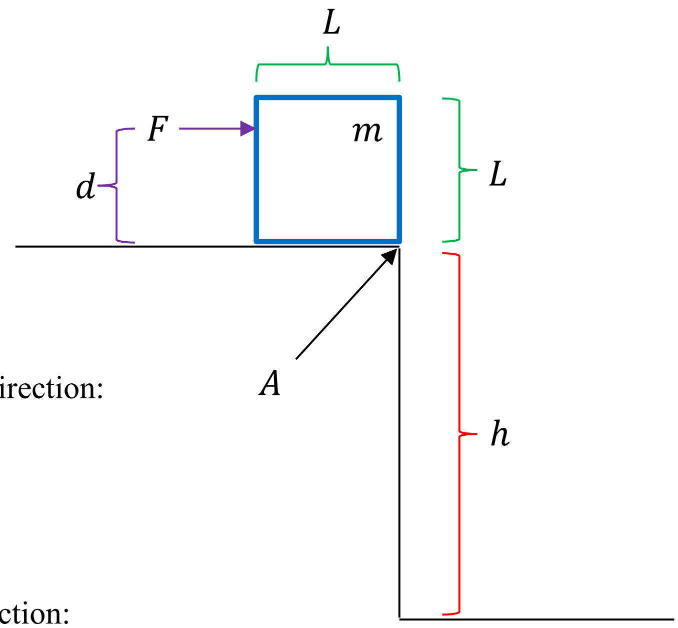
$$F_N = \frac{r_{bar} F_{w,bar}}{r_{F_N}} = \frac{(0.7m - 0.5m)\times 1kg \times 9.8\frac{m}{s^2}}{0.7m} = 2.8N \text{ vertically upwards.}$$

d. A fielder in baseball or softball may leap into the air to catch a ball and throw it quickly. As they throw the ball, the upper part of their body rotates. If you look quickly you will notice that their hips and legs also rotate. What is the direction of rotation of their hips and legs compared to their upper body and what is the physics being displayed?

1. Their hips and leges rotate in the same direction as their upper body and this is due to conservation of energy.
2. Their hips and leges rotate in the opposite direction as their upper body and this is due to conservation of energy.
3. Their hips and leges rotate in the same direction as their upper body and this is due to conservation of angular momentum.
4. Their hips and leges rotate in the opposite direction as their upper body and this is due to conservation of angular momentum.
5. None of the above are correct answers.

3. A cubic box with sides of length $L$ and mass $m$, sits near the edge of a cliff at a height $h$ above the ground below. Static friction exists between the box and the ground with coefficient of friction $\mu$. A constant force $F$ is applied to the right side of the cube as shown below at a height $d$ above the bottom of the cube.

a. Just as the box is about to tip about point A, what are the expressions for the sum of the forces in the horizontal and vertical directions (in terms of the quantities given in the problem) and starting from the general definition of torque, what is the expression for the sum of the torques about point A (in terms of the quantities given in the problem)? Take counterclockwise rotations to be positive.



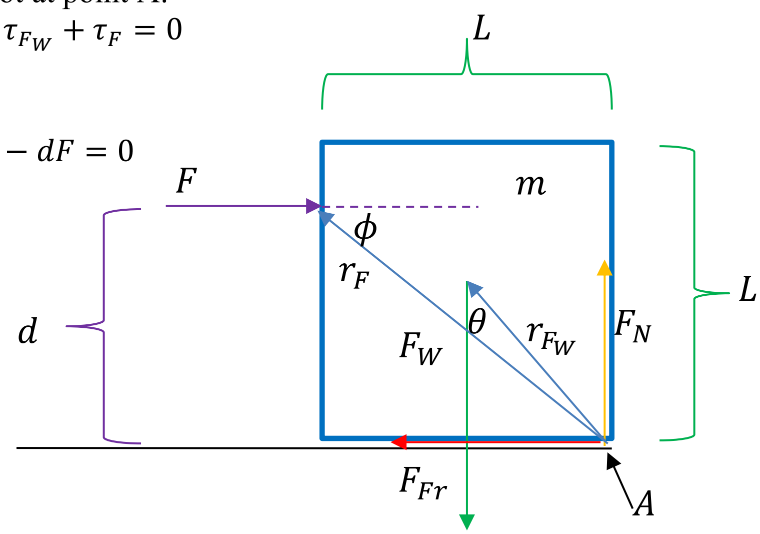

The sum of the forces in the horizontal direction:
$F - F_{fr} = ma_x = 0 \rightarrow F = F_{fr} = \mu F_N$

The sum of the forces in the vertical direction:
$F_N - F_W = ma_y \rightarrow F_N = F_w = mg$

The sum of the torques about the pivot at point A:
$\tau_{F_N} + \tau_{F_W} + \tau_{F_{fr}} + \tau_F = I\alpha = 0 \rightarrow \tau_{F_W} + \tau_F = 0$

$+r_{F_W} F_w \sin\theta - r_F F \sin\phi = 0$

$+r_{F_W} mg \left(\frac{L/2}{r_{F_W}}\right) - r_F F \left(\frac{d}{r_F}\right) = \frac{mgL}{2} - dF = 0$



b. In terms of the quantities given in the problem, with what minimum force $F$ would you need to apply to get the box to just start to tip about point A?

From the forces in vertical direction:
$F_N = F_W = mg$

From the forces in the horizontal direction:
$F = F_{fr} = \mu F_W = \mu mg$

c. In terms of the quantities given in the problem, what is the maximum height $d$ that you can apply this minimum force $F$ so that he box will just start to tip?

From the sum of the torques about the pivot:
$$\tau_F = \tau_w \rightarrow \frac{L}{2}mg = dF = d\mu mg \rightarrow d = \frac{L}{2\mu}$$

d. If the cube tips and falls off of the edge of the cliff, which of the following would give the correct statement of conservation of energy for the cube just before it strikes the ground? Assume the cube falls essentially from rest translationally and rotationally and that air resistance is negligible.

1. $\Delta E = \frac{1}{2}I\omega^2 - \frac{1}{2}mv^2 + mgh = 0.$
2. $\Delta E = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 + mgh = 0.$
3. $\Delta E = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 - mgh = 0.$
4. $\Delta E = -\frac{1}{2}I\omega^2 - \frac{1}{2}mv^2 + mgh = 0.$
5. None of the above statements are correct.

# Physics 110 Formulas

## Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

## Uniform Circular Motion

$$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

## Geometry /Algebra

*Circles* | *Triangles* | *Spheres*

$C = 2\pi r$ | $A = \frac{1}{2}bh$ | $A = 4\pi r^2$

$A = \pi r^2$ | | $V = \frac{4}{3}\pi r^3$

*Quadratic equation* : $ax^2 + bx + c = 0$,

*whose solutions are given by* : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

## Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x{}^2 + v^2{}_y}$

direction of a vector: $\phi = \tan^{-1}\left(\dfrac{v_y}{v_x}\right)$

## Useful Constants

$g = 9.8\, m/s^2 \qquad G = 6.67 \times 10^{-11}\, Nm^2/kg^2$

$N_A = 6.02 \times 10^{23}\, atoms/mole \qquad k_B = 1.38 \times 10^{-23}\, J/K$

$\sigma = 5.67 \times 10^{-8}\, W/m^2K^4 \qquad v_{sound} = 343\, m/s$

## Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F} \cdot dt$$

$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

$$\vec{F}_s = -k\vec{x}$$

$$\left|\vec{F}_{fr}\right| = \mu\left|\vec{F}_N\right|$$

## Work/Energy

$$K_T = \tfrac{1}{2}mv^2$$

$$K_R = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = Fd\,\mathrm{Cos}\,\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$$

## Heat

$$T_C = \tfrac{5}{9}[T_F - 32]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}(1 + \alpha\Delta T)$$

$$A_{new} = A_{old}(1 + 2\alpha\Delta T)$$

$$V_{new} = V_{old}(1 + \beta\Delta T) : \beta = 3\alpha$$

$$PV = Nk_BT$$

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

## Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$$

$$a_r = r\omega^2$$

## Sound

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1 \times 10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$$

## Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho gd$$

$$F_B = \rho gV$$

$$A_1v_1 = A_2v_2$$

$$\rho_1A_1v_1 = \rho_2A_2v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho gh_2$$

## Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2 \rho v A^2$$