Physics 110

Exam #2

May 21, 2021

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg) \times (2\frac{m}{s}) = 10\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 7 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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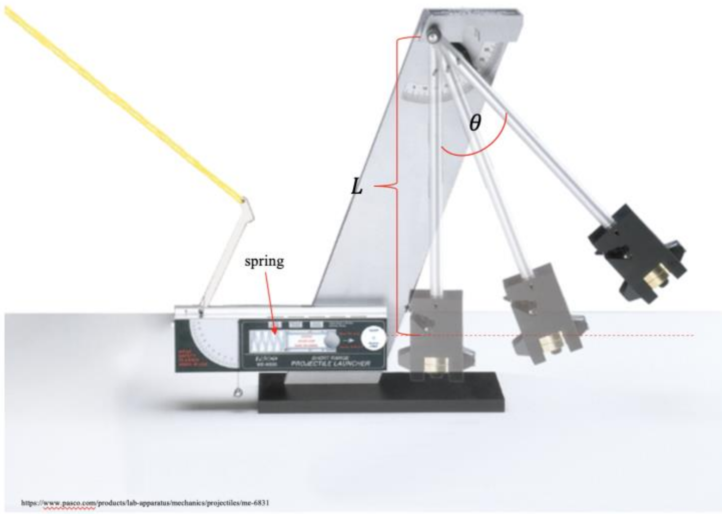
1. A ball of mass $m_b = 150g$ is compressed against a horizontal spring of stiffness k. The spring is compressed by an amount $x = 5cm$ from equilibrium and the ball is held at rest against the spring. The ball is then launched horizontally and collides with a pendulum of mass $m_p = 250g$ and length $L = 0.31m$ initially at rest.



a. If the ball and pendulum rise through an angle of $\theta = 58^0$ after the collision, what was the speed of the ball just before it collided with the pendulum?

$$\Delta E = 0 = \Delta K + \Delta U_g + \Delta U_s = \left(0 - \tfrac{1}{2}(m_b + m_p)V^2\right) + \left((m_b + m_p)gy_f - 0\right)$$
$$0 = -\tfrac{1}{2}(m_b + m_p)V^2 + (m_b + m_p)gL(1 - \cos\theta)$$
$$V = \sqrt{2gL(1 - \cos\theta)} = \sqrt{2 \times 9.8\tfrac{m}{s^2} \times 0.31m(1 - \cos 58)} = 1.69\tfrac{m}{s}$$

where, $L = y_f + L\cos\theta$

$$p_{ix} = p_{fx} \rightarrow m_b v_i = (m_b + m_p)V$$
$$\rightarrow v_i = \left(\frac{m_b + m_p}{m_b}\right)V = \left(\frac{0.15kg + 0.25kg}{0.15kg}\right) \times 1.69\tfrac{m}{s} = 4.5\tfrac{m}{s}$$

b. What is the stiffness constant of the spring?

$$\Delta E = 0 = \Delta K + \Delta U_g + \Delta U_s \rightarrow \left(\tfrac{1}{2}mv_i^2 - 0\right) + \left(0 - \tfrac{1}{2}kx^2\right) \rightarrow k = \frac{mv_i^2}{x^2}$$

$$k = \frac{0.15kg \times \left(4.5\frac{m}{s}\right)^2}{(0.05m)^2} = 1200\tfrac{N}{m}$$

c. Derive an expression for the fraction of the initial kinetic energy lost to the collision and show that the fraction depends on only the masses involved in the collision.

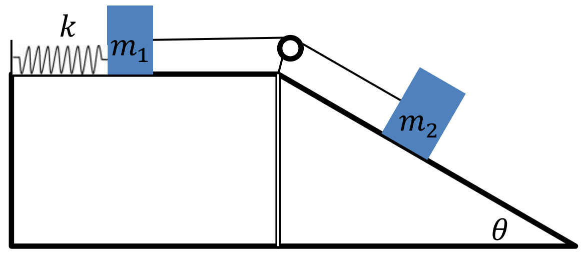
$$f = \frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = \frac{\frac{1}{2}(m_b+m_p)V^2 - \frac{1}{2}m_b v_i^2}{\frac{1}{2}m_b v_i^2} = \frac{(m_b+m_p)\left[\left(\frac{m_b}{m_b+m_p}\right)v_i\right]^2 - m_b v_i^2}{m_b v_i^2}$$

$$f = \frac{m_b}{m_b+m_p} - 1 = -\frac{m_p}{m_b+m_p}$$

d. Suppose that the pendulum arm is turned around and we again launch the ball from the spring. The ball again makes a collision with the pendulum at rest. This time however, the ball does not get caught by the pendulum. The ball bounces off the pendulum in a direction opposite to its original direction of motion and the pendulum arm swings through an angle ϕ. In this case, which of the following statements is true?

1. The collision is completely elastic and $\phi > \theta$.
2. The collision is completely elastic and $\phi < \theta$.
3. The collision is still inelastic and $\phi > \theta$.
4. The collision is still inelastic and $\phi < \theta$.
4. None of the above answers give the correct collision type or the relationship between ϕ and θ.

2. A block of mass $m_2 = 2kg$ is connected by a light rope to a block of mass $m_1 = 0.5kg$. The block of mass m_1 is connected to a horizontal, initially unstretched spring of stiffness $k = 40\frac{N}{m}$. The rope passes over a pulley with mass $m_p = 0.25kg$ and radius $r_p = 2.5cm$.



a. If the system is released from rest, what is the maximum extension (d_{max}) of the spring? Assume that all surfaces are frictionless, that there is no friction in the axle of the pulley, and $\theta = 52^0$.

$$\Delta E = 0 = \Delta K_1 + \Delta U_{g1} + \Delta K_2 + \Delta U_{g2} + \Delta U_s + \Delta K_R$$
$$\rightarrow 0 = \left(0 - m_2 g y_{2f}\right) + \left(\tfrac{1}{2}kd^2 - 0\right) \rightarrow -m_2 g d \sin\theta + \tfrac{1}{2}kd^2 = 0$$
$$d_{max} = \frac{2m_2 g \sin\theta}{k} = \frac{2\times 2kg\times 9.8\frac{m}{s^2}\times \sin 52}{40\frac{N}{m}} = 0.77m = 77cm$$

b. Suppose the system is reset to the initial condition in part a. The system is again released from rest and when the spring has been stretched by an amount $\frac{d_{max}}{2}$, where $d_{max}$is the maximum extension you found in part a, what is the rotational speed of the pulley? Assume that all surfaces are frictionless, there is no friction in the axle of the pulley, and that the moment of inertial of the pulley is given by $I_p = \frac{1}{2}m_p r_p^2$.

$$\Delta E = 0 = \Delta K_1 + \Delta U_{g1} + \Delta K_2 + \Delta U_{g2} + \Delta U_s + \Delta K_R$$
$$0 = \left(\tfrac{1}{2}m_1 v_{1f}^2 - 0\right) + \left(\tfrac{1}{2}m_2 v_{2f}^2 - 0\right) + \left(0 - m_2 g y_{2f}\right) + \left(\tfrac{1}{2}kd^2 - 0\right) + \left(\tfrac{1}{2}I_p \omega_{pf}^2 - 0\right)$$

$$0 = \tfrac{1}{2}m_1 v_{1f}^2 + \tfrac{1}{2}m_2 v_{2f}^2 - m_2 g y_{2f} + \tfrac{1}{2}kd^2 \tfrac{1}{2}I_p \omega_{pf}^2$$
$$0 = \tfrac{1}{2}\left[m_1 + m_2 + \tfrac{1}{2}m_p\right] r_p^2 \omega_f^2 - m_2 g \left(\frac{d_{max}}{2}\right)\sin\theta + \tfrac{1}{8}kd_{max}^2$$
$$\omega_f = \sqrt{\frac{m_2 g d_{max}\sin\theta - \frac{1}{4}kd_{max}^2}{\left(m_1 + m_2 + \frac{1}{2}m_p\right)r_p^2}} = \sqrt{\frac{2kg\times 9.8\frac{m}{s^2}\times 0.77m\times \sin 52 - \frac{1}{4}\left(40\frac{N}{m}\right)(0.77m)^2}{(0.5kg + 2kg + 0.125kg)(0.025m)^2}} = 60.3\frac{rad}{s}$$

where we have used the fact that $v = r\omega$.

c. When the spring has been stretched by an amount $\frac{d_{max}}{2}$ from equilibrium, what is the magnitude and direction of the net tension force in the rope about the pulley? Take counterclockwise as the positive direction for angular quantities. Hint: the moment of inertia of the pulley is given by: $I_p = \frac{1}{2}m_p r_p^2$

$$\tau_{net}: r_p F_{TL} - r_p F_{TR} = -I_p \alpha$$

$$\Delta x = r_p \Delta\theta \rightarrow \Delta\theta = \frac{\Delta x}{r_p} = \frac{d_{max}}{2r_p} = \frac{0.77m}{2\times 0.025m} = 15.4rad$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \rightarrow \alpha = \frac{\omega_f^2}{2\Delta\theta} = \frac{\left(60.3\frac{rad}{s}\right)^2}{2\times 15.4rad} = 118\frac{rad}{s^2}$$

$$F_{T,net} = F_{TL} - F_{TR} = -\frac{\frac{1}{2}m_p r_p^2 \alpha}{r_p} = -\frac{1}{2}m_p r_p \alpha$$

$$F_{T,net} = F_{TL} - F_{TR} = -\frac{1}{2}\times 0.25kg \times 0.025m \times 118\frac{rad}{s^2}$$

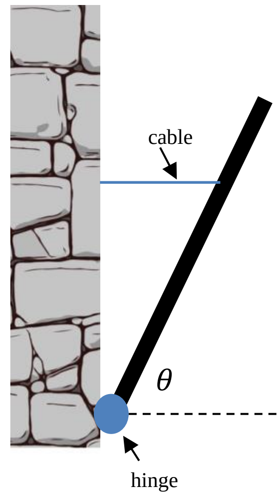
$F_{T,net} = F_{TL} - F_{TR} = 0.37N$ clockwise.

d. Suppose that you threw a ball of mass m vertically up into the air. Ignoring air resistance, at what point does the ball have the most energy?

1. At the highest point in its path.
2. When it was first thrown.
3. Just before it hits the ground.
4. Everywhere since the energy of the ball is the same everywhere.
5. None of the above answers are correct.

3. Drawbridges were used on medieval castles to keep invaders out and they could be raised or lowered at a moment's notice. Consider a square drawbridge with sides of length $L = 10m$ with a mass $M = 910kg$. The drawbridge is held at an angle of $\theta = 70^0$ measured with respect to the horizontal as shown below, by a single horizontal cable. The cable is attached at a distance of $\frac{2L}{3}$ of the way from the hinge to the edge of the drawbridge.

https://www.storyblocks.com/video/stock/castle-drawbridge-side-view-mediavel-structures-bridge-kzti7xh



a. What is the magnitude of the tension force in the horizontal cable?

$$\tau_{net}: -\frac{L}{2}F_W \sin(90-\theta) + \frac{2L}{3}F_T \sin\theta = I\alpha = 0$$

$$\frac{L}{2}F_W \sin(90-\theta) = \frac{L}{2}F_W \cos\theta = \frac{2L}{3}F_T \sin\theta$$

$$F_T = \frac{3}{4}F_W \cot\theta = \frac{3}{4} \times 910kg \times 9.8\frac{m}{s^2} \times \cot 70 = 2434N$$

b. What is the magnitude and direction of the reaction force on the hinge due to the drawbridge?

$$F_x:\ F_{Rx} - F_T = ma_x = 0 \rightarrow F_{Rx} = F_T = 2434N$$
$$F_y:\ F_{Ry} - F_w = ma_y = 0 \rightarrow F_{Ry} = F_W = 910kg \times 9.8\frac{m}{s^2} = 8918N$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(2434N)^2 + (8918N)^2} = 9244N$$

$$\tan\phi = \frac{F_{Ry}}{F_{Rx}} \rightarrow \phi = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{8918N}{2434N} = 74.7^0$$

c. Suppose that the cable holding the drawbridge breaks. What is the initial magnitude of the angular acceleration of the drawbridge about the hinge? Hint: The moment of inertia for a square slab of material spun about one end is $I = \frac{1}{3}ML^2$.

$$\tau_{net}: -\frac{L}{2}F_W \sin(90-\theta) = I\alpha \rightarrow \alpha = \left|\frac{L}{2I}F_W \cos\theta\right|$$

$$\alpha = \left|\frac{L}{2I}F_W \cos\theta\right| = \frac{Lmg\cos 70}{2\times\frac{1}{3}mL^2} = \frac{3g}{2L} = \frac{3\times 9.8\frac{m}{s^2}\cos 70}{2\times 10m} = 0.5\frac{rad}{s^2}$$

d. Suppose that you push a heavy crate down a ramp at a constant velocity with a force parallel to the ramp. There is friction between the crate and the ramp. Which of the following forces does the greatest magnitude of work on the crate?

1. The force of friction.
2. The force of gravity.
3. The normal force.
4. The force of you pushing.
5. The net force.

Physics 110 Formula sheet

Vectors

$v = \sqrt{v_x^2 + v_y^2}$

$\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Motion Definitions

Displacement: $\Delta x = x_f - x_i$

Average velocity: $v_{avg} = \frac{\Delta x}{\Delta t}$

Average acceleration: $a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Rotational Motion Definitions

Angular displacement: $\Delta s = R\Delta\theta$

Angular velocity: $\omega = \frac{\Delta\theta}{\Delta t} \rightarrow v = R\omega$

Angular acceleration: $\alpha = \frac{\Delta\omega}{\Delta t} \rightarrow \begin{cases} a_t = r\alpha \\ a_c = r\omega^2 \end{cases}$

Rotational Equations of Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$rF\sin\theta$

$\omega_f = \omega_i + \alpha t$

$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

Momentum & Force

$\vec{p} = m\vec{v} \rightarrow p_x = mv_x;\ p_y = mv_y$

$\Delta\vec{p} = \vec{F}\Delta t \rightarrow \vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a} \rightarrow F_x = ma_x; F_y = ma_y$

$F_{fr} = \mu F_N$

$F_w = mg$

$F_s = -kx$

$F_G = G\frac{M_1 M_2}{r^2}$

$F_c = ma_c = m\frac{v^2}{R}$

Work & Energy

$\begin{cases} W_T = \int \vec{F}\cdot d\vec{r} = Fdr\cos\theta = \Delta K_T \\ W_R = \int \vec{\tau}\cdot d\vec{\theta} = \tau d\theta = \Delta K_R \end{cases}$

$W_{net} = W_T + W_R = \Delta K_T + \Delta K_R = -\Delta U$

$K_T = \frac{1}{2}mv^2$

$K_R = \frac{1}{2}I\omega^2$

$U_g = mgy$

$U_s = \frac{1}{2}kx^2$

$\Delta E = \Delta E_R + \Delta E_T$

$\Delta E = \Delta K_R + \Delta K_T + \Delta U_g + \Delta U_s = \begin{cases} 0 \\ W_{fr} \end{cases}$

Rotational Momentum & Force

$\vec{\tau} = \vec{r}\times\vec{F};\ \tau = r_\perp F = rF_\perp =$

$\tau = \frac{\Delta L}{\Delta t} = I\alpha$

$L = I\omega$

$\Delta\vec{L} = \vec{\tau}\Delta t \rightarrow \vec{L}_f = \vec{L}_i + \vec{\tau}\Delta t$

Fluids

$\rho = \frac{m}{V}$

$P = \frac{F}{A}$

$P_y = P_{air} + \rho g y$

1,2,3, … open pipes

$F_B = \rho g V$

1,3,5, … closed pipes

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$; compressible

$A_1 v_1 = A_2 v_2$; incompressible

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

Simple Harmonic Motion

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_s = 2\pi\sqrt{\frac{m}{k}}; \quad \omega = \sqrt{\frac{k}{m}}$

$T_p = 2\pi\sqrt{\frac{l}{g}}; \quad \omega = \sqrt{\frac{g}{l}}$

Geometry/Algebra

Circles:	$A = \pi r^2$	$C = 2\pi r = \pi D$
Spheres:	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Triangles:	$A = \frac{1}{2}bh$	
Quadratics:	$ax^2 + bx + c = 0 \rightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$	

Sound

$v_s = f\lambda = (331 + 0.6T)\frac{m}{s}$

$\beta = 10\log\frac{I}{I_o}$

$f_n = nf_1 = n\frac{v}{2L}; n =$

$f_n = nf_1 = n\frac{v}{4L}; n =$

Waves

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L};\ n = 1,2,3, \ldots$

$I = 2\pi^2 f^2 \rho v A^2$

Equations of Motion for SHM

$$x(t) = \begin{cases} x_{max}\sin\left(\frac{2\pi}{T}t\right) \\ x_{max}\cos\left(\frac{2\pi}{T}t\right) \end{cases}$$

$$v(t) = \begin{cases} v_{max}\cos\left(\frac{2\pi}{T}t\right) \\ -v_{max}\sin\left(\frac{2\pi}{T}t\right) \end{cases}$$

$$a(t) = \begin{cases} -a_{max}\sin\left(\frac{2\pi}{T}t\right) \\ -a_{max}\cos\left(\frac{2\pi}{T}t\right) \end{cases}$$

$$v = \pm v_{max}\sqrt{1 - \left(\frac{x}{x_{max}}\right)^2}$$

$$v = \pm \omega x_{max}\sqrt{1 - \left(\frac{x}{x_{max}}\right)^2}$$

Periodic Table of the Elements