Physics 110

Exam #2

May 20, 2022

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times(2\,\frac{m}{s}) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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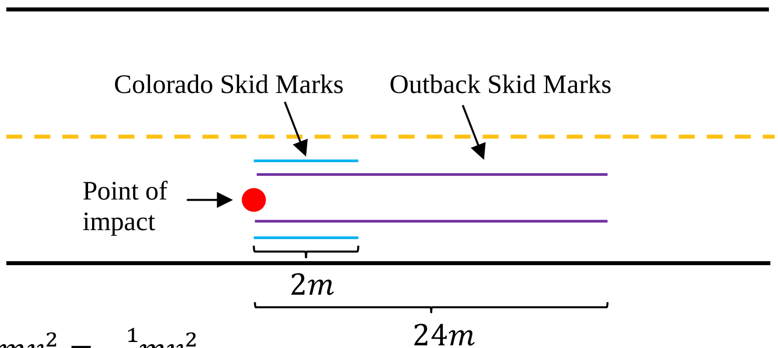
1. According to the National Highway Traffic Safety Association (https://www.nhtsa.gov/press-releases/2020-traffic-crash-data-fatalities) motor vehicle accidents accounted for about 40000 deaths in 2020 in the United. After an accident that involves severe injury or death, the accident will most likely be investigated and the accident reconstructed to determine what happened. As an accident investigator at the scene, you make the measurements shown below.

    a. The accident occurred in a $30mph$ $(13.5\frac{m}{s})$ zone on Union Street. A $4200lb$ $(1900kg)$ Chevy Colorado rear-ended a $3600lb$ $(1600kg)$ Subaru Outback at rest at a red light. From the measurements below, what were the speeds of the Colorado and Outback after the collision assuming that the accelerations of the Colorado and Outback were $-2\frac{m}{s^2}$ and $-3\frac{m}{s^2}$ respectively after the collision?



$$W_{fr} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -\frac{1}{2}mv_i^2$$

$$W_{fr} = F_{fr}d\cos\theta = -\mu mgd = -mad = -\frac{1}{2}mv_i^2$$

$$v_{i,outback} = \sqrt{2a_{outback}d_{outback}} = \sqrt{2 \times 3\frac{m}{s^2} \times 24m} = 12\frac{m}{s}$$

$$v_{i,colorado} = \sqrt{2a_{colorado}d_{colorado}} = \sqrt{2 \times 2\frac{m}{s^2} \times 2m} = 2.8\frac{m}{s}$$

    b. What was the speed of the Colorado before the collision?

$$p_{ix} = p_{fx} \rightarrow m_c v_{i,c} = m_c v_{f,c} + m_o v_{f,o}$$

$$v_{i,c} = \frac{m_c v_{f,c} + m_o v_{f,o}}{m_c} = \frac{\left(1900kg \times 2.8\frac{m}{s}\right) + \left(1600kg \times 12\frac{m}{s}\right)}{1900kg} = 12.9\frac{m}{s}$$

c. What type of collision was this? Justify your answer with a calculation.

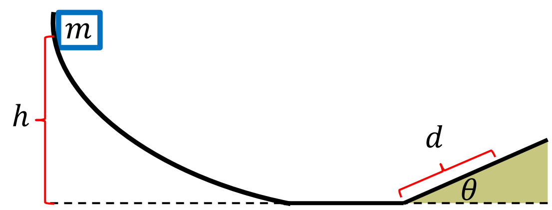
$$\Delta K = K_f - K_i = \left(\frac{1}{2}m_c v_{fc}^2 + \frac{1}{2}m_o v_{fo}^2\right) - \frac{1}{2}m_c v_{ic}^2$$
$$\Delta K = \left(\frac{1}{2} \times 1900kg\left(2.8\frac{m}{s}\right)^2 + \frac{1}{2} \times 1600kg\left(12\frac{m}{s}\right)^2\right) - \frac{1}{2} \times 1900kg\left(12.9\frac{m}{s}\right)^2$$
$\Delta K = -3.5 \times 10^4 J \neq 0$ therefore, the collision is inelastic

d. Suppose that the collision occurred over a time of $0.5s$. What is the magnitude and direction of the force exerted on the Outback from the Colorado?

$F_{o,c} = \frac{\Delta p_o}{\Delta t} = \frac{m_o v_{fo} - m_o v_{io}}{\Delta t} = \frac{1600kg\left(12\frac{m}{s}\right)}{0.5s} = 3.8 \times 10^4 N$ in the direction of the Outback.

2. A block of mass $m = 0.5kg$ is released from rest on the left ramp from a height $h = 5m$ above the ground. The block slides down the left ramp and makes a smooth transition to the horizontal surface. Both the left ramp and the horizontal surface are frictionless.

   a. The block then slides up the ramp on the right, where friction exists between the right ramp and the block. If the coefficient of friction between the right ramp and the block is $\mu = 0.2$ and if the right ramp is inclined at angle $\theta = 50^0$ with respect to the horizontal, how far along the ramp does the block slide, d? Use energy ideas.

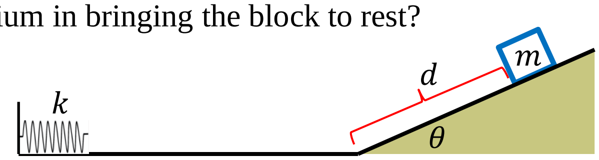


$$\Delta E = W_{fr} = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s$$
$$F_{fr} d \cos 180 = -\mu mgd \cos\theta = mgy_f - mgy_i = mgd \sin\theta - mgh$$
$$mgh = (mg \sin\theta + \mu mg \cos\theta)d \rightarrow d = \frac{h}{\sin\theta + \mu\cos\theta} = \frac{5m}{\sin 50 + 0.2\cos 50}$$
$$d = 5.59m$$

   b. The block momentarily comes to rest after sliding a distance d along the ramp. It then slides back down the ramp towards the horizontal surface. Assuming the system is the block of mass m, what is the net work done on the block by all external forces for the block sliding up and then back down the right ramp?

$$W_{net} = W_{g,up} + W_{g,down} + W_{fr,up} + W_{fr,down}$$
$$W_g = (mg\sin\theta)d\cos 180 + (mg\sin\theta)d\cos 0 = -mgd\sin\theta + mgd\sin\theta = 0$$
$$W_{fr} = (\mu mg\cos\theta)d\cos 180 + (\mu mg\cos\theta)d\cos 180 = -2\mu mgd\cos\theta$$
$$W_{fr} = -2 \times 0.2 \times 0.5m \times 9.8\frac{m}{s^2} \times 5.59m\cos 50 = -7.04J$$
$$W_{net} = -7.04J$$

c. When the block returns to the horizontal surface from the right ramp, it slides across the horizontal surface and at the end of the horizontal section there is a spring of stiffness $k = 250\frac{N}{m}$ as shown below. How far does the spring compress from equilibrium in bringing the block to rest?



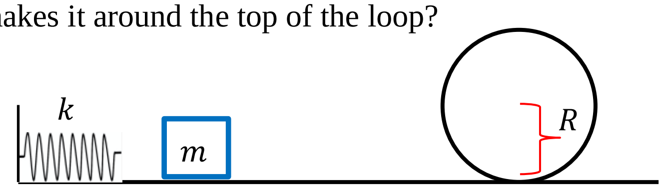
$$\Delta E = W_{fr} = \Delta K_R + \Delta K_T + \Delta U_g + \Delta U_s$$

$$(\mu mg\cos\theta)d\cos 180 = (0 - mgd\sin\theta) + \left(\tfrac{1}{2}kx_f^2 - 0\right)$$

$$x_f = \sqrt{\frac{2mgd\sin\theta - 2\mu mgd\cos\theta}{k}} = \sqrt{\frac{2mgd}{k}(\sin\theta - \mu\cos\theta)}$$

$$x_f = \sqrt{\frac{2 \times 0.2kg \times 9.8\frac{m}{s^2} \times 5.59m}{250\frac{N}{m}}(\sin 50 - 0.2\cos 50)} = 0.24m$$

d. The mass m momentarily comes to rest when the spring is fully compressed. The mass is then accelerated from rest and when the spring returns to equilibrium the m ass is released from the spring and slides to the right across the horizontal frictionless surface. At the end of the horizontal surface, the right ramp has been removed and a circular portion of track has been installed as shown below. What is the minimum radius R of the loop-the-loop portion of track such that the mass just barely makes it around the top of the loop?



$$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s$$

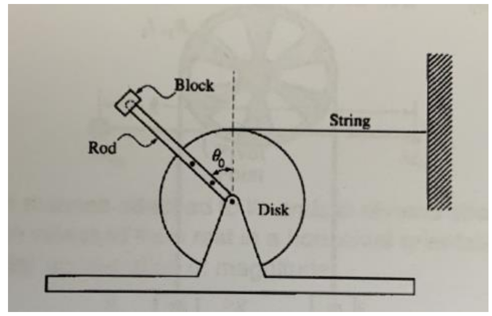
$$0 = \left(\tfrac{1}{2}mv_{top}^2 - 0\right) + (mg(2R) - 0) + \left(0 - \tfrac{1}{2}kx_i^2\right)$$

$$0 = \tfrac{1}{2}mv_{top}^2 + 2mgR - \tfrac{1}{2}kx_i^2$$

$$F_N + F_W = m\frac{v^2}{R} \rightarrow mg = m\frac{v^2}{R} \rightarrow v_{top}^2 = Rg$$

$$0 = \tfrac{1}{2}mRg + 2mgR - \tfrac{1}{2}kx_i^2 \rightarrow R = \frac{kx_i^2}{5mg} = \frac{250\frac{N}{m}(0.24m)^2}{5 \times 0.2kg \times 9.8\frac{m}{s^2}} = 1.4m$$

3. A uniform disk (of mass $3m$ and radius r) is mounted to an axle and is free to rotate without friction. Attached to the disk is a thin rod (of mass m and length $2r$) and attached to the end of the rod a block (of mass $2m$). The system is held at rest initially by a horizontal light rope with the rod making an angle of θ_0 measured with respect to the vertical as shown on the right.



a. In terms of m, g, r, and θ_0, what is the magnitude of the tension in the string? Hint: Consider the net torque in the system.

$$\tau_{net} = \tau_{F_T} + \tau_{rod} + \tau_{block} = I\alpha$$
$$\tau_{net} = -r_{disk}F_T \sin 90 + r_{rod}F_{w,rod} \sin\theta_0 + r_{block}F_{w,block} \sin\theta_0 = I\alpha$$
$$\tau_{net} = -rF_T + rmg\sin\theta_0 + 2r(2mg)\sin\theta_0 = I\alpha = 0$$
$$F_T = 5mg\sin\theta_0$$

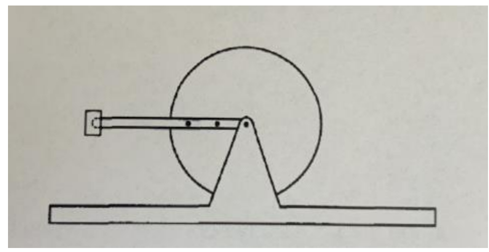
b. What is the total moment of inertia of the disk/rod/block system? Hints: The moment of inertia of a point mass is mR^2 and the moments of inertias of various shapes can be found in the table on the equation sheet.

$$I_{total} = I_{disk} + I_{rod} + I_{block} = \tfrac{1}{2}(3m)(r)^2 + \tfrac{1}{3}(m)(2r)^2 + (2m)(2r)^2$$
$$I_{total} = \frac{65}{6}mr^2$$

c. If the string is cut, the disk/rod/block system are free to rotate on the axle through the center of the disk. What is the angular acceleration of the system and what is the translational acceleration of the block?

$$\tau_{net} = +\tau_{rod} + \tau_{block} = I\alpha$$
$$\tau_{net} = +r_{rod}F_{w,rod}\sin\theta_0 + r_{block}F_{w,block}\sin\theta_0 = I\alpha$$
$$rmg\sin\theta_0 + 2r(2mg)\sin\theta_0 = 5rmg\sin\theta_0 = \frac{65}{6}mr^2\alpha$$
$$\alpha = \frac{30g}{65r}\sin\theta_0$$
$$a = R\alpha = 2r\left(\frac{30g}{65r}\sin\theta_0\right) = \frac{60}{65}g\sin\theta_0$$

d. When the system passes the horizontal position shown on the right, what is the linear speed of the block? Hint: Use conservation of energy and you may need $\sin(90-\theta_0) = \cos\theta_0$.



$$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s$$
$$0 = \left(\frac{1}{2}I\omega^2 - 0\right) + (0 - m_{rod}gh_{rod}) + (0 - m_{block}hh_{block})$$

$$\sin 90 - \theta_0 = \cos\theta_0 = \frac{h_{rod}}{r} \rightarrow h_{rod} = r\cos\theta_0$$
$$\sin 90 - \theta_0 = \cos\theta_0 = \frac{h_{block}}{2r} \rightarrow h_{block} = 2r\cos\theta_0$$

$$0 = \frac{1}{2}\left(\frac{65}{6}mr^2\right)\omega^2 - mgr\cos\theta_0 - 4mgr\cos\theta_0$$
$$5rg\cos\theta_0 = \frac{65}{12}r^2\omega^2 = \frac{65}{12}mr^2\left(\frac{v^2}{(2r)^2}\right) = \frac{65}{48}v^2$$
$$v = \sqrt{\frac{240}{65}rg\cos\theta_0}$$

Physics 110 Formula sheet

Vectors

$v = \sqrt{v_x^2 + v_y^2}$

$\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Motion Definitions

Displacement: $\Delta x = x_f - x_i$

Average velocity: $v_{avg} = \frac{\Delta x}{\Delta t}$

Average acceleration: $a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Rotational Motion Definitions

Angular displacement: $\Delta s = R\Delta\theta$

Angular velocity: $\omega = \frac{\Delta\theta}{\Delta t} \to v = R\omega$

Angular acceleration: $\alpha = \frac{\Delta\omega}{\Delta t} \to \begin{cases} a_t = r\alpha \\ a_c = r\omega^2 \end{cases}$

Rotational Equations of Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

Momentum & Force

$\vec{p} = m\vec{v} \to p_x = mv_x;\ p_y = mv_y$

$\Delta\vec{p} = \vec{F}\Delta t \to \vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a} \to F_x = ma_x; F_y = ma_y$

$F_{fr} = \mu F_N$

$F_w = mg$

$F_s = -kx$

$F_G = G\frac{M_1 M_2}{r^2}$

$F_c = ma_c = m\frac{v^2}{R}$

Work & Energy

$\begin{cases} W_T = \int \vec{F} \cdot d\vec{r} = Fdr\cos\theta = \Delta K_T \\ W_R = \int \vec{\tau} \cdot d\vec{\theta} = \tau d\theta = \Delta K_R \end{cases}$

$W_{net} = W_T + W_R = \Delta K_T + \Delta K_R = -\Delta U$

$K_T = \frac{1}{2}mv^2$

$K_R = \frac{1}{2}I\omega^2$

$U_g = mgy$

$U_s = \frac{1}{2}kx^2$

$\Delta E = \Delta E_R + \Delta E_T$

$\Delta E = \Delta K_R + \Delta K_T + \Delta U_g + \Delta U_s = \begin{cases} 0 \\ W_{fr} \end{cases}$

Rotational Momentum & Force

$\vec{\tau} = \vec{r} \times \vec{F};\ \tau = r_\perp F = rF_\perp = rF\sin\theta$

$\tau = \frac{\Delta L}{\Delta t} = I\alpha$

$L = I\omega$

$\Delta\vec{L} = \vec{\tau}\Delta t \to \vec{L}_f = \vec{L}_i + \vec{\tau}\Delta t$

Fluids

$\rho = \frac{m}{V}$

$P = \frac{F}{A}$

$P_y = P_{air} + \rho g y$

$F_B = \rho g V$

1,3,5, ... closed pipes

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$; compressible

$A_1 v_1 = A_2 v_2$; incompressible

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

Simple Harmonic Motion

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_s = 2\pi\sqrt{\frac{m}{k}}; \quad \omega = \sqrt{\frac{k}{m}}$

$T_p = 2\pi\sqrt{\frac{l}{g}}; \quad \omega = \sqrt{\frac{g}{l}}$

Geometry/Algebra

Circles:	$A = \pi r^2$	$C = 2\pi r = \pi D$
Spheres:	$A = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$
Triangles:	$A = \frac{1}{2}bh$	
Quadratics:	$ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	

Sound

$v_s = f\lambda = (331 + 0.6T)\frac{m}{s}$

$\beta = 10 \log \frac{I}{I_o}$

$f_n = n f_1 = n\frac{v}{2L}; n = 1,2,3, \ldots$ open pipes

$f_n = n f_1 = n\frac{v}{4L}; n =$

Waves

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = n f_1 = n\frac{v}{2L};\ n = 1,2,3, \ldots$

$I = 2\pi^2 f^2 \rho v A^2$

Equations of Motion for SHM

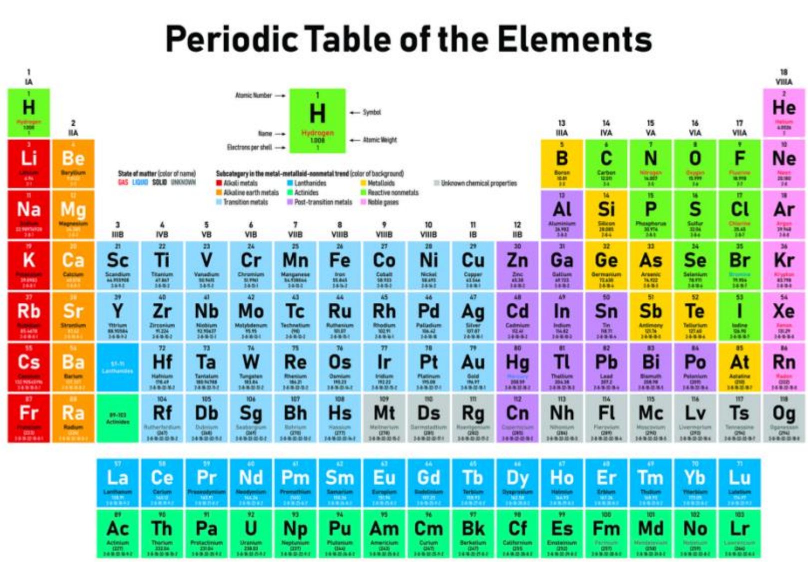
$$x(t) = \begin{cases} x_{max} \sin\left(\frac{2\pi}{T}t\right) \\ x_{max} \cos\left(\frac{2\pi}{T}t\right) \end{cases}$$

$$v(t) = \begin{cases} v_{max} \cos\left(\frac{2\pi}{T}t\right) \\ -v_{max} \sin\left(\frac{2\pi}{T}t\right) \end{cases}$$

$$a(t) = \begin{cases} -a_{max} \sin\left(\frac{2\pi}{T}t\right) \\ -a_{max} \cos\left(\frac{2\pi}{T}t\right) \end{cases}$$

$v = \pm v_{max}\sqrt{1 - \left(\frac{x}{x_{max}}\right)^2}$

$v = \pm \omega x_{max}\sqrt{1 - \left(\frac{x}{x_{max}}\right)^2}$



https://www.wuwm.com/post/periodic-table-elements-turns-150#stream/0