Physics 110

Exam #2

May 19, 2023

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\,\frac{m}{s}\right) = 10\,\frac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or reasonable value for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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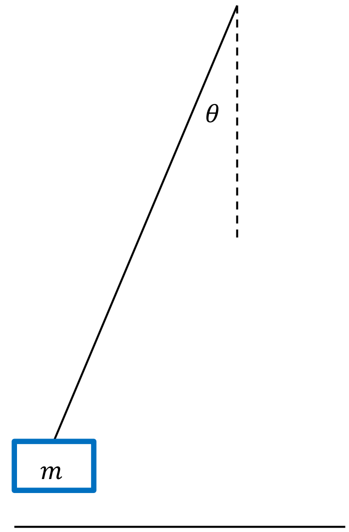
1. A block of mass $m = 0.5kg$ is attached to a string of length $l = 0.5m$. The block is pulled back from the vertical (taken to be $\theta = 0^0$) and is released from rest when the string makes an angle $\theta = 35^0$, as shown below.



   a. What is the speed of the mass when it passes through $\theta = 0^0$?

   $$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = \Delta K_T + \Delta U_g$$

   $$0 = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_f^2\right) + \left(mgy_f - mgy_i\right)$$

   $$v_f = \sqrt{2gl(1-\cos\theta)} = \sqrt{2 \times 9.8\frac{m}{s^2} \times 0.5m(1-\cos 35)}$$
   $$v_f = 1.33\frac{m}{s}$$

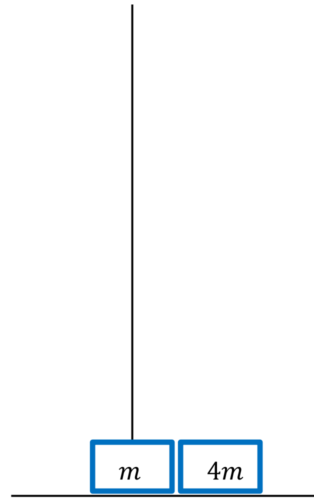
   Where from the geometry, $y_i = l - l\cos\theta$

   b. What is the magnitude and direction of the tension force in the string when the block passes trough $\theta = 0^0$?

   $$F_T - F_W = ma_c \rightarrow f_T = F_W + m\frac{v^2}{R} = mg + \frac{mv^2}{l}$$

   $$F_T = 0.5kg \times \left(9.8\frac{m}{s^2} + \frac{\left(1.33\frac{m}{s}\right)^2}{0.5m}\right) = 6.67N \text{ vertically up the string.}$$

c. As soon as the block of mass m passes through $\theta = 0^0$, it makes a head on collision with a second block of mass $4m$, initially at rest. After the collision it is found that the block of mass $4m$ moves to the right with a velocity $v_{f,4m} = 0.25\frac{m}{s}$. If the blocks do not stick together after the collision, what is the velocity of the block of mass m after the collision, $v_{f,m}$?



$$\Delta p_x = p_{fx} - p_{ix} = 0 \rightarrow p_{ix} = p_{fx}$$

$$mv_{im} = mv_{fm} + 4mv_{f4m}$$

$v_{fm} = v_{im} - 4v_{f4m} = 1.33\frac{m}{s} - 4 \times 0.25\frac{m}{s} = 0.33\frac{m}{s}$ in the positive x-direction (to the right)

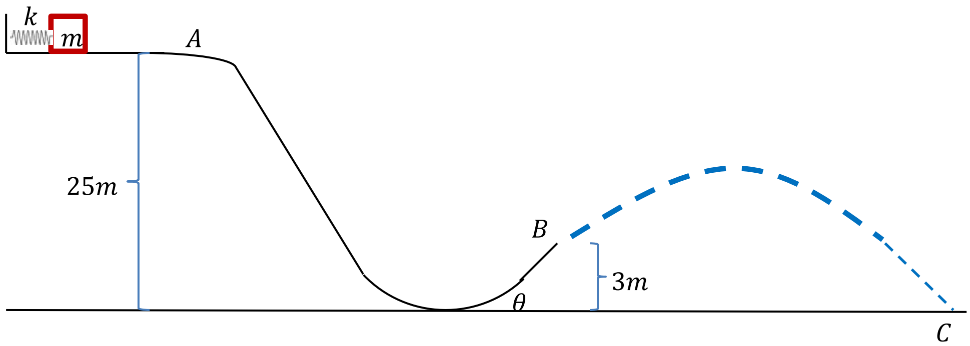
d. Is the collision of the two blocks elastic or inelastic? You will earn no credit for simply saying elastic or inelastic. You need to show that the collision is elastic or inelastic.

$$\Delta K = K_f - K_i = \left(\frac{1}{2}mv_{fm}^2 + \frac{1}{2}(4m)v_{f4m}^2\right) - \frac{1}{2}mv_{im}^2$$

$$\Delta K = \frac{1}{2} \times 0.5kg \times \left[\left(0.33\frac{m}{s}\right)^2 + \left(0.25\frac{m}{s}\right)^2 - \left(1.33\frac{m}{s}\right)^2\right] = -0.35J$$

Since $\Delta K \neq 0$ the collision is inelastic.

2. A $m = 70kg$ mass is launched from rest by a $x = 2m$ compressed spring of stiffness $k = 630\frac{N}{m}$. The mass m moves across the horizontal surface and then down the $25m$ tall hill and is launched from end of the $3m$ tall ramp on the right inclined at $\theta = 30^0$ measured with respect to the horizontal. All surfaces the mass moves on are frictionless.



a. Using the work-kinetic energy theorem, what is the speed of the mass at point A, just before the mass goes down the hill?

$$W = -\Delta U_S - \Delta U_g = -\Delta U_s = -\left(\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right) = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_A^2 \rightarrow v_A = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{630\frac{N}{m}}{70kg}} \times 2m = 6\frac{m}{s}$$

b. Using energy ideas between points A and B, what is the launch speed of the mass from the end of the ramp?

$$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = \Delta K_T + \Delta U_g$$

$$0 = \left(\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2\right) + (mgy_B - mgy_A) \rightarrow v_B = \sqrt{v_A^2 + 2g(y_A - y_B)}$$

$$v_B = \sqrt{\left(6\frac{m}{s}\right)^2 + 2 \times 9.8\frac{m}{s^2} \times (25m - 3m)} = 21.6\frac{m}{s}$$

c. The mass eventually lands at point C. How far horizontally is point C from point B?

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \rightarrow 0 = 3m + \left(21.6\frac{m}{s}\sin 30\right)t - \left(\frac{1}{2} \times 9.8\frac{m}{s^2}\right)t^2$$

$$t = \begin{cases} -0.25s \\ +2.45s \end{cases}$$

$$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \rightarrow x_f = v_{ix}t = \left(21.6\frac{m}{s}\cos 30\right) \times 2.45s = 45.8m$$

d. How much work was done by gravity between points B and C?

$$W = -\Delta U_f = -\left(mgy_f - mgy_i\right) = mgy_i = 70kg \times 9.8\frac{m}{s^2} \times 3m = 2058J$$

or

$$W = F_g \Delta y \cos\phi = mg\Delta y \cos 0 = mg\Delta y = 70kg \times 9.8\frac{m}{s^2} \times 3m = 2058J$$

or

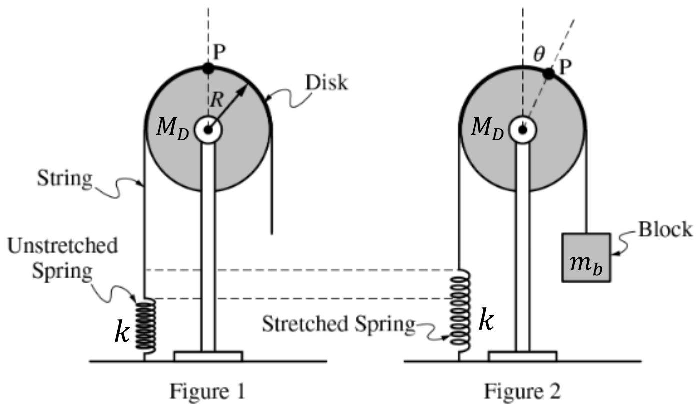
$$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2} \times 70kg \times \left(\left(22.9\frac{m}{s}\right)^2 - \left(21.6\frac{m}{s}\right)^2\right) = 2058J$$

where,

$$\Delta E = 0 = \left(\frac{1}{2}mv_C^2 - \frac{1}{2}mv_B^2\right) + \left(mgy_C - mgy_B\right) \rightarrow v_C = \sqrt{v_B^2 + 2gy_B}$$

$$v_C = \sqrt{\left(21.6\frac{m}{s}\right)^2 + 2 \times 9.8\frac{m}{s^2} \times 3m} = \frac{m}{s}$$

3. A solid disk of mass M_D and radius R is supported by a vertical stand. The disk can rotate about an axis through its center without friction. The rotational inertia of the disk is $I_D = \frac{1}{2}M_D R^2$ and there is a light string draped over the disk. The left end of the string is attached to a spring of stiffness k and the right end to a block of mass m_b. The mass m_b is slowly lowered until the spring/disk/block system comes into equilibrium. In equilibrium the disk has rotated through an angle θ and the spring has stretched by y from its unstretched length.



a. Derive an expression for the mass of the block m_b, in terms of M_D, R, k, θ, and any constant you need.

For the spring:

$$F_{TL} - F_s = ma_y = 0 \rightarrow F_{TL} = F_s = ky$$

For the block:

$$F_{Tr} - F_{Wb} = ma_y = 0 \rightarrow F_{TR} = F_{Wb} = m_b g$$

For the disk:

$$\tau_{F_{TR}} - \tau_{F_{TR}} + \tau_{F_N} + \tau_{F_W} = RF_{TR} - RF_{TL} = I_D\alpha = 0 \rightarrow F_{TL} = F_{TR}$$

$$\rightarrow ky = m_b g \rightarrow m_b = \frac{ky}{g} = \frac{kR\theta}{g}$$

b. At a time $t = 0$, the string on the right is cut and the mass m_b falls to the ground. As soon as the string is cut, the disk accelerates about the axis through its center. What is the expression for the initial maximum angular acceleration α about the axis through the center of the disk?

$$-\tau_L = -RF_{TL} = -Rky = I_D\alpha \rightarrow \alpha = -\frac{Rky}{I_D} = -\frac{kR^2\theta}{\frac{1}{2}M_D R^2} = -\frac{2k\theta}{M_D}$$

c. When the spring returns to its unstretched length, the disk reaches its maximum rotational speed. What is the maximum rotational speed ω of the disk?

$$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = \Delta K_R + \Delta U_s$$

$$0 = \left(\tfrac{1}{2}I_D\omega_f^2 - \tfrac{1}{2}I_D\omega_i^2\right) + \left(\tfrac{1}{2}ky_f^2 - \tfrac{1}{2}ky_i^2\right) = \tfrac{1}{2}I_D\omega_f^2 - \tfrac{1}{2}ky_i^2$$

$$\omega_f = \sqrt{\frac{k}{I_D}y_f^2} = \sqrt{\frac{k}{\frac{1}{2}M_D R^2}(R\theta)^2} = \sqrt{\frac{2k}{M_D}}\theta$$

d. Suppose that the system is adjusted so that the axis the disk rotates on does NOT pass through the center of the disk but is offset by an amount r. The block is again hung from the right side of the string and lowered until the spring/disk/block system are again in equilibrium. For each of the forces shown below, does the torque about the axis through the disk increase, decrease or remain due to each of these forces compared to those same forces from part a?

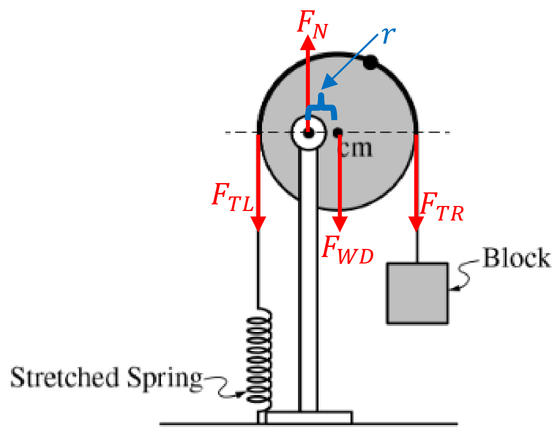


Figure 3

F_{WD}: The torque due to the weight of the disk increases from part a. In part a, the weight of the disk was located at the pivot, now it's a distance r.

F_N: The normal force acts at the pivot so its torque is the same as part a, namely zero.

F_{TR}: This is due to the weight of the block and since the distance from the pivot to the weight is larger $(R + r)$ compared to R, the torque increases from part a.

F_{TL}: This is due to the spring and since the torques due to F_{TR} and F_{WD} have increased this torque has to increase to compensate. This will result in the spring stretching farther from equilibrium than in part a.