Physics 110

Exam #2

May 7, 2025

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|p|$»$m|\vec{v}| = (5\text{kg})$´$\left(2\frac{m}{s}\right) = 10\frac{\text{kg}\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or reasonable value for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. A spring of stiffness k is suspended from the ceiling. Masses are then hung on the spring and the mass-spring system is allowed to come into equilibrium. Then for each mass hung from the spring, the mass is pulled from equilibrium by an amount y and released from rest. The period T of each mass' oscillation was measured and from a plot of T^2 versus m, a fit to the data was generated. The fit was found to be $T^2 = c \cdot m$, where the constant $c = 0.395\frac{s^2}{kg}$.

   a. What is the stiffness constant of the spring? Hint: For a mass oscillating on the end of a spring the period of its motion is given by $T = 2\pi\sqrt{\frac{m}{k}}$.

   $$T = 2\pi\sqrt{\frac{m}{k}} \rightarrow T^2 = \frac{4\pi^2}{k}m = 0.395\frac{s^2}{kg} \cdot m \rightarrow \frac{4\pi^2}{k} = 0.395\frac{s^2}{kg} \rightarrow k = \frac{4\pi^2}{0.395}\frac{s^2}{kg}$$

   $$k = 100\frac{N}{m}$$

   b. This spring is then used in the following experiment. In the Figure A the unstretched spring, or natural length L, has a mass m is attached to one end and the other end is attached to a rod that passes through a table. In Figure B the rod is spun at a constant rate. This causes the spring to stretch from equilibrium by an amount d. For the case in Figure B, what is the speed of the block and its period of revolution both expressed in terms of k, L, d, g, and m as required?

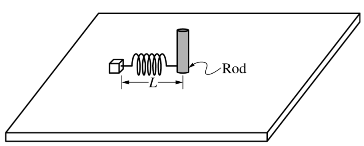


Figure A

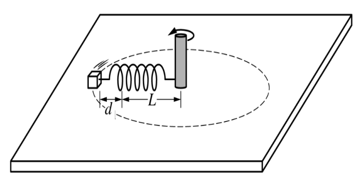
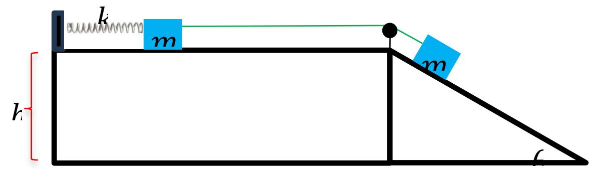


Figure B

   $$F_s = kd = ma_c = m\frac{v^2}{r} \rightarrow v = \sqrt{\frac{kdr}{m}} = \sqrt{\frac{kd(L+d)}{m}}$$

   $$v = \frac{2\pi r}{T} \rightarrow T = \frac{2\pi(L+d)}{v} = \sqrt{\frac{4\pi^2 m(L+d)^2}{kd(L+d)}} = \sqrt{\frac{4\pi^2 m(L+d)}{kd}}$$

c. Now consider the following experiment where two blocks of masses m_1 and m_2 are connected by a light string that passes over a massless pulley. Block of mass m_1 is connected to the wall by the spring with stiffness k. Suppose that the block of mass m_2 is released from rest and slides down the ramp a distance $\Delta x = x$ before coming momentarily to rest. In terms of ***only*** the forces that directly act on block of mass m_2, explain why the block of mass m_2 speeds up and then slows to a stop. Note, the spring in this part is not the spring from parts a and b



The forces that act directly on block 2 are the tension force and the weight. The sum of these two forces, by Newton's 2nd law, adds to the mass times the acceleration of M_2 down the ramp. The acceleration of M_2 is positive and looking at the forces that act we have: $F_T - M_2 g = -M_2 a$. Now the tension force comes from the block spring system of M_1. Here we have $F_T - kx = M_1 a$. At $x = 0$, we have $F_T = M_1 a$ and the block has a positive acceleration to the right both M_1 and M_2 speed up. As the spring stretches, the spring force increases and at some extension $F_T - kx = 0$ and the block M_2 (and M_1) reaches their maximum speeds. As the spring continues to stretch, the spring force becomes larger than the tension, $F_T - kx$, and the acceleration of M_1 and M_2 becomes negative. When the acceleration becomes negative, M_1 and M_2 slow down and will eventually come to rest.

d. In terms of m_1, m_2 , g , and k derive an expression for the maximum extension of the spring. Call this maximum extension x_{max}.

Let the system be both blocks, the spring and the earth. Then at maximum extension we know that $\Delta K_1 = \Delta K_2 = 0$ and block 1 never changes height so, $\Delta U_{g1} = 0$.

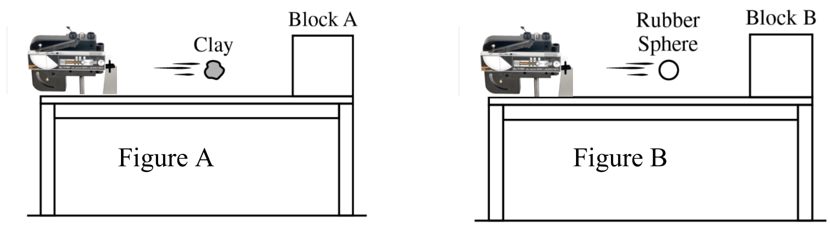
$$\Delta E_{system} = 0 = \Delta K_1 + \Delta K_2 + \Delta U_{g1} + \Delta U_{g2}$$

$$0 = (M_2 g y_{2f} - M_2 g y_{2i}) + \left(\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right) \rightarrow 0 = -M_2 g x \sin\theta + \frac{1}{2}kx^2$$

Minimum extension: $x_{min} = 0$

Maximum extension: $x_{max} = \frac{2M_2 g \sin\theta}{k}$

2. Two objects, a ball of clay and a ball of rubber, both with mass are launched from a horizontal projectile launcher. The projectile launcher consists of a horizontal spring of stiffness $k = 250\frac{N}{m}$. To launch both objects, the spring is compressed by an amount $x = 7.5cm$ from equilibrium. Both objects are launched from rest towards a blocks A and B, each of mass $M = 750g$. In Figure A, the ball of clay sticks to the block Awhile in Figure B, the rubber ball bounces off the block B.



a. What are the launch speeds of the ball of clay and the rubber ball if $m_{clayball} = m_{rubberball} = 250g$?

Let the system be the ball or clay, spring and the earth with $v_i = 0$ and $x_f = 0$ (spring returns to equilibrium from x_i.

$$\Delta E_{system} = \Delta K + \Delta U_g = 0 \rightarrow 0 = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + \left(\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right)$$

$$0 = \frac{1}{2}mv_f^2 - \frac{1}{2}kx_i^2 \rightarrow v_f = \sqrt{\frac{k}{m}x_i^2} = \sqrt{\frac{250\frac{N}{m}}{0.250kg} \times (0.075m)^2} = 2.37\frac{m}{s}$$

b. For the case shown in Figure A, what is the speed of the block A immediately after the collision with the clay and what fraction of the clay's initial kinetic energy is lost to the collision?

Since there are no outside forces, momentum is conserved.

$$p_{ix} = p_{fx} \rightarrow mv_i = (m + M)V \rightarrow V = \left(\frac{m}{m+M}\right)v_i$$

$$V = \left(\frac{0.25kg}{0.25kg+0.75kg}\right) \times 2.37\frac{m}{s} = 0.6\frac{m}{s}$$

$$f = \frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = \frac{K_f}{K_i} - 1 = \frac{\frac{1}{2}(m+M)V^2}{\frac{1}{2}mv_i^2} - 1 = \frac{m}{m+M} - 1$$

$f = \frac{0.25kg}{0.25kg+0.75kg} - 1 = -0.75$ or 75% is lost to the collision.

c. For the case shown in Figure B, what is the speed of the block B immediately after the collision with the rubber ball? Note: You need to derive an expression for the speed of the block after the collision with the rubber ball before you can evaluate it. You cannot simply state the equations from class. And to make the algebra easier use $\frac{M}{m} = 3$.

Since there are no outside forces, momentum is conserved and this collision is elastic so, kinetic energy is conserved. FYI: I did it out the long way.

$$p_{ix} = p_{fx} \rightarrow mv_{i,m} = mv_{f,m} + Mv_{f,M} \rightarrow v_{f,m} = v_{i,m} - \frac{M}{m}v_{fM}$$

$$K_i = K_f \rightarrow \frac{1}{2}mv_{i,m}^2 = \frac{1}{2}mv_{f,m}^2 + \frac{1}{2}Mv_{f,M}^2 \rightarrow v_{i,m}^2 = v_{f,m}^2 + \frac{M}{m}v_{f,M}^2$$

$$v_{f,m}^2 = v_{i,m}^2 + \frac{M^2}{M^2}v_{f,M}^2 - 2\frac{M}{m}v_{i,m}v_{f,M} \text{ and } v_{f,m}^2 = v_{i,m}^2 - \frac{M}{m}v_{f,M}^2.$$

$$v_{f,m}^2 = v_{i,m}^2 - \frac{M}{m}v_{f,M}^2 = v_{i,m}^2 + \frac{M^2}{M^2}v_{f,M}^2 - 2\frac{M}{m}v_{i,m}v_{f,M}$$

$$2\frac{M}{m}v_{i,m}v_{f,M} = \frac{M^2}{m^2}v_{f,M}^2 + \frac{M}{m}v_{f,M}^2 = \left(\frac{M^2}{m^2} + \frac{M}{m}\right)v_{f,M}^2$$

$$v_{f,M} = \left(\frac{2}{1+\frac{M}{m}}\right)v_{i,m} = \left(\frac{2}{1+\frac{0.75kg}{0.25kg}}\right) \times 2.37\frac{m}{s} = 1.19\frac{m}{s}$$

d. In both cases the block is launched horizontally from the table and allowed to fall thought a height $\Delta y = 1m$. Explain which block (the block A from Figure A or the block B from Figure B) lands closest to the table and which block lands farthest and why. Determine the distance between the two blocks when they land of the floor.

Block B has a higher launch velocity so it will travel farther than block A since the time of flight for both blocks is the same.

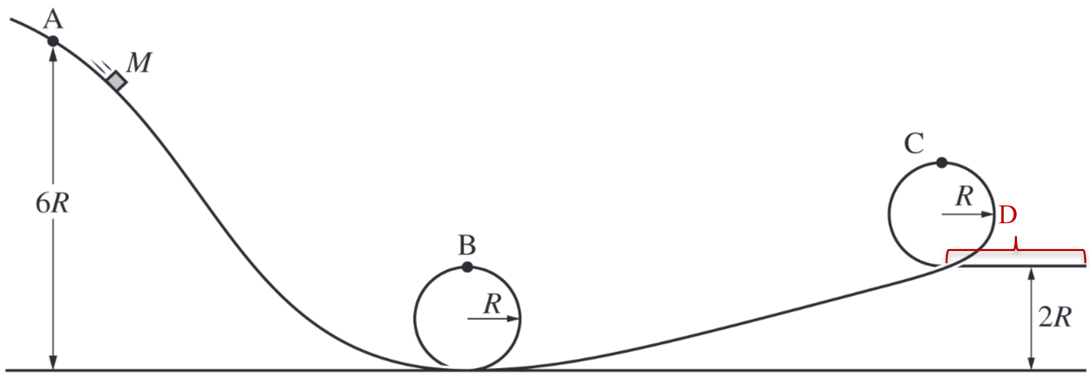
$$y_f = y_i + v_{iy}t - \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2\times 1m}{9.8\frac{m}{s^2}}} = 0.45$$

$$x_{fA} = v_{xA}t = 0.6\frac{m}{s} \times 0.45s = 0.27m$$

$$x_{fB} = v_{xB}t = 1.19\frac{m}{s} \times 0.45s = 0.53m$$

$$\Delta x = x_b - x_{fA} = 0.53m - 0.27 = 0.26m$$

3. A block of mass M starts from rest at point A on the top of a hill of height $6R$ above the ground. The entire track is frictionless except over the region of space labeled D.



a. What is the speed of the block at point B?

The system is the block and the earth.

$$\Delta E_{system} = 0 = \Delta K + \Delta U_g = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + \left(mgy_f - mgy_i\right)$$

$$\frac{1}{2}mv_{fA}^2 + (2mgR - 6mgR) = 0 \rightarrow v_{fA} = \sqrt{8gR}$$

b. What is the reaction force of the track on the block at point C?

$$-F_N - F_W = -ma_c = -m\frac{v_{fC}^2}{R} \rightarrow F_N = m\frac{v_{fC}^2}{R} - mg = \frac{4mgR}{R} - mg = 3mg$$

Where we calculated the speed at point C from conservation of energy by assuming the system is the block and the earth.

$$\Delta E_{system} = 0 = \Delta K + \Delta U_g = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + \left(mgy_f - mgy_i\right)$$

$$0 = \left(\frac{1}{2}mv_f^2 - v_i^2\right) + \left(mg_f - mg_i\right)$$

$$0 = \frac{1}{2}mv_{fc}^2 + 4mgR - 6mgR \rightarrow v_{fC}^2 = 4mgR$$

c. For the block to barely hang on at point C, from what minimum height h_{min}above the ground would the block have needed to be released.

To barely hang on means that the normal force must vanish.

$$F_N = m\frac{v_{fC}^2}{R} - mg = 0 \rightarrow v_{FC} = \sqrt{Rg}$$

To determine the minimum height, we must start the block, we assume the system is the earth and the block and apply conservation of energy.

$$\Delta E_{system} = 0 = \Delta K + \Delta U_g = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + \left(mgy_f - mgy_i\right)$$

$$0 = \frac{1}{2}mv_{fC}^2 + (4mgR - mgy_i) \rightarrow \frac{1}{2}mgR + 4mgR = mgy_i$$

$$y_i = 4.5R$$

d. Over region D, friction brings the block to rest after it has traveled a horizontal distance x. How much work was done by the force of friction bringing the block to rest and what is the expression for the coefficient of friction in terms of$g$, M, R, and x Assume that the block is released from the minimum height found in part c.?

To determine the speed of the block at the start of section D, we consider the system to be the block and the earth.

$$\Delta E_{system} = 0 = \Delta K + \Delta U_g = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + \left(mgy_f - mgy_i\right)$$

$$0 = \frac{1}{2}mv_f^2 + \left(mgy_f - mgy_i\right) = \frac{1}{2}mv_{fD}^2 + (2mgR - 4.5mgR)$$

$$v_{fD}^2 = 5gR$$

Friction is external to the system and does work. The work done is

$$W_{fr} = \mu F_N x cos180 = -\mu mgx = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{-1}{2}mv_{iD}^2$$

$$\mu = \frac{5mgR}{2mgx} = \frac{5R}{2x}$$