# Physics 110

## Exam #2

May 22, 2026

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. Erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg)\times\left(2\frac{m}{s}\right) = 10\frac{kg\cdot m}{s}$
- Give standard SI units with your results.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Formulas or constants not explicitly given on the formula sheet need to be derived before they can be used or full credit will not be given even if the formula or answer is correct.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 4 points.

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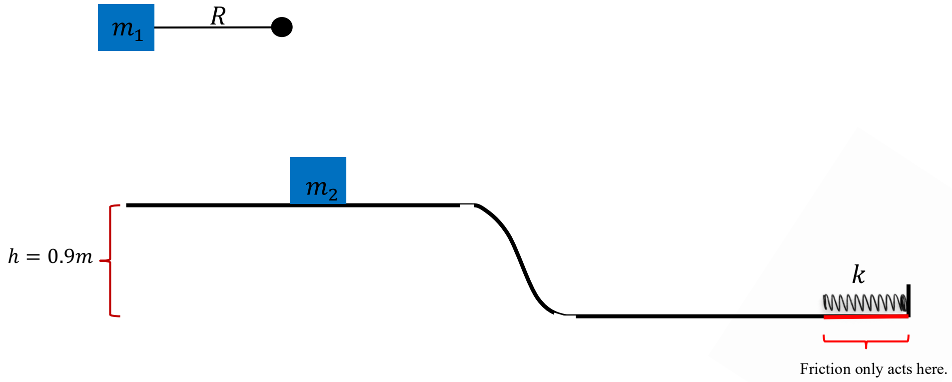
|---|---|
| Problem #1 | /24 |
| Problem #2 | /24 |
| Problem #3 | /24 |
| Total | /72 |

*I affirm that I have carried out my academic endeavors with full academic honesty.*

1. As mass $m_1 = 2kg$ is connected to a pendulum arm of length $R = 30cm$ and held horizontally at rest as shown below. The pendulum arm is released from rest and makes a head-on collision with a stationary mass $m_2 = 1kg$ when $m_1$ is at its lowest point.



a. When $m_1$ is at its lowest point, and just before it collides with mass $m_2$, what is its speed?

$$\Delta E_{system} = 0 = \Delta K + \Delta U_g + \Delta U_s = \left(\frac{1}{2}m_1 v_{1f}^2 - \frac{1}{2}m_1 v_{i1}^2\right) + \left(m_1 g y_{1f} - m_1 g y_{1i}\right)$$

$$0 = \frac{1}{2}m_1 v_{1f}^2 - m_1 g y_{1i} \rightarrow v_{1f} = \sqrt{2gy_i} = \sqrt{2 \times 9.8\frac{m}{s^2} \times 0.3m} = 2.42\frac{m}{s}$$

b. When $m_1$ is at its lowest point, and just before it collides with mass $m_2$, what is the magnitude of the tension force in the pendulum arm?

Taking up as the positive y-direction we have:

$$F_T - F_W = m_1 a_c \rightarrow F_T = F_W + m\frac{v_{1f}^2}{R} = m_1 g + m_1\frac{v_{1f}^2}{R} = m_1\left(g + \frac{v_{1f}^2}{R}\right)$$

$$F_T = 2kg\left(9.8\frac{m}{s^2} + \frac{\left(2.42\frac{m}{s}\right)^2}{0.3m}\right) = 58.6N$$

c. After mass $m_1$ undergoes the collision with mass $m_2$, we find that $m_2$ is launched horizontally forward with a speed, call it $v_{2C}$, and $m_1$ continues forward rising up through an angle $\theta = 27^0$ measured with respect to the vertical before coming to rest. What was the speed of $m_1$, call it $v_{1C}$, after the collision?

$$\Delta E_{system} = 0 = \Delta K + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}m_1 v_{1f}^2 - \tfrac{1}{2}m_1 v_{i1}^2\right) + \left(m_1 g y_{1f} - m_1 g y_{1i}\right)$$

$$0 = -\tfrac{1}{2}m_1 v_{1C}^2 + m_1 g y_{1f} \rightarrow v_{1C} = \sqrt{2 g y_{1f}} = \sqrt{2gR(1-\cos\theta)}$$

$$v_{1C} = \sqrt{2 \times 9.8\tfrac{m}{s^2} \times 0.3m \times (1-\cos 27)} = 0.8\tfrac{m}{s}$$

Where $y_{1f}$ is determined from the geometry in the problem.

d. What was the speed of mass $m_2$ after the collision?

In the collision momentum is conserved. Therefore, we have:

$$p_{ix} = p_{fx} \rightarrow m_1 v_{i1} = m_1 v_{1C} + m_2 v_{2C} \rightarrow v_{2C} = \frac{m_1 v_{i1} - m_1 v_{1C}}{m_2}$$

$$v_{2C} = \frac{2kg \times 2.42\frac{m}{s} - 2kg \times 0.8\frac{m}{2}}{1kg} = 3.24\frac{m}{s}$$

e. Was the collision between $m_1$ and $m_2$ elastic or inelastic? Explain. Hint: You will need to make a calculation in order to justify elastic or inelastic.

To determine if the collision was elastic or inelastic, we determine $\Delta K$. If $\Delta K$ is zero the collision is elastic. If $\Delta K$ is not zero, then the collision was inelastic.

$$\Delta K = \left(\tfrac{1}{2}m_1 v_{1C}^2 + \tfrac{1}{2}m_2 v_{2C}^2\right) - \tfrac{1}{2}m_1 v_{1i}^2$$

$$\Delta K = \left(\tfrac{1}{2} \times 2kg \times \left(0.8\tfrac{m}{s}\right)^2 + \tfrac{1}{2} \times 1kg \times \left(3.24\tfrac{m}{s}\right)^2\right) - \tfrac{1}{2} \times 2kg \times \left(2.42\tfrac{m}{s}\right)^2$$

$\Delta K = (0.64J + 5.25J) - 5.86J = 0.03J$ Since this is very close to zero (the difference is due to rounding), I'd say the collision is elastic.

f. The mass $m_2$ slides across the horizontal surface, down the curved part of the track, and across a second horizontal surface where it encounters a spring of stiffness $k = 20\frac{N}{m}$. If friction exists under the spring, with coefficient of friction $\mu = 0.4$, how far will the spring be compressed when the block comes to rest? Assume that the spring starts at its equilibrium position.

$$\Delta E_{system} = W_{fr} = \Delta K + \Delta U_g + \Delta U_s$$

$$W_{fr} = F_{fr} x_f \cos 180 = -\mu m_2 g x$$

$$-\mu m_2 g x = \left(\frac{1}{2} m_2 v_{2f}^2 - \frac{1}{2} m_2 v_{2C}^2\right) + \left(m_2 g y_{2f} - m_2 g y_{2i}\right) + \left(\frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2\right)$$

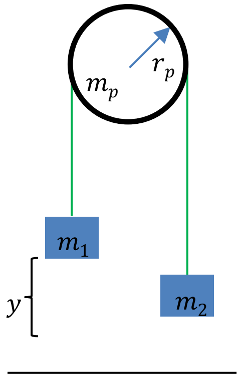
$$\rightarrow -\mu m_2 g x_f = -\frac{1}{2} m_2 v_{2C}^2 - m_2 g y_{2i} + \frac{1}{2} k x_f^2$$

$$\frac{1}{2}\left(20\frac{N}{m}\right) x^2 + \left(0.4 \times 1kg \times 9.8\frac{m}{s^2}\right) x - \frac{1}{2}(1kg)\left(3.24\frac{m}{2}\right)^2 - \left(1kg \times 9.8\frac{m}{s^2} \times 0.9m\right) = 0$$

$$10x^2 + 3.9x - 14.07 = 0 \rightarrow x = \begin{cases} 1.0m \\ \\ -1.4m \end{cases}$$

The mass was moving in the positive x-direction; thus, the spring is compressed by $x_f = 1m$.

2. Consider the arrangement of masses shown on the right, called an Atwood's machine, in which two masses $m_1$ and $m_2$ are connected by a light rope around a pulley. In the parts below assume that there is no friction in the axle about which the pulley spins and $m_1 > m_2$.



a. If the system is released from rest, what is the expression for the speed of mass $m_2$ after mass $m_1$ has fallen through a distance $y$ in terms of the quantities given in the problem? Assume the pulley has no mass and you must derive the expression using either forces/torques or energies to earn full credit.

$$\Delta E_{system} = 0 = \Delta K_T + \Delta U_g + \Delta U_s + \Delta K_R$$

$$0 = \Delta K_{T1} + \Delta U_{g1} + \Delta K_{T2} + \Delta U_{g2}$$

Since the masses are connected by the rope, they acquire the same final speed. And, we assume up is the positive y-direction, so the change in gravitational potential energy for mass $m_1$ is negative (it falls) and for mass $m_2$, positive (it rises). They both translate by an amount $y$.

$$0 = \tfrac{1}{2}m_1 v_f^2 + \tfrac{1}{2}m_2 v_f^2 - m_1 g y + m_2 g y \rightarrow \tfrac{1}{2}(m_1 + m_2)v_f^2 = (m_1 - m_2)gy$$

$$v_f = \sqrt{2\left[\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g\right]y}$$

b. If the system is released from rest, what is the expression for the speed of mass $m_2$ after mass $m_1$ has fallen through a distance $y$ if the pulley has mass $m_p$ and radius $r_p$ in terms of the quantities given in the problem? Hint: the moment of inertia of a disk spun about an axis through its center is $I = \tfrac{1}{2}mr^2$and you must derive the expression using either forces/torques or energies to earn full credit.

$$\Delta E_{system} = 0 = \Delta K_T + \Delta U_g + \Delta U_s + \Delta K_R$$

$$0 = \Delta K_{T1} + \Delta U_{g1} + \Delta K_{T2} + \Delta U_{g2} + \Delta K_R$$

$$0 = \tfrac{1}{2}m_1 v_f^2 + \tfrac{1}{2}m_2 v_f^2 - m_1 g y + m_2 g y + \tfrac{1}{2}I\omega_f^2$$

$$\rightarrow \tfrac{1}{2}(m_1 + m_2)v_f^2 + \frac{1}{2}\left(\frac{1}{2}m_p r_p^2\right)\left(\frac{v_f}{r_p}\right)^2 = (m_1 - m_2)gy$$

$$v_f = \sqrt{2\left[\left(\frac{m_1 - m_2}{m_1 + m_2 + \frac{1}{2}m_p}\right)g\right]y}$$

c. Either by using the results from parts a and b, or by deriving the two results asked for, what is the magnitude of the acceleration of the system of masses in the case where the pulley has no mass and the case when the pulley has mass? What can you say about the difference in the two accelerations and how does this affect the final speed of the system of masses? Be sure to fully explain your answer and simply quoting an expression without showing where it came from will earn limited credit.

Since the acceleration of the system is constant, the speed of the system is given by:

$$v_f = \sqrt{2ay}$$

For the case where the pulley has no mass, we have:

$$v_f = \sqrt{2\left[\left(\frac{m_1 - m_2}{m_1 + m_2}\right) g\right] y} \rightarrow a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

For the case where the pulley has mass, we have:

$$v_f = \sqrt{2\left[\left(\frac{m_1 - m_2}{m_1 + m_2 + \frac{1}{2}m_p}\right) g\right] y} \rightarrow a = \left(\frac{m_1 - m_2}{m_1 + m_2 + \frac{1}{2}m_p}\right) g$$

The case of the pulley with mass we see that the effect of the pulley's mass is in the denominator, so the acceleration with the massive pulley is smaller than the case when the pulley has no mass. Since this acceleration is smaller, the speed of the masses in the case with a massive pulley is smaller, due to the fact that we have to make the massive pulley spin.

d. How much work was done by the force of gravity on masses $m_1$ and on $m_2$ as each move through distance $y$, from rest? There should be two expressions here. One for the work done on $m_1$ and one for the work done on $m_2$.

$$W_{F_{W1}} = F_{W1}\Delta y \cos\phi = m_1 g y \cos 0 = m_1 g y$$

$$W_{F_{W2}} = F_{W2}\Delta y \cos\phi = m_2 g y \cos 180 = -m_2 g y$$

e. What is the net work done on the system of masses $m_1$and $m_2$ by the force of gravity and how does this work done show up in the system of masses? Hint: Consider separately the case where the pulley does not have mass and the case where the pulley has mass. Be sure to explain your answers for both cases.

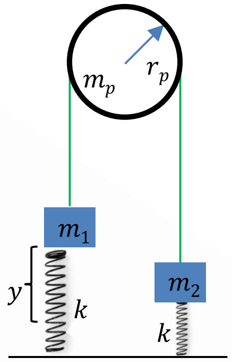
$$W_{net} = W_{F_{W1}} + W_{F_{W2}} = m_1 gy - m_2 gy$$

Whether the pulley has mass or not, the work done is the same as the pulley never changes height. The work done shows up as a change in the kinetic energy in the system.

For the case where the pulley has no mass, this change is pure translational kinetic energy.

For the case where the pulley has mass, the change is translational and rotational kinetic energy.

f. Consider the following situation where each of the masses are connected to identical springs of stiffness $k$. If both of the springs start at their equilibrium positions when the masses are released from rest, what is the maximum extension of the spring connected to mass $m_2$?



As mass $m_1$ falls and mass $m_2$ rises by $y$, the spring on the left compresses by $y$ and the spring on the right extends by $y$. Whether the pulley has mass or not, does not matter, as the system starts from rest and comes back to rest.

$$\Delta E_{system} = 0 = \Delta K_T + \Delta U_g + \Delta U_s + \Delta K_R$$

$$0 = \Delta K_{T1} + \Delta U_{g1} + \Delta K_{T2} + \Delta U_{g2} + \Delta U_{s1} + \Delta U_{s2} + \Delta K_R$$

$$0 = -m_1 gy + m_2 gy + \tfrac{1}{2}ky^2 + \tfrac{1}{2}ky^2$$

$$0 = (-m_1 gy + m_2 gy + ky)y \rightarrow \begin{cases} y_{min} = 0 \\ y_{max} = \left(\dfrac{m_1 - m_2}{k}\right) g \end{cases}$$

3. The rotator cuff is a group of four muscles and their associated tendons that are responsible for holding the humerus (the upper arm bone) in your shoulder socket. These muscles and tendons give stability to the shoulder joint and allow for the arm to be rotated. A cartoon image of two of the 4 main muscles/tendons are shown in Figure A below. The deltoid muscle is the major muscle responsible for you being able to hold your arm outstretched, at any angle above the horizontal. Consider the person shown below holding their arm (of mass $m_a = 3.5kg$ and total length $L$) at a $25^0$ angle measured with respect to the horizontal while holding a $m = 4kg$ mass at rest. In this situation, the deltoid muscle $F_M$ makes an angle $15^0$ measured with respect to the outstretched arm and acts at a point $0.3L$ measured from the shoulder joint, as shown below in Figure B.

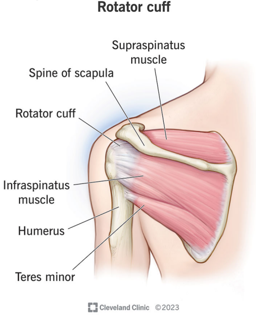


Figure A: Diagram of part of the rotator cuff, showing the deltoid muscle.

https://my.clevelandclinic.org/health/body/rotator-cuff

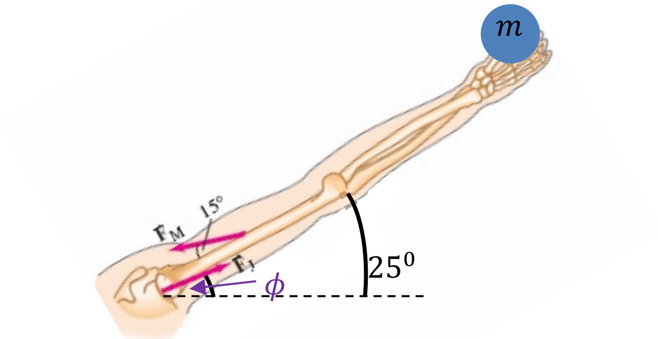


Figure B: Diagram of part of outstretched arm showing the location of the deltoid force application point and the shoulder joint reaction force. Figure adapted from Giancoli, Physics: Principles with Applications 7th ed.

a. Using Figure B, and applying the condition for translational equilibrium in the x-direction, write the sum of the forces in the horizontal direction. From this, what is the expression for the reaction force, $F_{Jx}$, on the shoulder joint in the horizontal direction?

$$F_J \cos\phi - F_M \sin(25 - 15) = ma_x = 0$$

$$F_{Jx} = F_J \cos\phi = F_M \sin(10)$$

b. Using Figure B, and applying the condition for translational equilibrium in the y-direction, write the sum of the forces in the vertical direction. From this, what is the expression for the reaction force, $F_{Jy}$, on the shoulder joint in the vertical direction?

$$F_J \sin\phi - F_M \sin 10 - F_{WA} - F_W = ma_y = 0$$

$$F_{Jy} = F_J \sin\phi = F_M \sin 10 + F_{WA} + F_W = F_M \sin 10 + m_A g + mg$$

c. Assuming that the pivot is taken at the shoulder joint and applying the condition for rotational equilibrium, what is the magnitude of the force exerted by the deltoid muscle, $F_M$?

Taking counterclockwise rotations as the positive direction for the torque we have:

$$+\tau_{F_M} - \tau_{F_{WA}} - \tau_{F_{Wm}} = I\alpha = 0$$

$$0.3L \times F_M \sin 15 - 0.5L \times F_{WA} \sin 65 - L \times F_{Wm} \sin 65 = 0$$

$$0.3F_M \sin 15 = 0.5F_{WA} \sin 65 + F_{Wm} \sin 65$$

$$\rightarrow F_M = \frac{0.5m_a g \sin 65 + mg \sin 65}{0.3 \sin 15}$$

$$F_M = \frac{\left(0.5 \times 3.5kg \times 9.8\frac{m}{s^2} \times \sin 65\right) + \left(4kg \times 9.8\frac{m}{s^2} \times \sin 65\right)}{0.3 \sin 15} = 657.3N$$

d. What is the magnitude of the reaction force of the shoulder joint, $F_J$?

$$F_{Jx} = F_J \cos\phi = F_M \cos 10 = 657.3N \times \sin 10 = 647.3N$$

$$F_{Jy} = F_M \sin 10 + m_A g + mg = 657.3N \times \sin 10 + (3.5Kg + 4kg)9.8\frac{m}{s^2}$$

$$F_{Jy} = 187.6N$$

$$F_J = \sqrt{F_{Jx}^2 + F_{Jy}^2} = \sqrt{(647.3N)^2 + (187.6N)^2} = 674.2N$$

e. In what direction, $\phi$, (measured with respect to the horizontal) does the reaction force of the shoulder joint point?

$$\tan\phi = \frac{F_{Jy}}{F_{Jx}} \rightarrow \phi = \tan^{-1}\left(\frac{F_{Jy}}{F_{Jx}}\right) = \tan^{-1}\left(\frac{187.6N}{647.3N}\right) = 16.2^0$$

f. What, if anything, would change if the deltoid muscle was attached at a point closer to the shoulder joint? That is, would the reaction force on the shoulder joint change? Would this change make supporting the weight in your hand easier or harder? Be sure to explain your thoughts.

There are probably two effects to consider if the muscle was attached at point closer to the shoulder joint. The two effects are a decrease in distance from the shoulder joint to the attachment point and the angle the muscle makes with the arm probably also becomes smaller and more aligned with the arm bone. The angle effect is probably a very small change, maybe a few degrees. This would not change the $\sin\theta$ term in the torque by a real amount, but the distance change would definitely affect the torque produced by the deltoid muscle. Here the torque would decrease significantly. Imagine the distance was one-half of what it is currently. This would cut the torque in half. But, to balance the torques (due to the weight of the arm and the mass) the deltoid muscle would have to create a much larger force, which would be twice as much. This would translate to a much larger force felt by the shoulder joint and it would make the mass feel harder to hold and probably would lead to fatigue much faster.