Physics 110

Fall 2011

Exam #2

October 14, 2011

Name____________________

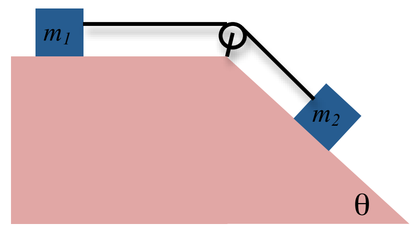
Multiple Choice	/ 12
Problem #1	/ 24
Problem #2	/ 32
Problem #3	/ 32
Total	/ 100

In keeping with the Union College policy on academic honesty, it is assumed that you will neither accept nor provide unauthorized assistance in the completion of this work.

Part I: Free Response Problems

The three problems below are worth 88 points total and each subpart is worth 8 points each. Please show all work in order to receive partial credit. If your solutions are illegible or illogical no credit will be given. A number with no work shown (even if correct) will be given no credit. Please use the back of the page if necessary, but number the problem you are working on.

1. Consider the arrangement of blocks shown below. Block #1 has a mass $m_1 = 2kg$, block #2 has a mass of $m_2 = 4kg$. A massless string that passes over a massless, frictionless pulley connects blocks #1 and #2 to each other. There is friction on the surfaces that both block are sitting on, with coefficient of friction $\mu_k = 0.2$ and the angle of inclination for the inclined plane is $\theta = 25^o$.



a. What is the magnitude of the acceleration of m_2 down the incline?

m_1 :

$$\sum F_x : \ F_T - F_{Fr1} = F_T - \mu_k F_{N1} = F_T - \mu_k m_1 g = m_1 a$$

$$\sum F_y : \ F_{N1} - F_{W1} = F_{N1} - m_1 g = 0 \rightarrow F_{N1} = m_1 g$$

m_2 :

$$\sum F_x : \ -F_T - F_{Fr1} + F_{W2x} = -F_T - \mu_k F_{N2} + m_2 g \sin\theta = -F_T - \mu_k m_2 g \cos\theta + m_2 g \sin\theta = m_2 a$$

$$\sum F_y : \ F_{N2} - F_{W2y} = F_{N2} - m_2 g \cos\theta = 0 \rightarrow F_{N2} = m_2 g \cos\theta$$

$$\Rightarrow -\mu_k m_1 g - \mu_k m_2 g \cos\theta + m_2 g \sin\theta = m_1 a + m_2 a$$

$$\therefore a = \frac{-\mu_k m_1 g - \mu_k m_2 g \cos\theta + m_2 g \sin\theta}{m_1 + m_2}$$

$$a = \frac{\left((-0.2 \times 2kg) - (0.2 \times 4kg \times \cos 25) + (4kg \times \sin 25)\right) \times 9.8 \frac{m}{s^2}}{6kg} = 0.92 \frac{m}{s^2}$$

b. Using the acceleration you calculated in part *a*, what is the speed of m_2 if the system is released from rest and m_2 travels a distance of $d = 1m$ along the ramp from its release point to the bottom of the ramp?

$$v_v^2 = v_i^2 + 2a\Delta x \rightarrow v_f = \sqrt{2a\Delta x} = \sqrt{2 \times 0.92 \frac{m}{s^2} \times 1m} = 1.36 \frac{m}{s}$$

c. Redo part *b*, but his time use energy methods to calculate the speed of m_2.

$$\Delta KE + \Delta U_g + \Delta U_s = \Delta KE_1 + \Delta KE_2 + \Delta U_{g1} + \Delta U_{g2} = \Delta E$$

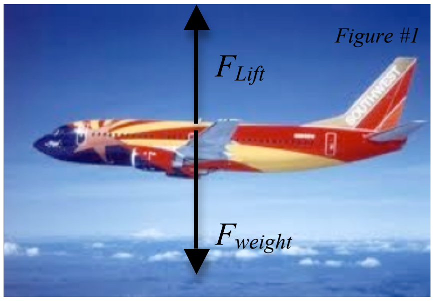
$$\left(\tfrac{1}{2} m_1 v_{1f}^2 - 0\right) + \left(\tfrac{1}{2} m_2 v_{2f}^2 - 0\right) + \left(m_2 g y_{2f} - m_2 g y_{2i}\right) = -F_{fr1} d - F_{fr2} d$$

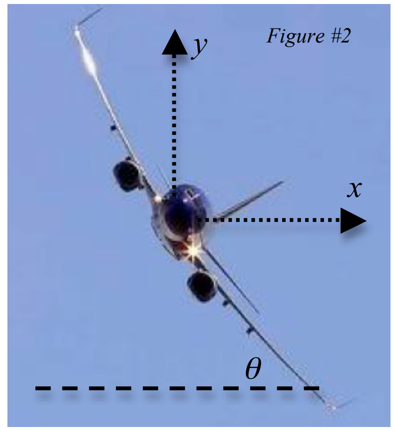
$$\tfrac{1}{2}(m_1 + m_2) v_f^2 - m_2 g d \sin\theta = -(m_1 g + m_2 g \cos\theta) d$$

$$v_f = \sqrt{\frac{2\left(m_2 g d \sin\theta - (m_1 g + m_2 g \cos\theta) d\right)}{m_1 + m_2}}$$

$$v_f = \sqrt{\frac{2 \times 9.8 \frac{m}{s^2} \times 1m \times \left(4kg \times \sin 25 - (2kg + 4kg \times \cos 25)\right)}{6kg}} = 1.36 \frac{m}{s}$$

2. When an airplane (flying in level flight as shown in *figure #1*) needs to turn, the plane must bank (as shown in *figure #2*.) While in level flight, the weight of the aircraft is balanced by an upward force called the lift and is the sum of the lifting forces due to each wing (each wing produces ½ of the total lifting force, F_{Lift}) and is always perpendicular to the wings. The net lift when in flight can be considered to act at the center of the airplane.





a. On *figure #2*, using the coordinate system shown, draw the forces that act on the airplane while it is banking. In addition, suppose air friction is negligible and that the airplane is banking such that the wings tip from the horizontal by an angle θ. From your force diagram what force (or forces) create the centripetal acceleration of that plane? How much of this force (or these forces) contribute to the centripetal acceleration?

The horizontal component of the lift force ($F_L \sin\theta$)creates the centripetal acceleration.

b. If the plane banks at an angle of $\theta = 20^o$ and makes one complete turn around a horizontal circle in the sky of radius R at a constant speed of $v = 166\ m/s$ (~*332 mi/hr*), what is the diameter of the planes circular orbit? (The plane here is a fully loaded Boeing 737-800 with a mass of *70,535kg* (~155,000lbs).) Express your answer in miles, where *1mile = 1600m*.

$$\sum F_x: \quad F_L \sin\theta = \frac{mv^2}{R}$$

$$\sum F_y: \quad F_L \cos\theta - mg = 0 \rightarrow F_L = \frac{mg}{\cos\theta}$$

$$\therefore R = \frac{mv^2}{F_L \sin\theta} = \frac{mv^2 \cos\theta}{mg \sin\theta} = \frac{v^2}{g \tan\theta} = \frac{\left(166 \frac{m}{s}\right)^2}{9.8 \frac{m}{s^2} \tan 20} = 7726m \times \frac{1mi}{1600m} = 4.8mi$$

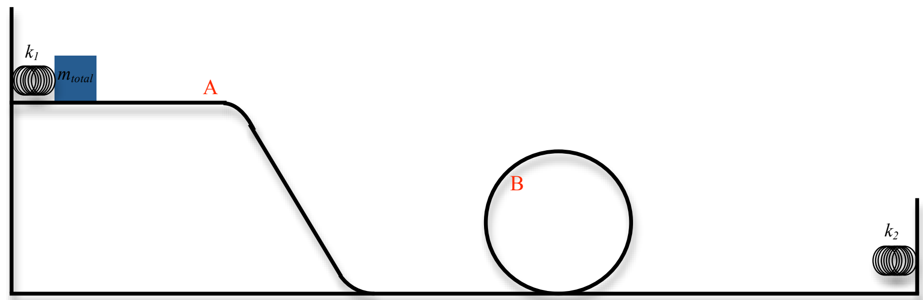
c. How much work is done by the force (or forces) responsible for the centripetal acceleration in part *a*, if the airplane makes one complete turn?

Since the horizontal component of the lift force is perpendicular to the displacement of the plane, no work is done. Further, since the plane flies at a constant, speed, there is no change in KE, so again no work is done.

d. What would happen to the force of lift and the diameter of the airplanes orbit (assuming all other quantities else remain constant) if the value of the acceleration due to gravity, *g* were *a factor of 2 smaller*?

Using the expressions in part b, we see that given everything else being equal, the lift force would decrease by a factor of 2 if g were a factor of 2 smaller. In addition, the turn radius would increase by a factor of 2 if g were to decrease.

3. Consider the amusement park thrill ride shown below. The cart has a mass of m_{cart} = *200kg* and 6 people at a time can ride. Assume that the average mass of a rider is m_{rider} = *70kg*, that the two springs in the ride have spring constants k_1 = *3247N/m* and k_2 = *7261 N/m* respectively, the ride starts out horizontal and is *25m* above the ground. The cart and riders go down over the hill and around the loop-the-loop (of radius R = *10m*) and strike the second spring, which brings the cart and riders to rest.



a. If the spring with spring constant k_1 is compressed by an amount *1.76m* from its equilibrium (or relaxed) length, how fast is the cart and its full complement of riders going at point *A* if the car starts from rest and the track is considered frictionless?

$$\Delta KE + \Delta U_g + \Delta U_s = \Delta E = 0$$

$$\left(\tfrac{1}{2}m_{total}v_A^2 - 0\right) + \left(0 - \tfrac{1}{2}k_1 x_i^2\right) = 0$$

$$v_A = \sqrt{\frac{k}{m_{total}}}x_i = \sqrt{\frac{k}{m_{cart} + 6m_{riders}}}x_i = \sqrt{\frac{3247\frac{N}{m}}{620kg}} \times 1.76m = 4.0\tfrac{m}{s}$$

b. How fast is the cart traveling at point *B,* if point *B* is located above ground level at an amount 75% of the diameter of the circular loop?

$$\Delta KE + \Delta U_g + \Delta U_s = \Delta E = 0$$

$$\left(\tfrac{1}{2}m_{total}v_B^2 - \tfrac{1}{2}m_{total}v_A^2\right) + \left(m_{total}gy_f - m_{total}gy_i\right) = 0$$

$$v_B = \sqrt{v_A^2 + 2g\left(y_i - y_f\right)} = \sqrt{\left(4\tfrac{m}{s}\right)^2 + 2 \times 9.8\tfrac{m}{s^2} \times \left(25m - (0.75 \times 20m)\right)} = 14.6\tfrac{m}{s}$$

c. If the cart strikes spring k_2 at the other end of the ride, initially at its relaxed length, how much work is done by the spring in bringing the cart and riders to rest?

$$W = \Delta KE = \left(0 - \tfrac{1}{2}m_{total}v_i^2\right) = -\tfrac{1}{2}m_{total}v_i^2 = -\tfrac{1}{2} \times 620kg \times \left(22.5\tfrac{m}{s}\right)^2 = -156860J$$

where, v_{bottom} before the spring: $\Delta KE + \Delta U_g = \left(\tfrac{1}{2}m_{total}v_{bottom}^2 - \tfrac{1}{2}m_{total}v_A^2\right) + \left(0 - mgy_A\right) = 0$

$$\rightarrow v_{bottom} = \sqrt{v_A^2 + 2gy_A} = \sqrt{\left(4\tfrac{m}{s}\right)^2 + 2 \times 9.8\tfrac{m}{s^2} \times 25m} = 22.5\tfrac{m}{s}$$

d. How much is the spring k_2 compressed when the cart and riders come to rest?

$$W_{k2} = -\Delta U_s = -\left(\tfrac{1}{2}k_2 x_f^2 - 0\right) \rightarrow x_f = \sqrt{\frac{2W_{k2}}{k_2}} = \sqrt{\frac{2 \times 156860J}{7261\frac{N}{m}}} = 6.6m$$

Part II: Multiple-Choice

Circle your best answer to each question. Any other marks will not be given credit. Each multiple-choice question is worth 3 points for a total of 12 points.

1. A spring of force constant k is stretched a certain distance. It takes twice as much work to stretch a second spring by half this distance. The force constant of the second spring is

   a. k b. $2k$ c. $4k$ d. $8k$

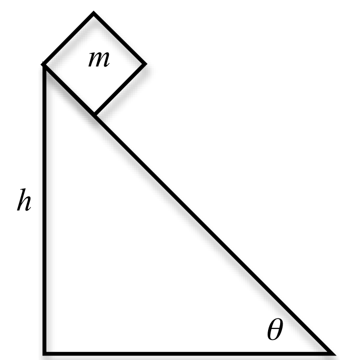
2. A block of mass m sliding down an incline at constant speed is initially at a height h above the ground. The coefficient of friction between the mass and the incline is μ. If the mass continues to slide down the ramp at constant speed, how much energy is dissipated by friction by the time the mass reaches the bottom of the incline?

   a. $\frac{mgh}{\mu}$

   b. mgh

   c. $\frac{\mu mgh}{\sin\theta}$

   d. $mgh\sin\theta$



3. Consider a ball of mass m connected to a spring of stiffness k. If the free end of the spring is rotated in a horizontal circle of radius R at a constant rate so that the ball has a constant speed v, the centripetal force is

a. constant and given by $\frac{mv^2}{R} = kx$.
b. zero because all of the forces are balanced.
c. not constant and is given by $\frac{mv^2}{R} = kx$.
d. unable to be determined since the time it takes the ball to complete one orbit is not known.

4. Suppose that you hang a spring of stiffness k from the ceiling and you hang a mass m on the other end of the spring and you watch as the mass stretches the spring and then comes to rest. Taking the position of the mass at this location as the equilibrium position, you then grab a hold of the mass and pull it down by an amount A measured from the equilibrium position (where the mass was hanging at rest) and release the mass from rest. You measure the time it takes for the mass to comeplete one comeplete oscillation and the resultant motion of the mass on the spring has a period T_1. You then repeat this same experiment (with the same spring and the same suspended mass) but this time you pull the mass down from its equilibrium position by an amount $\frac{A}{2}$ and again release it from rest. The period T_2 of the masses oscillation is given by

a. $T_2 = \frac{T_1}{2}$.
b. $T_2 = 2T_1$.
c. $T_2 = T_1$.
d. $T_2 = \frac{T_1}{\sqrt{2}}$.

Motion in the r = x, y or z-directions

$$r_f = r_i + v_{ir}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{ir} + a_r t$$

$$v_{fr}^2 = v_{ir}^2 + 2a_r\Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles: $C = 2\pi r$, $A = \pi r^2$

Triangles: $A = \tfrac{1}{2}bh$

Spheres: $A = 4\pi r^2$, $V = \tfrac{4}{3}\pi r^3$

Quadratic equation: $ax^2 + bx + c = 0$,

whose solutions are given by: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$\text{magnitude of a vector} = \sqrt{v_x^2 + v_y^2}$$

$$\text{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\ m/s^2$ $\quad G = 6.67\times10^{-11}\ Nm^2/kg^2$

$N_A = 6.02\times10^{23}\ atoms/mole$ $\quad k_B = 1.38\times10^{-23}\ J/K$

$\sigma = 5.67\times10^{-8}\ W/m^2K^4$ $\quad v_{sound} = 343\ m/s$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$$

Heat

$$T_C = \tfrac{5}{9}[T_F - 32]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}(1 + \alpha\Delta T)$$

$$A_{new} = A_{old}(1 + 2\alpha\Delta T)$$

$$V_{new} = V_{old}(1 + \beta\Delta T) : \beta = 3\alpha$$

$$PV = Nk_B T$$

$$\tfrac{3}{2}k_B T = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2_f = \omega^2_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$$

$$a_r = r\omega^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\ \tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\ \tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho gd$$

$$F_B = \rho gV$$

$$A_1v_1 = A_2v_2$$

$$\rho_1A_1v_1 = \rho_2A_2v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho gh_2$$

Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2 \rho v A^2$$