Physics 110

Exam #2

October 21, 2013

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg) \times (2\,\tfrac{m}{s}) = 10\,\tfrac{kg\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 8 points

Problem #1	/20
Problem #2	/28
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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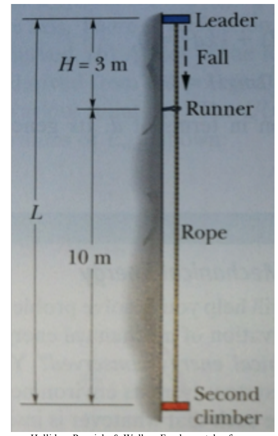
1. Rock-climbing is a fun but perhaps very dangerous sport. Suppose that the lead climber (the leader) has an accident and falls from a height H above a runner (a fixed metal loop through which the rope runs) while a second (lower) climber holds fast to limit the fall and does not move. When the falling leader reaches distance H below the runner, the rope begins to stretch. The stretch is a maximum when the leader has fallen an additional distance d and has come to a stop. The force on the rope is then a maximum at magnitude $F_{\max}$. This is the dangerous part of the fall because $F_{\max}$ could be large enough to snap the rope. For any particular rope, the spring constant k depends on the length of rope L and on the elasticity of the rope material to stretch e_{rope}, considered to be constant. Thus for this example we can write $k = \frac{e_{rope}}{L}$.



Halliday, Resnick, & Walker, Fundamentals of Physics, 7th Ed.

a. What is the maximum stretch of the rope, if $k = 1500\,\frac{N}{m}$ for this rope?

By conservation of energy we have, assuming that $\Delta KE = 0$ is zero because the leader starts and ends at rest and the zero of the gravitational potential energy is at the runner. Thus we have

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(mgy_f - mgy_i\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right) = \left(-mg(H+d) - mgH\right) + \left(\tfrac{1}{2}kd^2 - 0\right)$$

$$\rightarrow -2mgH - mgd + \tfrac{1}{2}kd^2 = 0$$

$$d = \frac{mg \pm \sqrt{(mg)^2 + 4mgkH}}{k} = \frac{\left(80kg \times 9.8\,\frac{m}{s^2}\right) \pm \sqrt{\left(80kg \times 9.8\,\frac{m}{s^2}\right)^2 + 4\left(80kg \times 9.8\,\frac{m}{s^2}\right) \times 1500 \times 3m}}{1500\,\frac{N}{m}} = \begin{cases} 3m \\ -2m \end{cases}$$

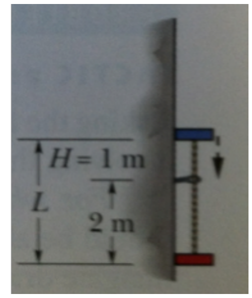
Therefore the maximum stretch is $d = 3m$.

b. What is the magnitude of the maximum force?

The maximum magnitude of the force is given by Hooke's law:

$$F_{\max} = kd = mg + \sqrt{(mg)^2 + 4mgkH} = 1500\,\tfrac{N}{m} \times 3m = 4500N$$

c. Suppose that you have the situation of a short fall. Compared to the maximum force for the longer fall ($F_{\max,long}$), the maximum force for the short fall ($F_{\max,short}$) is



Halliday, Resnick, & Walker, Fundamentals of Physics, 7th Ed.

1. greater than $F_{\max,long}$.
2. less than $F_{\max,long}$.
3. equal to $F_{\max,long}$.
4. unable to be determined from the information given.

Since $F_{\max} = kd = mg + \sqrt{(mg)^2 + 4mgkH}$, we have as L decreases in the expression for k, k increases as does d. Thus the maximum force increases since both d and k. You can also see this using the numbers in the problem and the result of part b.

2. When one thinks of bones in the human body, one doesn't normally consider bone as a compressible material, but human bones compress by different amounts when various loads (forces) are applied. Consider for example, an average adult male femur or thighbone. The femur has an average length of approximately $L = 48cm$ and a circular cross-sectional area with a diameter of $d = 2.3cm$. When a force is applied over the cross-sectional area of the bone, a stress on the bone is produced and the stress is given by $stress = \frac{F}{A}$, where A is the cross-sectional area of the bone. This stress causes the bone to strain under the load and the strain changes the length ΔL of the bone of length L. The strain is defined as $strain = \frac{\Delta L}{L}$. We've used Hooke's law do describe elastic materials, or materials that deform under an applied force and when the applied force is removed return to their original shape. The generalized form of Hooke's law is given as $Stress = Y \times Strain$, where Y is a constant called Young's modulus and Young's modulus for bone is $Y = 16 \times 10^{9} \frac{N}{m^2}$. (*Data are taken from Biomedical Applications of Introductory Physics, by J.A. Tuszynski and J.M. Dixon, Wiley, 2002 and Clinically Oriented Anatomy, Ed. by K.L. Moore & A.F. Dalley, LWW Publishing, 2005*)

   a. Using the generalized expression above for Hooke's law, under normal conditions, by how much do you compress a single femur when you stand upright at rest? Assume that you have a mass of $60kg$.

$$Stress = Y \times Strain \rightarrow \frac{F}{A} = Y \times \frac{\Delta L}{L} \rightarrow \Delta L = \frac{FL}{AY} = \frac{F_W L}{2AY} = \frac{mgL}{2AY}$$

$$\Delta L = \frac{60kg \times 9.8 \frac{m}{s^2} \times 0.48m}{2 \times \left(\pi\left(\frac{.0234m}{2}\right)^2\right) \times 16 \times 10^{9} \frac{N}{m^2}} = 2.1 \times 10^{-5} m = 0.021mm$$

   Here we are looking at calculating the change in length of a single femur. Each leg supports only one-half of your total weight.

   b. Suppose that you were to somehow compress your femur by $6mm$ (maybe by falling while rock climbing and by the way this amount is more than enough to break your femur). What is the ratio of the force needed to break your leg to your weight?

$$Stress = Y \times Strain \rightarrow \frac{F}{A} = Y \times \frac{\Delta L}{L} \rightarrow F = \frac{YA\Delta L}{L}$$

$$F = \frac{16 \times 10^{9} \frac{N}{m^2} \times \left(\pi\left(\frac{.0234m}{2}\right)^2\right) \times 0.006m}{0.48m} = 8.6 \times 10^{4} N = \alpha F_W$$

$$\alpha = \frac{F}{F_W} = \frac{8.6 \times 10^{4} N}{60kg \times 9.8 \frac{m}{s^2}} = 141$$

c. Starting from rest, through what height would you have to fall landing straight-legged on your feet, so that you could compress a femur by $6mm$? (Hint: Recall Hooke's law from class and you will need to apply this to the generalized form of Hooke's law above to determine a value for the stiffness of your femur.)

Using conservation of energy we have:

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(mgy_f - mgy_i\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right)$$

$$0 = \left(\frac{mg}{2}(-d) - \frac{mg}{2}h\right) + \tfrac{1}{2}k(-d)^2$$

$$h = \frac{kd^2}{mg} - d = \frac{1.43\times10^7\,\frac{N}{m}\times(0.006m)^2}{60kg\times9.8\,\frac{m}{s^2}} - (0.006m) = 0.88m$$

where the effective spring constant of a femur has been determined from the generalized form of Hooke's law,

$$Stress = Y\times Strain \rightarrow \frac{F}{A} = Y\times\frac{\Delta L}{L} \rightarrow F = \frac{AY}{L}\Delta L = k\Delta L$$

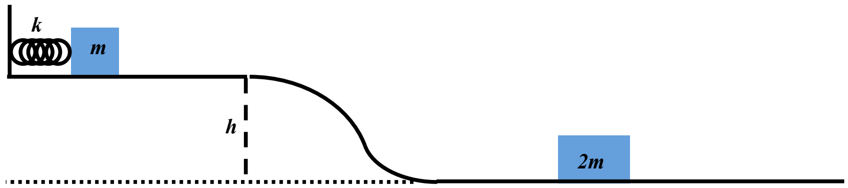
$$k = \frac{AY}{L} = \frac{\left(\pi\left(\frac{.0234m}{2}\right)^2\right)\times16\times10^9\,\frac{N}{m^2}}{0.48m} = 1.43\times10^7\,\frac{N}{m}$$

d. Suppose that instead of landing straight-legged as in the previous part you bend your knees while landing. In this case

1. the work done in bringing you to rest would be the same, but the force on your femur would be less because you decelerated to rest over a larger distance.
2. the work done in bringing you to rest would be the same, but the force on your femur would be greater because you decelerated to rest over a smaller distance.
3. the work done in bringing you to rest would be the greater, but the force on your femur would be less because you decelerated to rest over a larger distance.
4. the work done in bringing you to rest would be the smaller, but the force would be greater on your femur because you decelerated to rest over a smaller distance.

The work done is given by $W = \Delta KE = (-mgh) = F\Delta y$. Dropping from the same height produces the same change in kinetic energy. But bending your legs makes you decelerate over a much larger distance. Therefore the force on your femurs is less.

3. Suppose that you have a block of mass m attached to a spring of stiffness k and that the spring is compressed by a distance x from the equilibrium position of the spring. The mass is released from rest and when the block reaches the equilibrium position of the spring, the mass loses contact with the spring and the block slides towards and then down the hill of height h as shown below. Assume that all surfaces are frictionless.



a. What are the speed of when the block looses contact with the spring and when it is at the bottom of the hill in terms of the given quantities?

By conservation of energy we have:

$$spring: \quad \Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right) = v_{top} = \sqrt{\frac{k}{m}}x$$

$$hill: \quad \Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(mgy_f - mgy_i\right) + \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right)$$

$$0 = \left(0 - mgh\right) + \left(\tfrac{1}{2}mv_{bottom}^2 - \tfrac{1}{2}mv_{top}^2\right) \rightarrow v_{bottom} = \sqrt{\frac{k}{m}x^2 + 2gh}$$

b. Suppose that at the bottom of the hill, the block of mass m makes an elastic head on collision with a second block of mass $2m$ sitting at rest on a horizontal surface. What are the velocities of the two blocks after the collision? (***You may not use the formulas developed in class. Rather, you need to derive the actual results.***)

Using conservation of momentum and kinetic energy we have:

$$p_{ix} = p_{fx} \rightarrow mv_i = mv_{1f} + 2mv_{2f} \Rightarrow v_i = v_{1f} + 2v_{2f}$$

$$KE_i = KE_f \rightarrow \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_{1f}^2 + \tfrac{1}{2}(2m)v_{2f}^2 \Rightarrow v_i^2 = v_{1f}^2 + 2v_{2f}^2$$

where we define $v_{bottom} = v_i$. From conservation of momentum well solve for say $v_{1f} = v_i - 2v_{2f}$ and substituting the result in to the equation for conservation of kinetic energy to determine v_{2f}.

We get, $v_i^2 = \left(v_i - 2v_{2f}\right)^2 + 2v_{2f}^2 = v_i^2 - 4v_{2f}v_i + 4v_{2f}^2 + 2v_{2f}^2 \Rightarrow v_{2f} = \tfrac{4}{6}v_i$.

Therefore $v_{1f} = v_i - 2v_{2f} = v_i - 2\left(\tfrac{4}{6}v_i\right) = -\tfrac{1}{3}v_i$.

c. Defining y as the height the block of mass m rises after the collision, after the collision the block of mass m will

1. rise back up the entire hill sliding back toward the spring so that $y = h$.
2. will rise up the hill and come momentarily to rest at a point $h < y < \frac{h}{2}$.
3. will rise up the hill and come momentarily to rest at a point $\frac{h}{2} < y < 0$.
4. not rise up the hill at all so that $y = 0$.

Since the surface is frictionless, the block will rise to a height greater than zero. After the collision the block of mass m has a kinetic energy given by: $KE_f = \frac{1}{2}mv_{1f}^2 = \frac{1}{2}m\left(-\frac{1}{3}v_i\right)^2 = \frac{1}{9}KE_i$. Since this is less than the masses initial kinetic energy the block will not rise back up to the original height. Since kinetic energy is conserved the block of mass $2m$ will get of the remaining $\frac{8}{9}KE_i$, so the block of mass will stop at a height $\frac{h}{2} < y < 0$.

d. Suppose that the block of mass m collided with the block of mass $2m$ but in this case the two masses stuck together after the collision. The percent of the kinetic energy lost in the collision is given as

1. 0%
2. 20%
3. 33%
4. 67%

$$\% = \left[\frac{\Delta KE}{KE_i}\right] \times 100\% = \left[\frac{KE_i - KE_f}{KE_i}\right] \times 100\% = \left[1 - \frac{KE_f}{KE_i}\right] \times 100\%$$

$$\% = \left[1 - \frac{\frac{1}{2}(3m)\left(\frac{m}{3m}v_i\right)^2}{\frac{1}{2}mv_i^2}\right] \times 100\% = \left[1 - \left(\tfrac{1}{3}\right)\right] \times 100\% = 67\%$$

where the speed of the system after the collision was calculated using conservation of momentum.

Useful formulas:

Motion in the r = x, y or z-directions

$r_f = r_0 + v_{0r}t + \frac{1}{2}a_r t^2$

$v_{fr} = v_{0r} + a_r t$

$v_{fr}^2 = v_{0r}^2 + 2a_r\Delta r$

Uniform Circular Motion

$a_r = \frac{v^2}{r}$

$F_r = ma_r = m\frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0,$

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$magnitude\ of\ a\ vector = \sqrt{v_x^2 + v_y^2}$

$direction\ of\ a\ vector \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\ m/s^2 \qquad G = 6.67\times 10^{-11}\ Nm^2/kg^2$

$N_A = 6.02\times 10^{23}\ atoms/mole \qquad k_B = 1.38\times 10^{-23}\ J/K$

$\sigma = 5.67\times 10^{-8}\ W/m^2K^4 \qquad v_{sound} = 343\ m/s$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = \Delta E = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{dissipative}$

Heat

$T_C = \frac{5}{9}\left[T_F - 32\right]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T) : \beta = 3\alpha$

$PV = Nk_BT$

$\frac{3}{2}k_BT = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta :\ v = r\omega :\ a_t = r\alpha$

$a_r = r\omega^2$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1v_1 = A_2v_2$

$\rho_1A_1v_1 = \rho_2A_2v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = \sqrt{\frac{k}{m}} = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = x_{\max}\sin(\omega t)\ \ or\ \ x_{\max}\cos(\omega t)$

$v(t) = v_{\max}\cos(\omega t)\ \ or\ \ -v_{\max}\sin(\omega t)$

$a(t) = -a_{\max}\sin(\omega t)\ \ or\ \ -a_{\max}\cos(\omega t)$

$v_{\max} = \omega x_{\max};\ \ a_{\max} = \omega^2 x_{\max}$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2\rho vA^2$

Sound

$v = f\lambda = (331 + 0.6T)\ \frac{m}{s}$

$\beta = 10\log\frac{I}{I_0};\ \ I_o = 1\times 10^{-12}\ \frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L};\ \ f_n = nf_1 = n\frac{v}{4L}$