Physics 110

Exam #2

May 8, 2013

Name__________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg) \times \left(2\frac{m}{s}\right) = 10\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 8 points

Problem #1	/36
Problem #2	/36
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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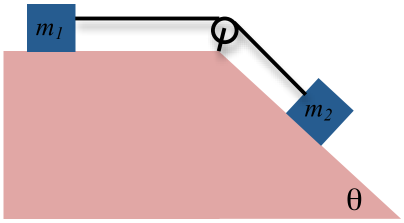
1. Two blocks $m_1 = 2kg$ and $m_2 = 3kg$ are connected by a light string that passes over a frictionless massless pulley and the system is used to study forces and the accelerations that the forces produce. Friction is present on all surfaces with coefficient of friction $\mu_k = 0.4$ and the ramp is inclined at $\theta = 47^o$.

   a. If m_2 is released from rest, what will be the magnitude of the tension force in the string and the magnitude of the acceleration of the system?



Assuming a standard Cartesian CS for block 1 and a titled CS for block 2, where the +x-axis points to the right for block #1 down the incline for block #2 we have for the forces:

m_1 :

$$\sum F_x : \quad F_T - F_{fr,1} = F_T - \mu_k F_{N,1} = F_T - \mu_k m_1 g = m_1 a_x = m_1 a$$

$$\sum F_y : \quad F_{N,1} - F_{W,1} = F_{N,1} - m_1 g = m_1 a_y = 0 \to F_{N,1} = m_1 g$$

m_2 :

$$\sum F_x : \quad -F_T - F_{fr,2} + F_{W,2,x} = -F_T - \mu_k F_{N,2} + F_{W,2} \sin\theta = -F_T - \mu_k m_2 g \cos\theta + m_2 g \sin\theta = m_2 a_x = m_2 a$$

$$\sum F_y : \quad F_{N,2} - F_{W,2,y} = F_{N,2} - m_2 g \cos\theta = m_2 a_y = 0 \to F_{N,2} = m_2 g \cos\theta$$

Adding the two equations for the x-directions we eliminate the tension force and can solve for the acceleration of the system. We have

$$-\mu_k m_1 g - \mu_k m_2 g \cos\theta + m_2 g \sin\theta = (m_1 + m_2) a$$

$$\therefore a = \frac{m_2 g \sin\theta - \mu_k m_1 g - \mu_k m_2 g \cos\theta}{m_1 + m_2}$$

$$a = \frac{\left(3kg \times 9.8 \tfrac{m}{s^2} \times \sin 47\right) - \left(0.4 \times 2kg \times 9.8 \tfrac{m}{s^2}\right) - \left(0.4 \times 3kg \times 9.8 \tfrac{m}{s^2} \times \cos 47\right)}{5kg}$$

$$a = 1.13 \tfrac{m}{s^2}$$

And from either horizontal equation we can calculate the tension force. Thus,

$$F_T = m_1 a + \mu_k m_1 g = \left(2kg \times 1.13 \tfrac{m}{s^2}\right) + \left(0.4 \times 2kg \times 9.8 \tfrac{m}{s^2}\right) = 10.1N$$

b. How long will it take and what will be the speed of m_2 if m_2 slides down the ramp a distance $d = 1m$?

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2 \rightarrow t = \sqrt{\frac{2x_f}{a_x}} = \sqrt{\frac{2 \times 1m}{1.13\frac{m}{s^2}}} = 1.33s$$

$$v_{fx} = v_{ix} + a_x t = 1.13\tfrac{m}{s^2} \times 1.33s = 1.50\tfrac{m}{s} \text{ or}$$
$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x \rightarrow v_{fx} = \sqrt{2a_x\Delta x} = \sqrt{2 \times 1.13\tfrac{m}{s^2} \times 1m} = 1.50\tfrac{m}{s}$$

c. Suppose that you to add some springs to your experiment above. But before you can use the springs you need to determine its stiffness. To do this you take two identical springs and suspend one from ceiling and to this spring you hang a series of masses. You then measure the displacement of the spring from its equilibrium position for each mass and then make a plot of the mass versus the stretch of the spring. The slope of the resulting plot is equal to

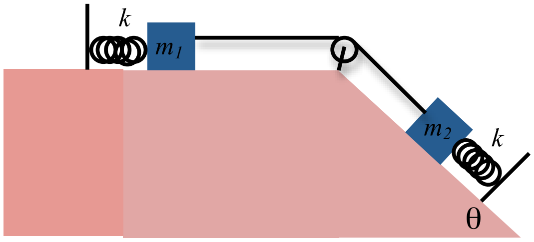
1. k.
2. $\frac{1}{k}$.
3. $\frac{g}{k}$.

[✓ 4.] $\frac{k}{g}$.

A plot of the mass versus the stretch has mass on the vertical axis and the stretch on the horizontal axis. Thus we have

$$\sum F_y: \quad ky - mg = ma_y = 0 \rightarrow ky = mg \Rightarrow m = \frac{k}{g}y \rightarrow \text{mass} = \text{slope} \times \text{stetch}$$

d. Consider the following system in which the same masses are used in the same order, but now each mass is connected to one of the identical springs from above and each spring has stiffness $k = 100\,\frac{N}{m}$. If m_2 is again released from rest with the springs at their equilibrium length, what will be the maximum extension of the spring connected to m_1? (Hint: Use energy methods to solve this problem and remember that there is friction on all of the surfaces.)



Using energy methods we have (where at maximum extension of the left spring (and maximum compression of the right spring) the velocity of the blocks being zero) and block #1 having no change in gravitational potential energy

$$\Delta E = W_{fr,1} + W_{fr,2} = \Delta KE_1 + \Delta U_{g,1} + \Delta KE_2 + \Delta U_{g,2} + \Delta U_{s,left} + \Delta U_{s,right}$$

$$-\mu_k m_1 g x - \mu_k m_2 (g\cos\theta)x = (0-0) + (0-0) + (0-0) + (0 - m_2(g\sin\theta)x) + \left(\tfrac{1}{2}kx^2 - 0\right) + \left(\tfrac{1}{2}kx^2 - 0\right)$$

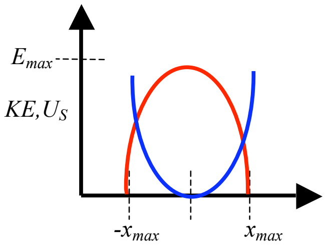
$$0 = (m_2 g\sin\theta - \mu_k m_1 g - \mu_k m_2 g\cos\theta - kx)x$$

$$\rightarrow \begin{cases} x_{\min} = 0 \\ x_{\max} = \dfrac{m_2 g\sin\theta - \mu_k m_1 g - \mu_k m_2 g\cos\theta}{k} \end{cases}$$

$$x_{\max} = \frac{\left(3kg \times 9.8\,\frac{m}{s^2} \times \sin 47\right) - \left(0.4 \times 2kg \times 9.8\,\frac{m}{s^2}\right) - \left(0.4 \times 3kg \times 9.8\,\frac{m}{s^2} \times \cos 47\right)}{100\,\frac{N}{m}} = 0.056m = 5.6cm$$

e. In the last experiment you had the springs attached to the blocks. Suppose that you remove the springs from each mass and that you take one of the springs and place it on a horizontal frictionless surface. You attach one end to a wall and to the other end you put mass m_2. You then stretch the spring by an amount $x_{\max}$ from equilibrium (defined to be $x_{eq} = 0$). When you release the mass from rest, the kinetic and potential energies change according to which of the following graph? (Key: Red = KE and Blue = U$_s$)

[✓ 1.]



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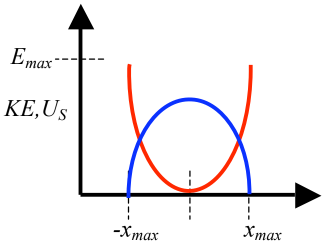

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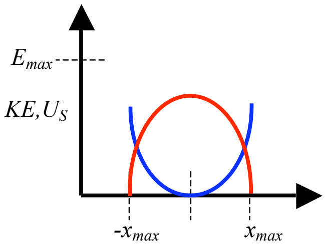

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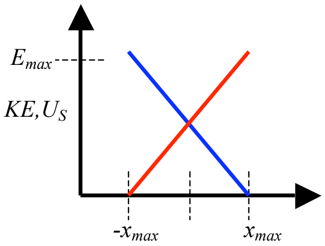


Since there's no friction, at any point the sum of the kinetic and potential energies have to sum to the total energy by:

$$\Delta E = 0 = \Delta Ke + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}mv^2 - 0\right) + \left(\tfrac{1}{2}kx^2 - \tfrac{1}{2}kx_{\max}^2\right)$$

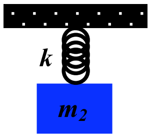
$$\rightarrow \tfrac{1}{2}kx_{\max}^2 = E_{\max} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

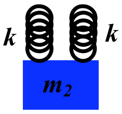
f. As a last experiment with the springs you decide to perform the two experiments shown below. On the left is one of the springs with stiffness k attached to mass m_2 and this stretches the spring from its relaxed length. Now, suppose that you perform the experiment on the right in which you attach both springs to the mass m_2 as shown. The effective stiffness of the system on the right is defined as k_{eff}. The effective spring constant k_{eff} is given as

1. $k_{eff} = k$.
2. $k_{eff} = \frac{k}{2}$.

[✓ 3.] $k_{eff} = 2k$.

4. $k_{eff} = \frac{1}{2k}$.





$$\sum F_y : \quad ky + ky - mg = ma_y = 0 \rightarrow 2ky = k_{eff}\,y = mg \Rightarrow k_{eff} = 2k$$

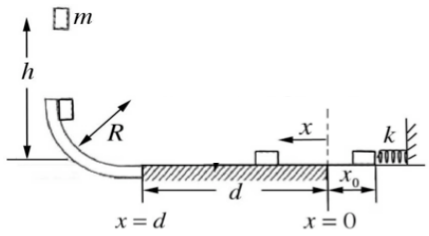
2. An object of mass m is released from rest from a spring of stiffness that has been compressed by x_o. After leaving the spring at $x_{eq} = 0$ when the spring is unstretched the mass travels a distance along a horizontal track that has a coefficient of kinetic friction μ_k. Following the horizontal section of track the object enters a ¼-turn loop of radius R. After exiting the ¼-turn loop the mass travels vertically upward to a maximum height h.

   a. What is the speed of the mass at the end of the horizontal section of track just before the ¼-turn loop? Express your answers in terms of the variables stated in the problem or in the diagram below.

From x_0 to zero there is no friction, so we have

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}mv^2 - 0\right) + \left(0 - \tfrac{1}{2}kx_0^2\right)$$

$$v = \sqrt{\frac{k}{m}}x_0$$



From zero to d there is friction, so friction does work and this changes the kinetic energy of the mass. We have

$$\Delta E = W_{fr} = F_{fr}d\cos(180) = -\mu_k mgd = \Delta KE = \left(\tfrac{1}{2}mv_d^2 - \tfrac{1}{2}mv^2\right)$$

$$v_d = \sqrt{\frac{k}{m}x_0^2 - 2\mu_k gd}$$

   b. What is the maximum height h? Express your answers in terms of the variables stated in the problem or in the diagram above.

The remainder of the track is frictionless and using energy methods we have

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(0 - \tfrac{1}{2}mv_d^2\right) + \left(mgh - 0\right)$$

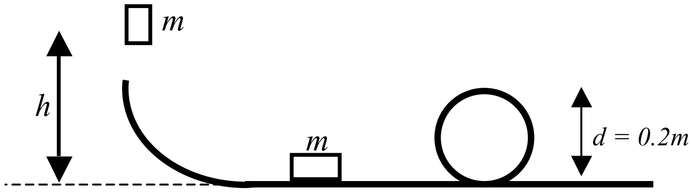
$$h = \frac{v_d^2}{2g} = \frac{\frac{k}{m}x_0^2 - 2\mu_k gd}{2g}$$

c. Once the mass reaches its maximum height it falls back down and reconnects with the ¼-turn loop and then slides back across the horizontal section of track and strikes compresses the spring. If the process were to repeat itself where the mass compresses the spring, the spring launches the mass and the mass travels across the horizontal section of track and around the ¼-turn loop and rises again to a new maximum height, compared to the maximum height h found in part b, this new maximum height h_{new} follows the relation

1. $h_{new} > h$.
2. $h_{new} = h$.

[✓ 3.] $h_{new} < h$. Since the mass travels (a few times?) across the horizontal portion with friction it loses energy each time and thus will not travel to the same maximum height successively.

d. Suppose instead of the mass repeating the motion as in part c, that at the very instant the mass reaches its maximum height as in part b, someone replaces the original track with the one shown below. The mass will fall vertically and enter the ¼-turn loop and when the falling mass becomes horizontal, the falling mass makes a collision with a second identical mass initially at rest. After the collision the two masses stick together. What is the fraction of the kinetic energy lost in the collision? Assume that the entire track is frictionless.



Since there are no external forces doing work between the maximum height and the point where the masses collide, energy is conserved and the speed is the same as it was in part a, namely

$$\Delta E = 0 = \Delta KE + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}mv_{ground}^2 - 0\right) + \left(0 - mgh\right) \to v_{ground}^2 = 2gh = \frac{k}{m}x_0^2 - 2\mu_k gd$$

The objects collide and momentum is conserved.

$$p_{ix} = p_{fx} \to mv_{ground} = 2mV \to V = \frac{v_{ground}}{2} = \tfrac{1}{2}\sqrt{\frac{k}{m}x_0^2 - 2\mu_k gd}$$

The fraction of the energy lost:

$$fraction = \frac{KE_i - KE_f}{KE_i} = \frac{\frac{1}{2}mv_{ground}^2 - \frac{1}{2}(2m)V^2}{\frac{1}{2}mv_{ground}^2} = \frac{v_{ground}^2 - 2V^2}{v_{ground}^2}$$

$$= 1 - 2\left(\frac{V^2}{v_{ground}^2}\right) = 1 - 2\left(\frac{\frac{1}{2}\sqrt{\frac{k}{m}x_0^2 - 2\mu_k gd}}{\sqrt{\frac{k}{m}x_0^2 - 2\mu_k gd}}\right)^2 = \frac{1}{2}$$

$$\therefore\ fraction = 0.5$$

e. After the collision, the combined system then travels toward a loop-the-loop portion of the track. The combined system then travels around the loop-the-loop portion of the track. The work done by gravity on the two-mass system between the bottom and the top of the loop is given by

1. $W_g = 2mgd$.
[✓ 2.] $W_g = -2mgd$. $\quad W_g = F_g \Delta y \cos(180) = -2mgd$
3. $W_g = 2mg\cos d$.
4. $W_g = -2mg\cos d$.
5. $W_g = -2mgR$

f. As the two-mass system travels in one complete loop around the loop-the-loop the work done by the normal force is given by

1. $W_N = -2\pi Rmg$.
2. $W_N = 2\pi Rmg$.
3. $W_N = 2\pi mv^2$.
4. $W_N = -2\pi mv^2$.
[✓ 5.] $W_N = 0$. $\quad W_N = F_N \Delta y \cos(90) = 0$; F_N is always perpendicular to the displacement, which is tangent to the circle.

Useful formulas:

Motion in the r = x, y or z-directions

$$r_f = r_0 + v_{0r}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{0r} + a_r t$$

$$v_{fr}^{\ 2} = v_{0r}^{\ 2} + 2a_r \Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \tfrac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \tfrac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$\textit{magnitude of a vector} = \sqrt{{v_x}^2 + {v^2}_y}$$

$$\textit{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$$g = 9.8\,{}^{m}\!/\!_{s^2} \qquad G = 6.67\times10^{-11}\,{}^{Nm^2}\!/\!_{kg^2}$$

$$N_A = 6.02\times10^{23}\,{}^{atoms}\!/\!_{mole} \qquad k_B = 1.38\times10^{-23}\,{}^{J}\!/\!_{K}$$

$$\sigma = 5.67\times10^{-8}\,{}^{W}\!/\!_{m^2K^4} \qquad v_{sound} = 343\,{}^{m}\!/\!_{s}$$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$$

Heat

$$T_C = \tfrac{5}{9}\left[T_F - 32\right]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}\left(1 + \alpha\Delta T\right)$$

$$A_{new} = A_{old}\left(1 + 2\alpha\Delta T\right)$$

$$V_{new} = V_{old}\left(1 + \beta\Delta T\right):\ \beta = 3\alpha$$

$$PV = Nk_BT$$

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta:\ v = r\omega:\ a_t = r\alpha$$

$$a_r = r\omega^2$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho g d$$

$$F_B = \rho g V$$

$$A_1v_1 = A_2v_2$$

$$\rho_1A_1v_1 = \rho_2A_2v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho gh_2$$

Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2f^2\rho vA^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0};\quad I_o = 1\times10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L};\quad f_n = nf_1 = n\frac{v}{4L}$$