Physics 110

Exam #3

November 9, 2026

Name_____

Please read and follow these instructions carefully:

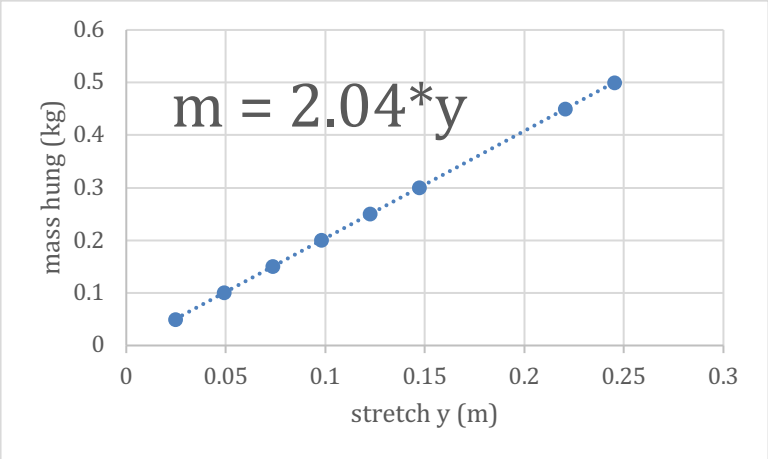
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. Erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|\vec{p}| \approx m|\vec{v}| = (5kg) \times \left(2\,\frac{m}{s}\right) = 10\,\frac{kg \cdot m}{s}$
- Give standard SI units with your results.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Formulas or constants not explicitly given on the formula sheet need to be derived before they can be used or full credit will not be given even if the formula or answer is correct.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 5 points.

Problem #1	/25
Problem #2	/25
Problem #3	/25
Total	/75

I affirm that I have carried out my academic endeavors with full academic honesty.

_____

1. A spring of stiffness k is suspended from the ceiling. Various masses m (in kilograms) are hung from the end of the spring and the stretch y (in meters) of the spring from equilibrium is measured. A plot of the mass hung from the end of the spring versus the stretch of the spring is constructed and shown on the right. A best fit line to the data is also provided.



a. From the plot and a carefully labeled free-body diagram, what is the stiffness k of the spring? If you cannot determine a value for the stiffness, use $k = 10\frac{N}{m}$ in any calculations below.

$$F_S - F_W = ma_y = 0 \rightarrow F_S = F_W \rightarrow ky = mg \rightarrow m = \frac{k}{g}y = \left(2.04\frac{kg}{m}\right) \cdot y$$

$$\frac{k}{g} = 2.04\frac{kg}{m} \rightarrow k = 2.04\frac{kg}{m} \cdot g = 2.04\frac{kg}{m} \cdot 9.8\frac{m}{s^2} = 20\frac{N}{m}$$

b. The spring is removed from the ceiling and laid horizontally with the left end connected to a wall and the right end to a block of mass $m = 500g$ as shown on the right. The block of mass m is compressed against the spring by an amount $x = 75cm$ measured from the spring's equilibrium position taken to be zero. The block of mass m is launched from rest and when the spring reaches its equilibrium position the mass leaves the end of the spring. What is the launch speed of the mass m spring if the horizontal surface is frictionless?

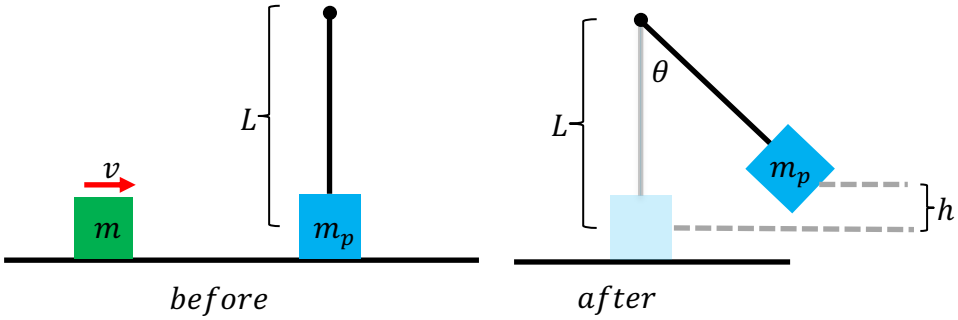


$$\Delta E_{sys} = W_{fr} = 0 = \Delta K_R + \Delta K_T + \Delta U_S + \Delta U_g = \Delta K_T + \Delta U_s$$

$$0 = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right) = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}kx_i^2$$

$$v_f = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{20\frac{N}{m}}{0.5kg}} \cdot 0.75m = 4.7\frac{m}{s}$$

c. The block of mass m slides across the horizontal frictionless surface and collides with a stationary pendulum of mass $m_p = 1.5kg$ and length $L = 30cm$. After the collision *the pendulum* rises through a height h by sweeping out an angle $\theta = 37^0$ measured with respect to the vertical. The mass and the pendulum do not stick together after the collision. What was the speed of *the pendulum arm* immediately after the collision?



$$\Delta E_{sys} = W_{fr} = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s = \Delta K_T + \Delta U_g$$

$$0 = \left(\tfrac{1}{2}m_p v_f^2 - \tfrac{1}{2}m_p v_i^2\right) + \left(m_p g y_f - m_p g y_i\right) = -\tfrac{1}{2}m_p v_i^2 + m_p g h$$

$$v_{im_p} = \sqrt{2gh} = \sqrt{2hL(1-\cos\theta)} = \sqrt{2 \cdot 9.8\tfrac{m}{s^2} \cdot 0.3m \cdot (1-\cos 37)} = 1.1\tfrac{m}{s}$$

d. What is the magnitude of and the direction for the velocity of the *block of mass m* immediately after the collision?

Taking to the right as the positive direction, we have:

$$\Delta p_x = 0 \rightarrow p_{ix} = p_{fx} \rightarrow mv_{im} = mv_{fm} + m_p v_{im_p}$$

$$v_{fm} = \frac{mv_{im} - m_p v_{im_p}}{m} = \frac{0.5kg \cdot 4.7\frac{m}{s} - 1.5kg \cdot 1.1\frac{m}{s}}{0.5kg} = 1.4\frac{m}{s}$$

Since this is positive, the block of mass moves to the right in the positive x-direction.

e. Was the collision between the block and the pendulum arm elastic? You need to do a calculation to answer this and be sure to explain the result you get in terms of whether the collision is elastic or not.

To determine whether the collision was elastic or not, we'll calculate ΔK. If ΔK is zero the collision is elastic, otherwise it is inelastic.

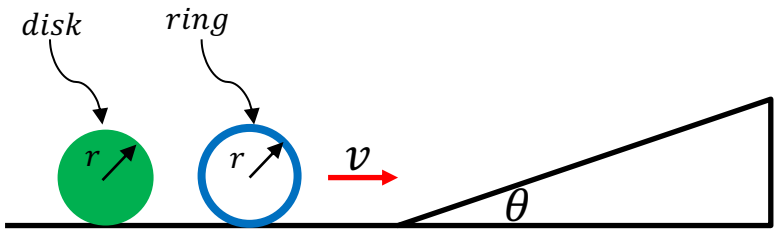
$$\Delta K = K_f - K_i = \left(\frac{1}{2}mv_{fm}^2 + \frac{1}{2}m_p v_{fm_p}^2\right) - \frac{1}{2}mv_{im}^2$$

$$\Delta K = \left(\frac{1}{2} \cdot 0.5kg \cdot \left(1.4\frac{m}{s}\right)^2 + \frac{1}{2} \cdot 1.5kg \cdot \left(1.1\frac{m}{s}\right)^2\right) - \frac{1}{2} \cdot 0.5kg \cdot \left(4.7\frac{m}{s}\right)^2$$

$$\Delta K = -4.1J$$

Since this is not zero the collision was inelastic.

2. A uniform disk and ring each of mass m and radius r roll without slipping along a horizontal surface each with a constant speed v as shown below. They both then encounter a ramp inclined at angle θ measured with respect to the horizontal and continue to roll without slipping up the ramp until they momentarily come to rest before rolling back down the ramp.



a. Explain how the angular acceleration vector $\vec{\alpha}$ points with respect to the angular velocity vector $\vec{\omega}$ as either object rolls up the incline. Be sure to clearly explain how the vectors must point relative to each other and be sure to use standard directions for vectors for rotating quantities.

Since the objects are rotating clockwise, the angular velocity vector points into the page and we take this a negative for the direction of the vector.

As the objects roll up the ramp, they are slowing down translationally and rotationally. To slow down rotationally, this means that the angular acceleration must point opposite to the angular velocity. Since the angular velocity points into the page, the angular acceleration must point out of the page, leading to a counterclockwise angular acceleration.

Thus, the angular velocity and angular acceleration vectors must point opposite to each other for the object to slow down rotationally.

This is also true translationally. Since the translational velocity is up the incline, the translational acceleration must point down the incline, opposite the translational velocity.

b. As either object rolls up the incline without slipping explain which way static friction between the object and the incline must point and why for the objects to slow down and come to rest momentarily.

Since the angular acceleration must point out of the page, leading to a counterclockwise rotation, the net torque must also point out of the page and produce a counterclockwise rotation of the objects about their centers. To achieve this, the static frictional force must point up the ramp.

c. From a carefully labeled free-body diagram what is the expression for the magnitude of the static frictional force expressed in terms of m, r, I, θ, and any constants you may need? Do not enter any expressions for the moment of inertia for either the disk or ring. Leave their moments of inertia as I in the expression for the static frictional force as the expression for the static frictional force is the same for both the disk and ring.

Taking up the ramp as the positive x-direction and perpendicular to the ramp as the positive y-direction we have:

From the forces in the x-direction:

$$-F_{w,x} + F_{fr,s} = ma_x = -ma \rightarrow F_{fr,s} = -ma + F_{w,x} = mg\sin\theta - ma$$

From the torques about the axis of rotation:

$$\tau_{net} = \tau_{F_N} + \tau_{F_W} + \tau_{F_{fr}} = \tau_{F_{fr,s}} = rF_{fr,s}\sin 90 = rF_{fr,s} = I\alpha = I\left(\frac{a}{r}\right) \rightarrow a = \frac{r^2}{I}F_{fr,x}$$

$$\rightarrow F_{fr,s} = mg\sin\theta - ma = mg\sin\theta - \frac{mr^2}{I}F_{fr,s}$$

$$F_{fr,s} = \frac{mg\sin\theta}{1 + \frac{mr^2}{I}}$$

d. What is the magnitude of the translational acceleration of either object up the incline expressed in terms of m, r, I, θ, and any constants you may need? Again, leave their moments of inertia as I as the expression for the translational acceleeration is the same for both the disk and ring.

$$a = \frac{r^2}{I}F_{fr,x} = \frac{r^2}{I}\left(\frac{mg\sin\theta}{1 + \frac{mr^2}{I}}\right) = \frac{g\sin\theta}{1 + \frac{I}{mr^2}}$$

e. Which object, the disk or ring, goes the farthest up the ramp before coming to rest? You will need to do a calculation to answer this and be sure to explain your result. That is, determine the expression for the displacement Δx along the ramp for both the disk and ring and see which is greater. Simply saying disk or ring will earn no credit.

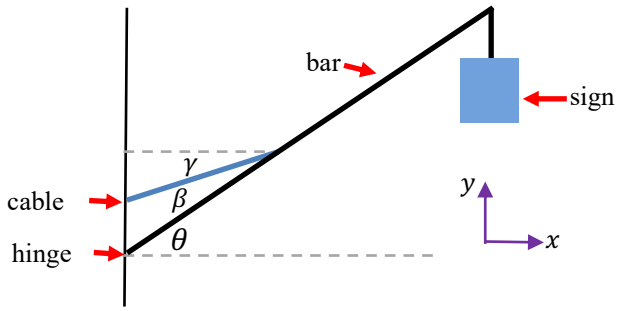
$$v_{fx}^2 = 0 = v_{ix}^2 - 2a_x\Delta x \rightarrow \Delta x = \frac{v_{ix}^2}{2a_x} = \left(1 + \frac{I}{mr^2}\right) \cdot \frac{v^2}{2g\sin\theta}$$

$$\Delta x_{ring} = \left(1 + \frac{mr^2}{mr^2}\right) \cdot \frac{v^2}{2g\sin\theta} = \frac{v^2}{g\sin\theta}$$

$$\Delta x_{disk} = \left(1 + \frac{\frac{1}{2}mr^2}{mr^2}\right) \cdot \frac{v^2}{2g\sin\theta} = \frac{3v^2}{4g\sin\theta} = \frac{3}{4}\Delta x_{ring}$$

Since $\Delta x_{disk} < \Delta x_{ring}$ the ring goes farther up the incline than the disk.

3. A uniform bar of weight $F_{WB} = 500N$ and length $L = 5m$ is connected to a wall by a hinge on its left end and on its right end has a sign of weight $F_{Ws} = 110N$ suspended. To keep the system from rotating, a cable is attached to the wall on its left end and on its right end to the bar at a point 40% of the length of the bar measured from the hinge. The cable makes an angle $\beta = 15^0$ between it and the bar and an angle γ measured with respect to the horizontal as shown below. The bar is angled at an angle $\theta = 55^0$measured with respect to the horizontal.



a. Using the coordinate system shown in the figure, and from a carefully labeled free-body diagram what are the expressions for the sum of the forces in the x- and y-directions? Your expressions should contain no numbers (except constants like $\frac{1}{2}$ or 90^0, etc.), just symbols at this point. Express your answers in terms of the angle γ.

Forces in the x-direction:

$$F_{Rx} - F_{Tx} = F_{Rx} - F_T \cos\gamma = ma_x = 0$$

$$\rightarrow F_{Rx} = F_T \cos\gamma$$

Forces in the y-direction:

$$F_{Ry} - F_{wB} - F_{wS} - F_{Ty} = F_{Ry} - F_{wB} - F_{wS} - F_T \sin\gamma = ma_y = 0$$

$$\rightarrow F_{Ry} = F_{wB} + F_{wS} + F_T \sin\gamma$$

b. From a carefully labeled free-body diagram, what is the expression for the sum of the torques about the hinge and from this expression, what is the magnitude of the tension force in the cable? Your expression for the sum of the torques should also contain no numbers (except constants like $\frac{1}{2}$ or 90^0, etc.), just symbols at this point. Your answer for the magnitude of the tension force will be a number.

$$\tau_{net} = \tau_{F_T} + \tau_{F_{Wb}} + \tau_{F_{Ws}} = I_{system}\alpha = 0$$

$$r_{F_T}F_T \sin\beta - r_{F_{Wb}}F_{Wb}\sin(90-\theta) - r_{F_{Ws}}F_{Ws}\sin(90-\theta) = 0$$

$$0.4LF_T \sin\beta - 0.5LF_{Wb}\sin(90-\theta) - LF_{Ws}\sin(90-\theta) = 0$$

$$F_T = \frac{0.5F_{Wb}\sin(90-\theta) + F_{Ws}\sin(90-\theta)}{0.4\sin\beta}$$

$$F_T = \frac{0.5 \cdot 500N \cdot \sin(35) + 110N \cdot \sin(35)}{0.4\sin 15} = 1995N$$

c. What is the magnitude of and the direction for the reaction force in the hinge, measured with respect to the horizontal?

From the forces in the x-direction:

$$F_{Rx} = F_T\cos\gamma = F_T\cos(\theta-\beta) = 1995N \cdot \cos(55-15) = 1528N$$

From the forces in the y-direction:

$$F_{Ry} = F_{wB} + F_{wS} + F_T\sin\gamma = 500N + 110N + 1994N \cdot \sin(55-15) = 1892N$$

The magnitude of the reaction force of the hinge:

$$F_R = \sqrt{F_{Rx}^2 + F_{ry}^2} = \sqrt{(1528N)^2 + (1892N)^2} = 2432N$$

The direction of the reaction force of the hinge:

$$\tan\phi = \frac{F_{Ry}}{F_{Rx}} \rightarrow \phi = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{1892N}{1528N}\right) = 51^0$$

d. Suppose the bar was lowered so that $\theta = 30^0$ measured with respect to the horizontal. For this situation, explain how the tension force in the cable changes. That is does it increase, decrease, or remain the same. Assume that the angle β between the bar and the cable does not change and simply stating the tension increases, decreases or remains the same with no explanation will earn minimal credit.

The tension force goes as (from part b):

$$F_T = \frac{0.5F_{Wb}\sin(90-\theta\theta) + F_{Ws}\sin(90-\theta)}{0.4\sin\beta} = \left(\frac{0.5F_{Wb}+F_{Ws}}{0.4\sin\beta}\right)\sin(90-\theta)$$

As $\theta \downarrow$, $\sin(90-\theta) \uparrow$ and thus, the tension force in the cable would increase.

e. Suppose we return to the original setup shown in part a. If the cable were cut, what would be the initial magnitude of and direction for the angular acceleration of the bar about the hinge?

$$\tau_{net} = \tau_{F_{Wb}} + \tau_{F_{Ws}} = I_{system}\alpha = 0$$

$$-r_{F_{Wb}}F_{Wb}\sin(90-\theta) - r_{F_{Ws}}F_{Ws}\sin(90-\theta) = (I_b + I_s)\alpha$$

$$\alpha = -\left(\frac{0.5F_{Wb}+F_{Ws}}{I_b+I_s}\right)L\sin(90-\theta) = -\left(\frac{0.5F_{Wb}+F_{Ws}}{\frac{1}{3}m_b r^2 + m_s r^2}\right)L\sin(90-\theta)$$

$$\alpha = -\left(\frac{0.5\cdot 500N + 110N}{\frac{1}{3}\cdot\left(\frac{500N}{9.8\frac{m}{s^2}}\right)\cdot\left(\frac{5m}{2}\right)^2 + \left(\frac{110N}{9.8\frac{m}{s^2}}\right)\cdot(5m)^2}\right)\cdot 5m\cdot\sin(90-55) = -2.67\frac{rad}{s^2}$$

Or $2.67\frac{rad}{s^2}$ clockwise.