Physics 110

Exam #3

May 30, 2025

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So, erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example, $|p|»m|\vec{v}| = (5\text{kg})´\left(2\frac{m}{s}\right) = 10\frac{\text{kg}\cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or reasonable value for the quantity you cannot calculate (explain that you are doing this), and use it to do the rest of the problem.
- Each free-response part is worth 6 points

Problem #1	/24
Problem #2	/24
Problem #3	/24
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. A block of mass m_1 is on a ramp inclined at an angle θ, measured with respect to the horizontal. To this block a light string is attached and the string passes over a pulley of mass m_p and radius r_p. To the other end of the string a block of mass m_2 is suspended.

   a. If the system is released from rest, what are the expression for the tension forces in the string on the left and right sides of the pulley? Assume that the block of mass m_1 slides up the ramp.

$$F_{TL} - F_{Wx1} = m_1 a \rightarrow F_{TL} = F_{Wx} + m_1 a = m_1 g \sin\theta + m_1 a$$

$$F_{TR} - F_{W2} = -m_2 a \rightarrow F_{TR} = F_{W2} - m_2 a = m_2 g - m_2 a$$

   b. By examining the torques that act on the pulley, derive an expression for the angle θ that will produce an acceleration equal to $\frac{g}{3}$ up the incline? For this part, let $m_1 = 8kg$, $m_2 = 6kg$, $m_p = 2kg$, and $r_p = 10cm$. Note, you need to derive the expression before you can evaluate it. You can't simply state a result.

$$\tau = r_p F_{TL} - r_p F_{TR} = -I\alpha = -I\left(\frac{a}{r_p}\right)$$

$$m_1 g \sin\theta + m_1 a - (m_2 g - m_2 a) = -\left(\tfrac{1}{2} m_p r_p^2\right)\left(\frac{a}{r_p^2}\right)$$

$$\sin\theta = \frac{m_2 - \left(m_1 + m_2 + \frac{1}{2}m_p\right)\frac{a}{g}}{m_1} = \frac{6kg - (8kg + 6kg + 1kg)\frac{\frac{1}{3}g}{g}}{8kg} = 0.125$$

$$\theta = \sin^{-1} 0.125 = 7.18^0$$

c. Using energy ideas, what is the speed of the block of mass m_2 after m_2 has traveled a vertical distance $y = 0.5m$, starting from rest translationally? Again, you have to derive a result before you can evaluate it. Simply stating a formula will earn very minimal credit.

$$\Delta E_{system} = 0 = \Delta K_T + \Delta K_R + \Delta U_g + \Delta U_s$$

$$0 = \tfrac{1}{2}m_1 v_f^2 + \tfrac{1}{2}m_2 v_f^2 + \tfrac{I}{2}I\omega_f^2 + m_1 g y' - m_2 g y$$

$$0 = \tfrac{1}{2}m_1 v_f^2 + \tfrac{1}{2}m_2 v_f^2 + \tfrac{1}{2}\left(\tfrac{1}{2}m_p r_p^2\right)\left(\frac{v_f}{r_p}\right)^2 + (m_1 \sin\theta - m_2)gy$$

$$\left(m_1 + m_2 + \tfrac{1}{2}m_p\right)v_f^2 = 2(m_2 - m_1 \sin\theta)gy$$

$$v_f = \sqrt{2\left(\frac{m_2 - m_1 \sin\theta}{m_1 + m_2 + \frac{1}{2}m_p}\right)gy} = \sqrt{2\left(\frac{g}{3}\right)y}$$

$$v_f = \sqrt{2\left(\frac{6kg - 8kg\sin 7.18}{8kg + 6kg + 1kg}\right)} \times 9.8\frac{m}{s^2} \times 0.5m = 1.81\frac{m}{s}$$

d. How much work was done by the string on the pulley?

$$W_R = \Delta K_R = \tfrac{1}{2}I\omega_f^2 = \tfrac{1}{2}\left(\tfrac{1}{2}m_p r_p^2\right)\left(\frac{v_f}{r_p}\right)^2 = \tfrac{1}{4}m_p v_f^2 = \tfrac{1}{4} \times 2kg \times \left(1.81\tfrac{m}{s}\right)^2 = 1.64J$$

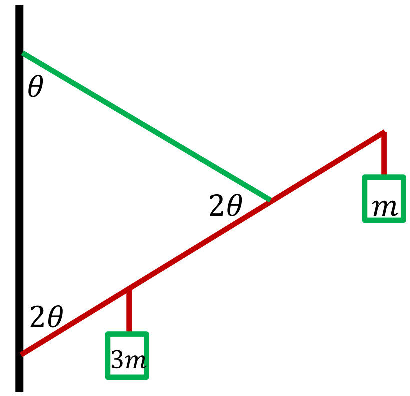
Or by

$$W_R = \tau\Delta\theta = r_p(F_{TL} - F_{TR})\left(\frac{-\Delta s}{r_p}\right) = -\big(m_1 g \sin\theta + m_1 a - (m_2 g - m_2 a)\big)\Delta s$$

$$W_R = -\big((m_1 \sin\theta - m_2)g + (m_1 + m_2)a\big)y$$

$$W_R = -\left((10kg\sin 5.74 - 6kg) + \frac{1}{3}(10kg + 6kg)\right) \times 9.8\frac{m}{s^2} \times 0.5m = 1.63J$$

2. A bar of mass $5m$ and length L is suspended by a light wire attached at a point $\frac{3}{4}L$, as shown below, measured from the point where the rod attaches to the wall. Two masses $3m$ and m are suspended at points $\frac{3}{8}L$ and L, respectively.



a. What is the tension force in the wire in terms of mg?

$$\tau = I_{system}\alpha = 0$$

$$0 = -\frac{3}{8}L(3mg)\sin 2\theta - \frac{1}{2}L(5mg)\sin 2\theta - L(mg)\sin 2\theta + \frac{3}{4}LF_T \sin 2\theta$$

$$F_T = \frac{4}{3}\left(\frac{9}{8} + \frac{5}{2} + 1\right)\frac{mg\sin 2\theta}{\sin 2\theta} = \frac{4}{3}\left(\frac{9+20+8}{8}\right)mg = 6.17mg$$

b. What are the magnitude and direction of the reaction force at the point where the bar attaches to the wall?

$$F_{Rx} - F_{Tx} = m_{system}a_x = 0 \to F_{Rx} = F_{Tx} = F_T\cos\theta$$

$$F_{Rx} = 6.17mg\cos 36 = 5mg$$

$$F_{Ry} - 3mg - 5mg - mg + F_{Ty} = m_{system}a_y = 0$$

$$F_{Ry} = 9mg - F_T\sin\theta = 9mg - 6.17mg\sin 36 = 5.37mg$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(5)^2 + (5.37)^2}mg = 7.3mg$$

$$\phi = \tan^{-1}\left(\frac{F_{Ryy}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{5.37mg}{5mg}\right) = 47^0$$

c. Suppose that for some reason the block of mass m (and density ρ_m) becomes loose and falls into some water below it. The block is a cube with sides of length d and is made out of a material whose density is 60% of that of water. What is the buoyant force on the block from the water in terms of ρ_m and d?

$$F_B = mg = \rho_m d^3 g$$

d. The block, since it's density is less than that of the water, floats in the water. How much of the block's height, d, is above the water? That is what fraction of the block is above the waterline?

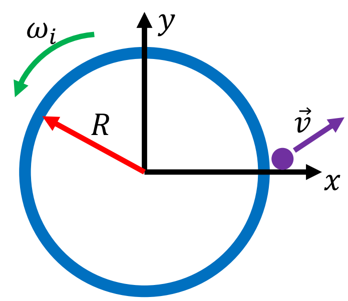
$$F_B = \rho_m d^3 g = \rho_w d^2 h g$$

Let h be the amount of the block under water.

$$h = \frac{\rho_m}{\rho_w} d = \frac{0.6\rho_w}{\rho_w} d = 0.6d$$

Since the block is 60% submerged, 40% must be above the waterline.

3. A space station is constructed in the shape of a hoop of radius R and mass M. The space station is not initially translating through space, but it is spinning with an angular speed ω_i. The crew decides that it needs to eject a small package of mass m from the space station to space shuttle located nearby. Is launched with a speed v at an angle θ measured with respect to the horizontal as shown in the diagram below.



a. Since the space station has a uniform distribution of mass, we can assume that all of its mass is concentrated at a special spot, called the center-of-mass. The center-of-mass of this space station is located at its center, or at the origin. Using this idea, what is the recoil speed of the space station due to the ejection of the package? For this problem, assume $m = 500kg$, $v = 40\frac{m}{s}$, $M = 5000kg$, $R = 10m$, and $\theta = 30^0$.

$$p_{ix} = p_{fx} \rightarrow 0 = mv_x - MV_x \rightarrow V_x = \frac{m}{M}v_x = \frac{m}{M}v\cos\theta$$

$$V_x = \frac{500kg}{5000kg} \times 40\frac{m}{s}\cos 30 = 3.5\frac{m}{s}$$

$$p_{iy} = p_{fy} \rightarrow 0 = mv_y - MV_y \rightarrow V_y = \frac{m}{M}v_y = \frac{m}{M}v\sin\theta$$

$$V_y = \frac{500kg}{5000kg} \times 40\frac{m}{s}\sin 30 = 2\frac{m}{s}$$

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{\left(3.5\frac{m}{s}\right)^2 + \left(2\frac{m}{s}\right)^2} = 4\frac{m}{s}$$

b. What is the direction of recoil of the space station measured with respect to the negative x-axis? Comment on the result you get in light of conservation of momentum?

$$\tan\phi = \frac{V_y}{V_x} = \frac{2\frac{m}{s}}{3.5\frac{m}{s}} = 0.5714 \rightarrow \phi = 29.7^0 \sim 30^0$$

Since momentum is conserved, the space station has to recoil exactly opposite the direction that the package was ejected.

c. Assuming that there are no external torques that act on the system, what is the new rotational speed ω_f of the space station if the initial rotational speed was $\omega_i = 1\frac{rad}{s}$? That is, at what rotational speed is the space station spinning after we eject the package? Hint: In the absence of any external torques, angular momentum is conserved and the angular momentum of the package is defined as $L_{package} = Rp_{package,\perp} = Rp\sin\theta = mvR\sin\theta$.

$$L_i = L_f \rightarrow I_i\omega_i = I_f\omega_f + L_{package}$$

$$\omega_f = \frac{I_i\omega_i - L_{package}}{I_f} = \frac{I_i\omega_i - mvR\sin\theta}{I_f}$$

$$\omega_f = \frac{MR^2\omega_i - mvR\sin\theta}{MR^2} = \omega_i - \frac{mv}{MR}\sin\theta = \omega_i - \frac{500kg \times 40\frac{m}{s}}{5000kg \times 10m}\sin 30$$

$$\omega_f = 1\frac{rad}{s} - 0.2\frac{rad}{s} = 0.8\frac{rad}{s}$$

d. Explain in as much detail as you can, why the rotational speed of the space station changes. That is, why does the space station spin at a different rate than it was originally spinning?

For the system of the space station and the package, there are no external torques. But, if we look only at the space station then there is an external torque that acts on the space station from the ejected package. The package exerts a force opposite to its motion (by Newton's 3rd law) and this creates a clockwise torque. Since the space station was originally spinning counter clockwise, this clockwise torque should slow the space station down, which it does in part c.