Physics 110

Fall 2011

Exam #3

November 7, 2011

Name____________________

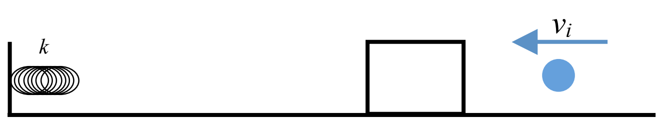
Multiple Choice	/ 16
Problem #1	/ 28
Problem #2	/ 28
Problem #3	/ 28
Total	/ 100

In keeping with the Union College policy on academic honesty, it is assumed that you will neither accept nor provide unauthorized assistance in the completion of this work.

Part I: Free Response Problems

The three problems below are worth 84 points total and each subpart is worth 7 points each. Please show all work in order to receive partial credit. If your solutions are illegible or illogical no credit will be given. A number with no work shown (even if correct) will be given no credit. Please use the back of the page if necessary, but number the problem you are working on.

1. Consider the system shown below in which a ball of mass $m_{ball} = 200g$ is fired at a speed v_i and subsequently collides with a stationary block of mass $m_{block} = 1.7kg$. The bullet and block slide across a frictionless surface until they collide with a horizontal spring of stiffness $k = 18N/m$ initially is at its equilibrium length.



a. Derive an expression for the initial speed of the ball, v_i before the collision, in terms of the given masses and the speed of the ball and block after the collision?

$$\Delta p = p_f - p_i = 0 \rightarrow p_f = p_i \Rightarrow m_{ball} v_i = (m_{bal} + m_{block})V$$

$$\therefore v_i = \left(\frac{m_{bal} + m_{block}}{m_{ball}}\right)V$$

b. If the ball and block strike the spring and eventually come to rest over a distance of $x = 0.87m$, how fast were the block and ball traveling before they struck the spring assuming that friction ($\mu_k = 0.18$) is present *only over the distance* the block and ball travel while in contact with the spring?

$$\Delta E = -W_{fr} = \Delta KE + \Delta U_s$$

$$-\mu(m_{ball} + m_{block})g = \left(0 - \tfrac{1}{2}(m_{ball} + m_{block})V^2\right) + \left(\tfrac{1}{2}kx_f^2 - 0\right)$$

$$V = \sqrt{\left(\frac{k}{m_{ball} + m_{block}}\right)x_f^2 + 2\mu g x} = \sqrt{\left(\frac{18\frac{N}{m}}{1.9kg}\right)\times(0.87m)^2 + \left(2\times 0.18\times 9.8\tfrac{m}{s^2}\times 0.87m\right)}$$

$$V = 3.2\tfrac{m}{s}$$

c. What was the initial speed of the ball before it collided with the block?

$$v_i = \left(\frac{m_{bal} + m_{block}}{m_{ball}}\right)V = \left(\frac{0.2kg + 1.7kg}{0.2kg}\right)\times 3.2\tfrac{m}{s} = 30.4\tfrac{m}{s}$$

d. Derive an expression for (and then evaluate) the fraction of the energy lost in the collision.

$$\% = \left[\frac{KE_f - KE_i}{KE_i}\right]\times 100 = \left[\frac{(m_{ball} + m_{block})V^2 - m_{ball}v_i^2}{m_{ball}v_i^2}\right]\times 100 = \left[\frac{(m_{ball} + m_{block})\left(\frac{m_{ball}v_i}{m_{ball} + m_{block}}\right)^2 - m_{ball}v_i^2}{m_{ball}v_i^2}\right]\times 100$$

$$\% = \left|\frac{m_{ball}}{m_{ball} + m_{block}} - 1\right|\times 100 = \left|\frac{0.2kg}{1.9kg} - 1\right|\times 100 = 89.5\%$$

2. Consider a bicycle that has been turned upside down and is sitting on its seat and handlebars. Suppose that one of the wheels of mass $M = 1.0kg$ and diameter $D = 0.66m$, is spinning at an initial, constant rate of $\omega = 27rad/s$, and you decide that you want to stop the tire from spinning and bring it to rest using a set of hand brakes. You squeeze a lever on the handlebars of the bike and this causes a brake located on each side of the tire to engage and slow the tire down. Friction created between the tire and the brakes (with coefficient of friction $\mu_k = 0.02$) causes the tire to slow down. The brakes are located toward the outer edge of the wheel at a distance of *90%* of the distance between the axle and the outer edge of the tire.

   a. What net torque was created by the application of the brakes, if the force of friction is $F_{Fr} = 0.04N$?

$$\tau_{net} = -2(rF_\perp) = -2 \times [(0.9 \times R) \times F_{fr}] = -2 \times 0.9 \times 0.33m \times 0.04N = -0.024Nm$$

   b. How much work was done by the external torque in bringing the tire to rest? (Hint: The moment of inertia of a tire is given as $I = MR^2$.)

$$W_R = \Delta KE_R = \left(0 - \tfrac{1}{2} I\omega_i^2\right) = -\tfrac{1}{2}\left(MR^2\right)\omega_i^2 = -\tfrac{1}{2}\left(1kg \times (0.33m)^2\right)\left(27\tfrac{rad}{s}\right)^2 = -39.7J$$

   c. How many revolutions does the tire make in coming to rest and how long does it take the tire to stop rotating?

$$W_R = \tau\Delta\theta \rightarrow \Delta\theta = \frac{W_R}{\tau} = \frac{-39.7J}{-0.024Nm} = 1654rad$$

$$\omega_f = \omega_i - \alpha t$$

$$t = \frac{\omega_i}{\alpha} = \frac{\omega_i}{\tau/I} = \frac{I\omega_i}{\tau} = \frac{MR^2\omega_i}{\tau} = \frac{1kg \times (0.33m)^2 \times 27\frac{rad}{s}}{0.024Nm} = 122.5s$$

   d. If the brakes are rectangular and measure *5cm by 1cm*, what pressure was created by one of the brakes on the tire?

$$P = \frac{F_\perp}{A} = \frac{F_N}{A} = \frac{F_{fr}}{\mu A} = \frac{0.04N}{0.02 \times (0.05m \times 0.01m)} = 4000\tfrac{N}{m^2}$$

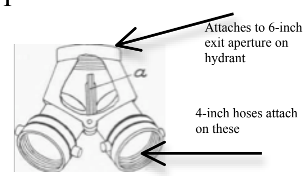
3. Consider the fire hydrant shown below where the exit aperture of the hydrant is *6 inches* (*15.24cm*) in diameter.

   a. Suppose that you connect one *6-inch* hose to the hydrant and hold it horizontally, so that the hose is at the same height as the exit aperture of the hydrant. Assuming that the pressure just before the water exits the hose is *150 pounds per square inch* (*$1.03x10^6$ N/m^2*), with what force (magnitude and direction) would you need to apply to the hose in order to hold onto the hose as the water comes out of the open end?

   As the water exits, the hose would recoil to conserve momentum. Thus the force applied by you should be in the direction of the water's velocity. The magnitude of the force is given by

$$F = PA = 1.03\times10^6 \tfrac{N}{m^2} \times \pi(0.0762m)^2 = 1.88\times10^4 N$$

   b. The *6-inch* hose is probably too big to hold onto in a practical situation, so suppose that you want to fight a fire and to the *6-inch* exit aperture of the hydrant above you attach a coupling (shown below) that can accommodate two hoses, each of diameter *4 inches* (*10.16cm*.) If the pressure inside of the hydrant is *150 pounds per square inch* (*$1.03x10^6$ N/m^2*) what is the exit speed of the fluid out of each hose when the hydrant is opened? (Hints: Assume that each hose is held horizontally at the same height as the exit aperture, that air pressure is *$1.01x10^5$ N/m^2* and the density of water is $\rho_{water} = 1000 kg/m^3$.)?



$$P_h + \tfrac{1}{2}\rho v_h^2 = P_h + \tfrac{1}{2}\rho\left(\frac{2A_{hose}v_{hose}}{A_h}\right)^2 = P_{air} + \tfrac{1}{2}\rho v_{hose}^2 + P_{air} + \tfrac{1}{2}\rho v_{hose}^2$$

$$v_{hose} = \sqrt{\frac{(P_h - 2P_{air})}{\rho\left[1 - \frac{2A_{hose}^2}{A_h^2}\right]}} = \sqrt{\frac{\left(1.03\times10^6 \tfrac{N}{m^2} - 2\times1.01\times10^5 \tfrac{N}{m^2}\right)}{1000 \tfrac{kg}{m^3}\left[1 - \frac{2(0.051m)^2}{(0.0762m)^2}\right]}} = 89.2 \tfrac{m}{s}$$

   c. What would be the speed of the water out of the *6inch* aperture of the hydrant and the flow rate out of the hydrant?

$$Q = A_h v_h = 2A_{hose}v_{hose} \rightarrow v_h = \frac{2A_{hose}v_{hose}}{A_h} = \frac{2\times(0.051m)^2\times89.2\tfrac{m}{s}}{(0.0762m)^2} = 79.9\tfrac{m}{s}$$

$$Q = A_h v_h = \pi(0.0762m)^2 \times 79.9\tfrac{m}{s} = 1.46\tfrac{m^3}{s}$$

   d. Suppose that you wanted to fight a fire on the third floor of a building and that you drag your fire hose up the 3-flights of stairs (a vertical height of *10m* above the ground) to the fire. Assuming that the flow rate is continuous, what is the exit speed of the water from a single hose on the third floor, if the hose is held horizontally when you reach the third floor?

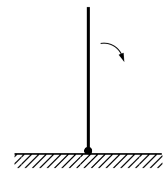
$$P_h + \tfrac{1}{2}\rho v_h^2 = 2P_{air} + 2\times\tfrac{1}{2}\rho v_{hose}^2 + 2\rho g y_{hose}$$

$$v_{hose} = \sqrt{\frac{(P_h - 2P_{air})}{\rho} + \tfrac{1}{2}v_h^2 + 2gy_{hose}} = \sqrt{\frac{\left(1.03\times10^6 \tfrac{N}{m^2} - 2\times1.01\times10^5 \tfrac{N}{m^2}\right)}{1000\tfrac{kg}{m^3}} + \tfrac{1}{2}\left(79.9\tfrac{m}{s}\right)^2 + 2\times9.8\tfrac{m}{s^2}\times10m} = 61.8\tfrac{m}{s}$$

Part II: Multiple-Choice

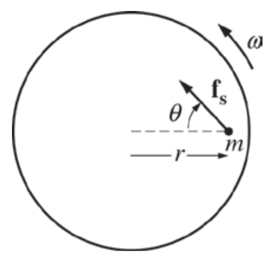
Circle your best answer to each question. Any other marks will not be given credit. Each multiple-choice question is worth 3 points for a total of 12 points.

1. A thin uniform rod of mass M and length L is positioned vertically above an anchored frictionless pivot point, as shown on the right, and then allowed to fall to the ground. With what speed, v does the free end of the rod strike the ground? (Hint: The moment of inertial of a thin rod pivoted about one end is $I = \frac{1}{3}ML^2$.)



a. $\sqrt{\frac{3g}{L}}$ b. $\sqrt{3gL}$ c. $\sqrt{12gL}$ d. $\sqrt{gL}$

2. A small particle of mass m is at rest on a horizontal circular platform that is free to rotate about a vertical axis through its center. The particle is located at a radius r from the axis, as shown in the figure on the right. The platform begins to rotate with constant angular acceleration α. Because of friction between the particle and the platform, the particle remains at rest with respect to the platform. When the platform has reached angular speed ω, the angle θ between the static frictional force f_s and the inward radial direction is given by which of the following?



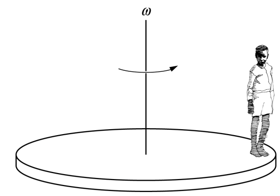
a. $\theta = \frac{\omega^2}{\alpha}$

b. $\theta = \frac{\omega^2 r}{g}$

c. $\theta = \tan^{-1}\left(\frac{\omega^2}{\alpha}\right)$

d. $\theta = \tan^{-1}\left(\frac{\alpha}{\omega^2}\right)$

3. A child is standing on the edge of a merry-go-round that has the shape of a solid disk, as shown in the figure above. The mass of the child is m_{child}. The merry-go-round has a mass of M_{mgr}, radius of R, and it is rotating with an initial angular velocity of ω_i radians per second. The child then walks slowly toward the center of the merry-go-round. Treating the child as a point mass ($I_{child} = m_{child} r^2$), the final angular velocity of the merry-go-round ($I_{mgr} = \frac{1}{2} M_{mgr} R^2$) when the child reaches the center is



   a. constant and given by $\omega_f = \omega_i$.
   b. increasing and is given by $\omega_f = \left(\dfrac{m_{child} R^2 + \frac{1}{2} M_{mgr} R^2}{\frac{1}{2} M_{mgr} R^2} \right) \omega_i$.
   c. increasing and is given by $\omega_f = \left(\dfrac{\frac{1}{2} M_{mgr} R^2}{m_{child} R^2 + \frac{1}{2} M_{mgr} R^2} \right) \omega_i$.
   d. decreasing and is given by $\omega_f = \left(\dfrac{\frac{1}{2} M_{mgr} R^2}{m_{child} R^2 + \frac{1}{2} M_{mgr} R^2} \right) \omega_i$.

4. Suppose that a fluid of constant density is flowing through a horizontal pipe with circular cross section and where the cross sectional area of the pipe is decreasing. At a point where the diameter is a maximum, the speed of the fluid is v_1 and the pressure at this point is P_1, while at a point where the diameter is ½ of the maximum diameter, the speed of the fluid is v_2 and the pressure there is P_2. Comparing the pressure P_2 to the pressure P_1 we find that P_2 is

   a. smaller than P_1 by an amount $\frac{15}{2} \rho v_1^2$.
   b. is equal to P_1.
   c. larger than P_1 by an amount $\frac{15}{2} \rho v_1^2$.
   d. unable to be related to P_1 since we are missing too much information.

Motion in the r = x, y or z-directions

$$r_f = r_i + v_{ir}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{ir} + a_r t$$

$$v_{fr}^2 = v_{ir}^2 + 2a_r\Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles — $C = 2\pi r$, $A = \pi r^2$

Triangles — $A = \tfrac{1}{2}bh$

Spheres — $A = 4\pi r^2$, $V = \tfrac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$\text{magnitude of a vector} = \sqrt{v_x^2 + v_y^2}$$

$$\text{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\,{}^{m}/_{s^2}$ $\quad G = 6.67\times10^{-11}\,{}^{Nm^2}/_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}/_{mole}$ $\quad k_B = 1.38\times10^{-23}\,{}^{J}/_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}/_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}/_{s}$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$$

Heat

$$T_C = \tfrac{5}{9}\left[T_F - 32\right]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}\left(1 + \alpha\Delta T\right)$$

$$A_{new} = A_{old}\left(1 + 2\alpha\Delta T\right)$$

$$V_{new} = V_{old}\left(1 + \beta\Delta T\right):\ \beta = 3\alpha$$

$$PV = Nk_BT$$

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta:\ v = r\omega:\ a_t = r\alpha$$

$$a_r = r\omega^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0};\quad I_o = 1\times10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L};\ f_n = nf_1 = n\frac{v}{4L}$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho gd$$

$$F_B = \rho gV$$

$$A_1v_1 = A_2v_2$$

$$\rho_1A_1v_1 = \rho_2A_2v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho gh_2$$

Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2f^2\rho vA^2$$