Physics 110

Exam #3

November 11, 2013

Name___________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg) \times \left(2\,\frac{m}{s}\right) = 10\,\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 8 points

Problem #1	/28
Problem #2	/24
Problem #3	/20
Total	/72

I affirm that I have carried out my academic endeavors with full academic honesty.

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1. As high as eighty percent of the population at one time or another will suffer from some form of lower back pain, especially during bending and lifting activities. The stresses, which the mussels apply to the disks located between each of the vertebrae, can be very large and these stresses can, along with degeneration of the structure of the disks produce pain, muscle spasm, and immobilization of the lower back. Consider the diagram of the spinal column shown in Figure 1. We'll specifically look at the lumbo-sacral disk intersection of the spine (the dark line in the Figure 1 below) as the point at which we bend, to say pick something up when you keep your legs straight. We'll look at a specific case of just bending over with your arms hanging vertically to determine how large the reaction force on the lumbo-sacral disk can be in this case.

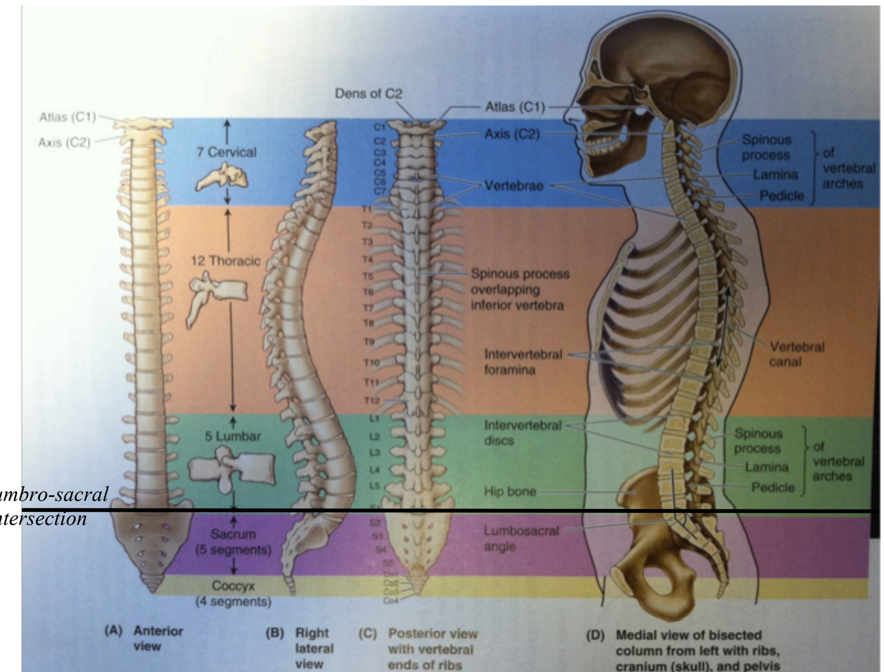


Figure 1: Views of the human spine. Figure from Clinically Oriented Anatomy, by Morre & Dalley. Note, anterior means from the front, lateral means in a line, posterior means from the back and medial means a slice through the middle.

On the next page are Figures 2 and 3. Figure 2 is a cartoon picture of the situation in question and Figure 3 is a stick figure drawing of your spine of length L shown as you are bending over with your arms hanging vertically down along with the forces that act on your spine. Your two arms compose approximately 20% of your body's total weight, while your torso composes approximately 40% of the body's total weight. Assuming W is the weight of your body, we have $F_{W,arms} = 0.2W$ and $F_{W,torso} = 0.4W$ as the weights of your arms and torso respectively. F_e is the force exerted by the *erector spinae* muscles that are in part responsible for "picking" you back up and/or holding you in place at this bend. The force exerted by the *erector spinae* muscles acts at an angle 12^o with respect to the spinal column as shown by the angle α and acts a distance of $\frac{2}{3}L$ from the lumbo-sacral disk. F_R is the reaction force of the lumbo-sacral disk and this force is directed at an angle ϕ. Angle ϕ is unknown but is larger than angle $\theta = 30^o$, which is the angle between the spinal column and the horizontal.

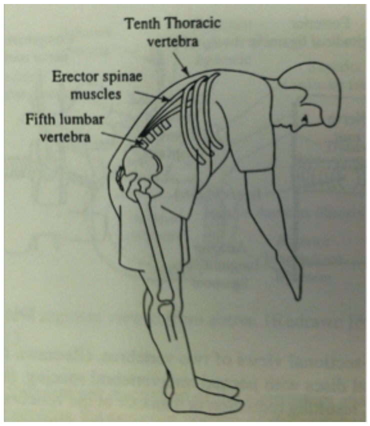


Figure 2: Cartoon illustration of the problem. Drawing from *Physics with Illustrative Examples from Medicine and Biology*, by Benedek & Villars.

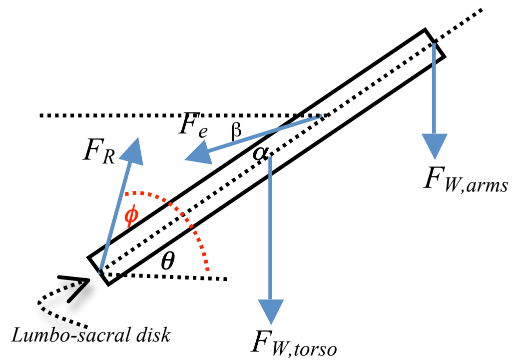


Figure 3: Stick figure drawing of your spinal column showing the various forces that act.

a. What are the expressions for the sum of the forces in the vertical and horizontal directions? Assume a standard Cartesian coordinate system.

$$\sum F_x: \quad F_{Rx} - F_e \cos\beta = F_R \cos\phi - F_e \cos\beta = ma_x = 0$$

$$\sum F_y: \quad F_{Ry} - F_e \sin\beta - F_{W,torso} - F_{W,arms} = F_R \sin\phi - F_e \sin\beta - F_{W,torso} - F_{W,arms} = ma_y = 0$$

where β is the angle between the horizontal and force F_e.

b. What is the expression for the sum of the torques about the lumbo-sacral disk? Assume that you have bent over so that your spinal column makes a $\theta = 30^o$ angle with respect to the horizontal and are not moving.

$$\sum \tau: \quad \tfrac{2}{3} LF_e \sin\alpha - \tfrac{1}{2} LF_{W,torso} \sin(90-\theta) - LF_{W,arms} \sin(90-\theta) = I\alpha = 0$$

c. What is the magnitude of F_e and what are the magnitude and direction of F_R both expressed in terms of your weight W?

From the sum of the torques we can determine F_e. Thus we have:

$$\tfrac{2}{3}F_e\sin(12) = \tfrac{1}{2}F_{W,torso}\sin(60) + F_{W,arms}\sin 60$$

$$F_e = \frac{3}{2}\left[\frac{\tfrac{1}{2}F_{W,torso}\sin(60) + F_{W,arms}\sin 60}{\sin(12)}\right] = \frac{3}{2}\left[\frac{\tfrac{1}{2}(0.4W)\sin(60) + (0.2W)\sin 60}{\sin(12)}\right] = 2.5W$$

$$F_e = 2.5W$$

Then from the sum of the forces in the horizontal and vertical directions we can determine the magnitude and direction of the reaction force F_R.

$$\sum F_x: \quad F_{Rx} - F_e\cos\beta = 0 \rightarrow F_R\cos\phi = F_{Rx} = F_e\cos\beta = 2.5W\cos(18) = 2.38W$$

$$\sum F_y: \quad F_{Ry} - F_e\sin\beta - F_{W,torso} - F_{W,arms} = 0 \rightarrow F_R\sin\phi = F_{Ry} = F_e\sin\beta + F_{W,torso} + F_{W,arms}$$

$$F_{Ry} = F_e\sin\beta + F_{W,torso} + F_{W,arms} = 2.5W\sin(18) + 0.4W + 0.2W = 1.37W$$

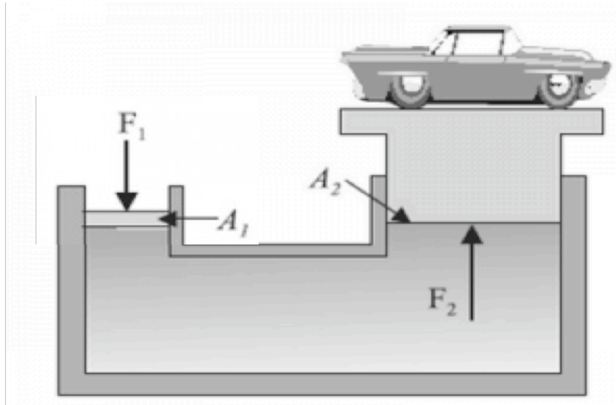
$$\therefore F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \ @\ \phi = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = 2.75W \ @\ \phi = 30.1^o$$

d. Suppose that the compressive force F_R were three times your weight and directed along the spinal column and that this force compresses the disk (modeled to look like a donut) between your 5th lumbar vertebrae and you sacrum (the lumbo-sacral disk). If the total volume of the disk were to remain constant the radius of the disk would tend to

1. increase and the disk would bulge outward away from your spinal column.
2. increase and the disk would compress inward toward your spinal column.
3. decrease and the disk would bulge outward away from your spinal column.
4. decrease and the disk would compress inward toward your spinal column.

Since the volume is given by $\text{volume} = \text{area} \times \text{thickness}$ and the volume is constant, a decrease in the thickness leads to an increase in the area and consequently an increase in the radius. There fore the disk bulges outward from your spine.

2. Suppose that you bring your car to an auto mechanic. The auto mechanic, to get a better look at what's broke, puts your car on a hydraulic lift, shown below in which there is an incompressible fluid of density $\rho = 900\,\frac{kg}{m^3}$. A force F_1 is applied to a piston of area A_1. This supports the weight of the car F_2 on the second piston of area A_2. Assume that both of the pistons have negligible masses.



http://www.miniphysics.com/wp-content/uploads/2012/05/hydralic-lift.png

a. If the car has a mass of $m = 500kg$ what magnitude of force F_1 is needed to support the car so that the bottoms of each piston are at the same height? Assume that the radii of piston 1 and piston 2 are $r_1 = \frac{1}{8}m$ and $r_2 = \frac{1}{2}m$ respectively and that each piston is negligibly thin.

The pressure under each piston is the same (since both are at the same level). The pressure is give as the force per unit area. Thus we have:

$$P_1 = P_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow F_1 = \left(\frac{A_1}{A_2}\right)F_2 = \left(\frac{A_1}{A_2}\right)m_{car}g = \left(\frac{r_1}{r_2}\right)^2 m_{car}g$$

$$F_1 = \left(\frac{0.125m}{0.5m}\right)^2\left(500kg \times 9.8\,\tfrac{m}{s^2}\right) = 306.3N$$

b. Suppose that you wanted to lift the car up so that you could see under it. If you wanted to lift the car by a distance $d_2 = 2m$ above the car's initial height, how far down d_1 below piston 1's starting height, would you have to push piston 1? Assume that piston 1 can move as far down as you would like.

The volume of fluid moved is the same since there are no leaks. So we have:

$$\text{Volume}_1 = \text{Volume}_2 \rightarrow A_1 d_1 = A_2 d_2 \rightarrow d_1 = \left(\frac{A_2}{A_1}\right)d_2 = \left(\frac{r_2}{r_1}\right)^2 d_2 = \left(\frac{0.5m}{0.125m}\right)^2 2m = 32m$$

This is a really unrealistic distance, but the method is fine. To make it more realistic, I'd have to choose different piston sizes, especially the piston that the car is sitting on.

c. How much extra force would be required to push piston 1 down by distance d_1?

If you push piston 1 down a distance d_1 piston 2 rises by a height d_2. Taking zero to be where piston 1 ends up, we have the pressure at this depth, say $P_1^{'}$ (corresponding to a force $F_1^{'}$) equal at all points along a line at this depth. The pressure under the car $P_2^{'}$ at this depth is due to the weight of the car plus the weight of the column of fluid of depth $d_1 + d_2$. Thus we have:

$$P_1^{'} = P_2^{'} \rightarrow \frac{F_1^{'}}{A_1} = \frac{F_2^{'}}{A_2} = \frac{m_{car}g + \rho g(d_1 + d_2)}{A_2} \rightarrow F_1^{'} = \left[\frac{m_{car}g + \rho g(d_1 + d_2)}{A_2}\right]A_1.$$

The difference in force is given as:

$$\Delta F = F_1^{'} - F_1 = \left[\frac{m_{car}g + \rho g(d_1 + d_2)}{A_2}\right]A_1 - \left[\frac{A_1}{A_2}\right]m_{car}g = \frac{\rho g(d_1 + d_2)A_1}{A_2}$$

$$\Delta F = \frac{\rho g(d_1 + d_2)A_1}{A_2} = \frac{900\,\frac{kg}{m^3} \times 9.8\,\frac{m}{s^2}(32m + 2m)(0.125m)^2}{(0.5m)^2} = 18743N$$

Note: You can also get these results from Bernoulli's equation:
For part a, since there is no difference in heights for the pistons and the fluid is at rest we have:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \rightarrow P_1 = P_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

For part c, the fluid again is at rest and we have:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \rightarrow P_1 + \rho g y_1 = P_2 + \rho g y_1$$

$$\rightarrow P_1^{'} - \rho g d_1 = P_2 + \rho g d_2 \Rightarrow P_1^{'} = P_2 + \rho g d_2 + \rho g d$$

3. The human heart can be modeled as a mechanical pump. The aorta is a large artery

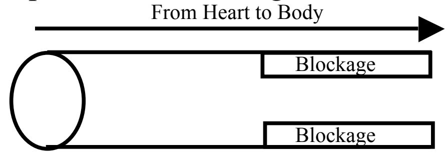
that carries oxygenated blood away from the heart to various organs in the body. For an individual at rest, the blood ($\rho_{blood} = 1050 \frac{kg}{m^3}$) in the aorta of radius $r_{aorta} = 1.25cm$ flows at a rate of $5 \times 10^{-4} \frac{m^3}{\text{min}}$.

a. With every beat, the heart does work moving the blood into the aorta. The heart does work at a rate of $0.5 \frac{J}{s}$. Derive an expression for the energy per unit volume of blood associated with the blood flow into the aorta? (Hint: The power P, or the rate at which work done or energy is transferred is given as $P = \frac{\Delta E}{\Delta t}$.)

$$P = \frac{\Delta E}{\Delta t} \times \frac{\text{Volume}}{\text{Volume}} = \frac{\Delta E}{\text{Volume}} \times \frac{\text{Volume}}{\Delta t} = \frac{\Delta E}{\text{Volume}} \times Q$$

$$\therefore \frac{\Delta E}{\text{Volume}} = \frac{P}{Q} = \frac{0.5 \frac{J}{s}}{\frac{5\times10^{-4} m^3}{60s}} = 60000 \frac{J}{m^3} = 60000 \frac{N}{m^2}$$

b. The energy per unit volume of a moving fluid corresponds to a difference in pressure between two different points in space. Suppose you have the condition called *atherosclerosis*. What are the radius of the opening that remains and what percent of the aorta is blocked? Assume that the buildup on the walls of the aorta is uniform so that the opening that remains is circular and that the patient is lying horizontal. (*Atherosclerosis* is a disease in which plaque builds up inside your arteries. Arteries are blood vessels that carry oxygen-rich blood to your heart and other parts of your body. Plaque is made up of fat, cholesterol, calcium, and other substances found in the blood. Over time, plaque hardens and narrows your arteries. This limits the flow of oxygen-rich blood to your organs and other parts of your body. Atherosclerosis can lead to serious problems, including hear attack, stroke, or even death.).



Starting with Bernoulli's equation, we have, for a person lying horizontally, the difference in pressure is given by the result for part a and we also need the speed of the blood in the aorta.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \rightarrow P_{aorta} + \tfrac{1}{2}\rho v_{aorta}^2 = P_{blockage} + \tfrac{1}{2}\rho v_{blockage}^2$$

$$\rightarrow v_{blockage} = \sqrt{\frac{2}{\rho}\left(P_{aorta} - P_{blockage}\right) + v_{aorta}^2} = \sqrt{\frac{2\left(60000 \frac{N}{m^2}\right)}{1050 \frac{kg}{m^3}} + \left(0.017 \tfrac{m}{s}\right)^2} = 10.07 \tfrac{m}{s}$$

This is given from the flow rate where

$$Q = A_{aorta} v_{aorta} \rightarrow v_{aorta} = \frac{Q}{A_{aorta}} = \frac{\frac{5.4\times10^{-4} m^3}{60s}}{\pi(0.0125m)^2} = 0.017 \tfrac{m}{s}$$

Now we can calculate the speed of the blood in the blockage and from the fact that the flow rate is continuous we can determine the area of the blockage. From the area we can determine the radius of the blockage and what percent is blocked.

Thus the radius is:

$$Q = A_{blockage} v_{blockage} \rightarrow A_{blockage} = \frac{Q}{v_{blockage}} = \frac{\frac{5.4\times10^{-4}\,m^3}{60s}}{10.7\,\frac{m}{s}} = 8.41\times10^{-7}\,m^2$$

$$r_{blockage} = \sqrt{\frac{A_{blockage}}{\pi}} = \sqrt{\frac{8.41\times10^{-7}\,m^2}{\pi}} = 0.0005m = 0.05cm$$

The percent of blockage $\% = \left[\frac{r_i - r_f}{r_i}\right]\times100\% = \left[\frac{1.25cm - 0.05cm}{1.25cm}\right]\times100\% = 96\%$.

c. In the human body, the aorta actually rises a short distance after exiting the heart. At this point the aorta arches and descends towards your legs. In your pelvis the descending aorta branches into two arteries, called the external iliac arteries, which become the femoral arteries when these arteries enter your legs. These arteries carry oxygenated blood to your lower limbs as shown in the figure below. Suppose that you are standing upright on your feet. If you wanted to calculate the flow velocity of the blood in either one of your femoral arteries, what *additional* quantity or quantities below would you need to know? Assume that there is no blockage in the aorta and no friction in the artery walls.

1. The size of the femoral arteries.
2. The speed of the blood in the aorta and the pressure in the aorta.
3. The flow rate of the blood in the aorta.
4. The pressure of the blood flowing in the aorta and the vertical drop of the blood between the aorta and the spot where you want to measure the speed in the femoral artery.
5. The pressure difference between the aorta and the spot you want to measure the speed in the femoral artery as well as the vertical drop of the blood.
6. The flow rate, along with the pressure difference between the aorta and the spot you want to measure the speed in the femoral artery and the vertical drop of the blood.
7. The size of the femoral arteries, the pressure difference between the aorta and the spot you want to measure the speed in the femoral artery and the vertical drop of the blood.
8. The pressure of the blood in the femoral artery and the vertical drop of the blood.
9. The vertical drop of the blood between the aorta and the where you want to measure the speed in the femoral artery.
10. The flow rate, size of the femoral arteries, the pressure difference between the aorta and the spot you want to measure the speed in the femoral artery and the vertical drop of the blood.

(As an aside, the blood returns to the heart, after passing through the capillaries in the lower extremities, through the femoral veins to the iliac veins and finally to the vena cava back to your heart as shown in the figure on the far right.)

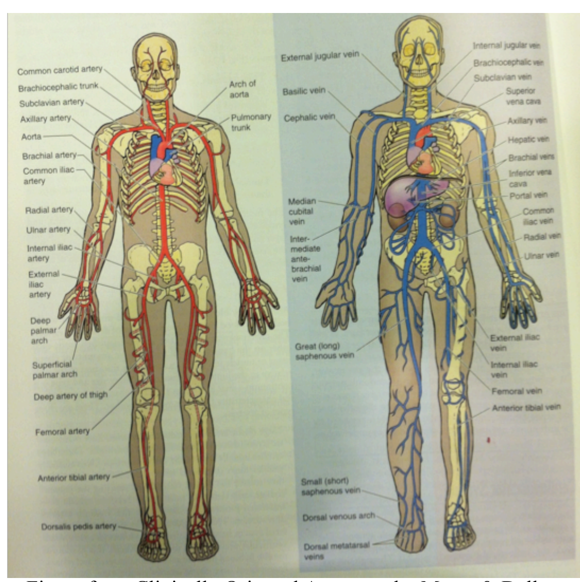


Figure from Clinically Oriented Anatomy, by Morre & Dalley.

Useful formulas:

Motion in the r = x, y or z-directions

$$r_f = r_0 + v_{0r}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{0r} + a_r t$$

$$v_{fr}^{\ 2} = v_{0r}^{\ 2} + 2a_r \Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \tfrac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \tfrac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$\textit{magnitude of a vector} = \sqrt{v_x^{\ 2} + v_y^{\ 2}}$$

$$\textit{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\,{}^{m}\!/_{s^2}$ $\quad G = 6.67\times10^{-11}\,{}^{Nm^2}\!/_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}\!/_{mole}$ $\quad k_B = 1.38\times10^{-23}\,{}^{J}\!/_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}\!/_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}\!/_{s}$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = \Delta E = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{dissipative}$$

Heat

$$T_C = \tfrac{5}{9}\left[T_F - 32\right]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}\left(1 + \alpha\Delta T\right)$$

$$A_{new} = A_{old}\left(1 + 2\alpha\Delta T\right)$$

$$V_{new} = V_{old}\left(1 + \beta\Delta T\right) : \beta = 3\alpha$$

$$PV = Nk_BT$$

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$$

$$a_r = r\omega^2$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho gd$$

$$F_B = \rho gV$$

$$A_1v_1 = A_2v_2$$

$$\rho_1A_1v_1 = \rho_2A_2v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho gh_2$$

Simple Harmonic Motion/Waves

$$\omega = \sqrt{\frac{k}{m}} = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = x_{max}\sin(\omega t) \ \ or \ \ x_{max}\cos(\omega t)$$

$$v(t) = v_{max}\cos(\omega t) \ \ or \ -v_{max}\sin(\omega t)$$

$$a(t) = -a_{max}\sin(\omega t) \ \ or \ -a_{max}\cos(\omega t)$$

$$v_{max} = \omega x_{max}; \ \ a_{max} = \omega^2 x_{max}$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2 \rho v A^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0}; \ \ I_o = 1\times10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L}; \ \ f_n = nf_1 = n\frac{v}{4L}$$