Physics 110

Exam #3

May 31, 2013

Name____________________________________

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg) \times \left(2\frac{m}{s}\right) = 10\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 8 points

Problem #1	/20
Problem #2	/20
Problem #3	/20
Problem #4	/20
Total	/80

I affirm that I have carried out my academic endeavors with full academic honesty.

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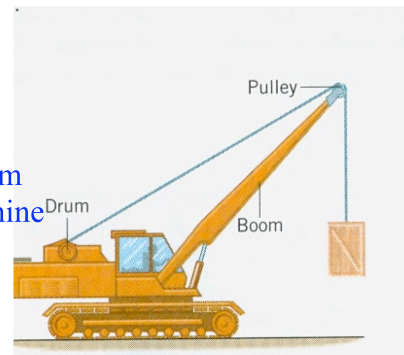
1. Suppose that a crane is lifting a $180kg$ crate upward with an acceleration of $1.2\frac{m}{s^2}$. The cable from the crate passes over a solid cylindrical pulley at the top of the boom. The pulley has a mass of $130kg$, radius $r_p = 0.5m$ and moment of inertia $I_p = \frac{1}{2}mr^2$. The cable is then wound onto a hollow cylindrical drum mounted on the deck of the crane. The drum has a mass of $150kg$, radius $r_d = 0.76m$ and moment of inertia $I_d = mr^2$. The engine attached to the drum is used to wind up the cable.

   a. What is the torque created by the engine?

   Taking ccw rotations as the positive direction for torques we have,

   $$\tau_{net,drum} = \tau_{engine} - r_d F_{T,d-p} = I_{drum}\alpha = m_d r_d^2\left(\frac{a}{r_d}\right) = m_d r_d a$$

   $$\rightarrow \tau_{engine} = m_d r_d a + r_d F_{T,d-p}$$

   Here we have an unknown tension force between the drum and the pulley. To determine this tension force, we examine the forces (and torques) on the pulley.

   $$\tau_{net,p} = I_p\alpha \rightarrow r_p F_{T,p-d} - r_p F_{T,p-c} = \frac{1}{2}m_p r_p^2\left(\frac{a}{r_p}\right) \rightarrow F_{T,p-d} - F_{T,p-c} = \frac{1}{2}m_p a.$$

   Lastly we need the tension force between the pulley and the crate in order to determine the tension force between the drum and the pulley. Then we can solve for the torque due to the engine. For the tension force between the pulley and the crate we have

   $$F_{T,p-c} - m_c g = m_c a \rightarrow F_{T,p-c} = m_c g + m_c a = m_c(g+a) = 180kg\left(9.8\frac{m}{s^2} + 1.2\frac{m}{s^2}\right) = 1980N$$

   Now, the tension force between the drum and pulley is

   $$F_{T,p-d} = F_{T,p-c} + \frac{1}{2}m_p a = 1980N + \left(\frac{1}{2}\times 180kg \times 1.2\frac{m}{s^2}\right) = 2088N$$

   And, therefore the torque produced by the engine is

   $$\tau_{engine} = m_d r_d a + r_d F_{T,d-p} = \left(150kg \times 0.76m \times 1.2\frac{m}{s^2}\right) + \left(0.76 \times 2088N\right) = 1724Nm$$

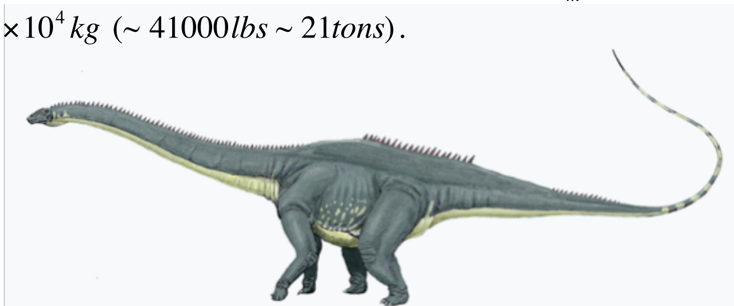
b. What are the tension forces in the cable between the pulley and drum and between the pulley and the crate?

$$F_{T,p-c} = 1980N$$
$$F_{T,d-p} = 2088N$$

c. Suppose that you wanted to raise the crate from the ground to a height h above the ground. The angular momentum of the pulley

1. increases because there is an external torque on the system. (the external torque is produced by the engine.)
2. increases because there is more than one external torque on the system.
3. decreases because gravity does work on the system.
4. decreases because the pulley does work to oppose the motion of the crate.
5. remains the same since angular momentum has to be conserved.

2. The dinosaur *Diplodocus* was enormous, with a long neck and tail and a mass that was large enough to really test its leg strength. According to conjecture, *Diplodocus* waded in water, perhaps in water that was up to its head, so that it could use the water to help lighten the load on its legs. To check this conjecture, take the density of *Diplodocus* to be 0.9 times that of water ($\rho_{water} = 1000\frac{kg}{m^3}$) and assume its mass is about $1.85\times 10^4\,kg$ $(\sim 41000 lbs \sim 21 tons)$.



http://upload.wikimedia.org/wikipedia/commons/0/04/Diplodocus_carng1DB.jpg

a. If the dinosaur were submerged 80%, what would be the dinosaur's apparent weight? In other words, how much would the dinosaur appear to weigh when it is standing in the water and 80% of it its body is under water so that only say part of its neck and head are above the water?

$$F_{app} = F_{actual} - F_{up,water} = m_{dino}g - m_{water,disp}g = m_{dino}g - \rho_{water}V_{water,disp}g = m_{dino}g - \rho_{water}\left(fV_{dino}\right)g$$

$$F_{app} = m_{dino}g - \rho_{water}\left(f\frac{m_{dino}}{\rho_{dino}}\right)g$$

$$F_{app} = m_{dino}g\left(1 - f\frac{\rho_{water}}{\rho_{dino}}\right) = 1.85\times 10^4\,kg\times 9.8\,\tfrac{m}{s^2}\left(1 - 0.8\left(\frac{\rho_{water}}{0.9\rho_{water}}\right)\right) = 20144N$$

b. When it is about 80% submerged (with only about it's head above water) its lungs would have been about $8m$ below the surface of the water. At this depth, what would the difference in pressure be between the water and the dinosaur's lungs? For the dinosaur to breath in, its lung muscles would have to expand its lungs against this pressure difference and it probably could not do so against a pressure difference of more than $8kPa = 8\frac{kN}{m^2}$. Do you think *Diplodocus* did as the conjecture claims? Justify your answer.

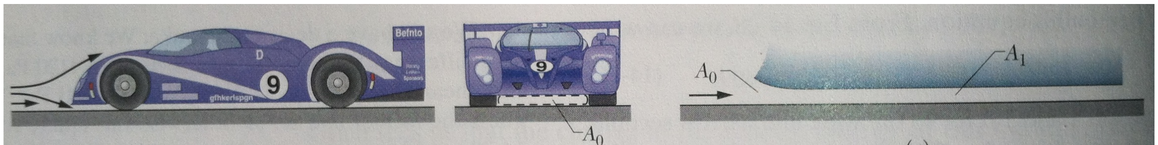
$$P_d = P_{lungs} - \rho g d \rightarrow P_d - P_{lungs} = \rho_{water} g d = 1000\tfrac{kg}{m^3} \times 9.8\tfrac{m}{s^2} \times 8m = 78400\tfrac{N}{m^2}.$$

Since this is about ten times greater than what the dinosaur could probably sustain, the dinosaur probably couldn't breath by submerging 80% of it's body underwater. So the conjecture is probably false.

c. Suppose that instead of a dinosaur you have a cup of water with an ice cube floating in it. When the ice cube is added to the water, the water level in the cup is right at the brim of the cup. When the ice melts the water

1. overflows the cup and water spills.
2. decreases and the water remains in the cup with no spill.
3. remains at the brim of the cup and no water spills. (Since the ice cube floats, it displaces a volume of water equal to its weight. (When it melts it becomes water, and its weight is the same. So the melted ice fills exactly the same volume that the ice cube displaced when floating.)
4. level will change but you cannot tell how because the density of ice is not known.
5. level will change but you cannot tell how because the density of ice and water are not known.

3. As a racecar moves forward at $56\frac{m}{s}(\sim 125mph)$, air is forced to flow over and under the car as shown below. The air forced to flow under the car enters through a vertical cross-sectional area $A_0 = 0.033m^2$ at the front of the car and then flows beneath the car where the vertical cross-sectional area is $A_1 = 0.031m^2$. Assume that air pressure and density given to be $P_{air} = 1.01\times10^5\frac{N}{m^2}$ and $\rho_{air} = 1.3\frac{kg}{m^3}$ respectively.



(a) (b) (c)

a. What is the pressure of the air as it moves through the region of cross-sectional area A_1?

$$P_0 + \tfrac{1}{2}\rho v_0^2 + \rho g y_0 = P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1$$

$$P_0 + \tfrac{1}{2}\rho v_0^2 = P_1 + \tfrac{1}{2}\rho v_1^2$$

$$\rightarrow P_1 = P_0 + \tfrac{1}{2}\rho\left(v_0^2 - v_1^2\right), \text{ where } A_0 v_0 = A_1 v_1 \rightarrow v_1 = \frac{A_0}{A_1}v_0$$

$$\therefore P_1 = P_0 + \tfrac{1}{2}\rho v_0^2\left(1-\left(\frac{A_0}{A_1}\right)^2\right) = 1.01\times10^5\tfrac{N}{m^2} + \tfrac{1}{2}\times1.3\tfrac{kg}{m^3}\times\left(56\tfrac{m}{s}\right)^2\left[1-\left(\frac{0.033m^2}{0.031m^2}\right)^2\right]$$

$$P_1 = 100729\tfrac{N}{m^2} = 1.007\times10^5\tfrac{N}{m^2}$$

b. Suppose that the car can be modeled as a rectangular box with dimensions $length = 3.24m$, $width = 1.5m$, and $height = 1.5m$. What is the net vertical force on the car due to the air pressures above and below the car?

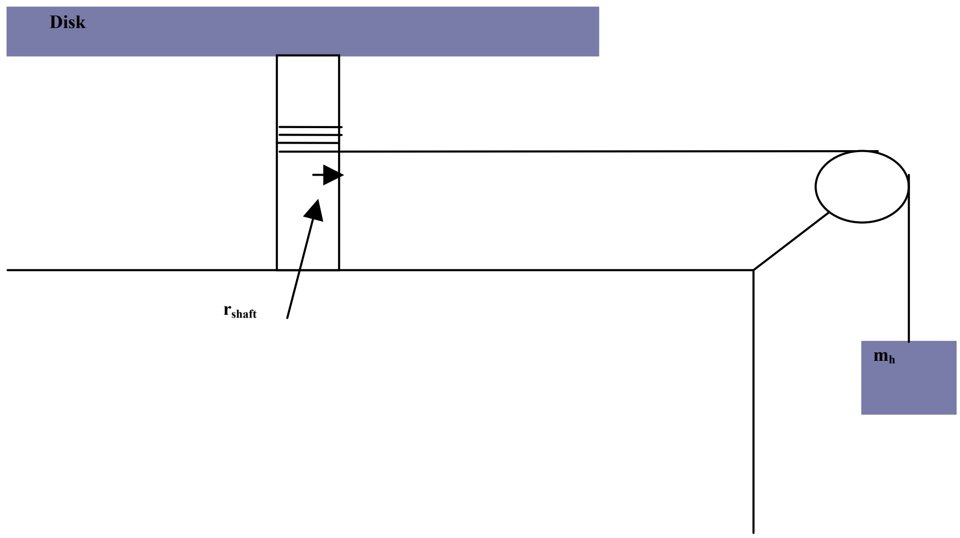
If $P_T > P_B$ then F_{net} points vertically down.
If $P_T < P_B$ then points F_{net} vertically up.

$F_{net} = F_{lift} = (P_T - P_B)A_T = \left(1.01\times10^5 \frac{N}{m^2} - 1.007\times10^5 \frac{N}{m^2}\right)\times(3.24m \times 1.5m) = 1320N$
vertically down since $P_T > P_B$ and where the area of the top and bottom of the car are the same.

c. As the racecar increases its speed from rest to its maximum speed $v_{\max}$ (which could be much greater than the speed it is traveling) the net vertical force on the car

1. increases because the pressure on the top of the car increases and on the bottom of the car decreases.
2. decreases because the pressure on the top of the car increases and the pressure on the bottom of the car increases.
3. increases because the pressure on the top of the car remains the same and the pressure on the bottom of the car decreases. (from part a, as the speed increases the pressure on the bottom decreases while the pressure over the top is just air pressure which remains constant.)
4. decreases because the pressure on the top of the car decreases and the pressure on the bottom decreases.
5. remains the same because the net vertical force is independent of the speed of the racecar.

4. A typical laboratory experiment consists of a rotating disk and an attached mass and can be used to study rotational motion. Suppose that we have the set-up shown below consisting of a shaft of radius r_{shaft} around which a massless string is wound and a disk of moment of inertia I_{disk} is placed. The shaft is frictionless. The string is passed over a massless pulley where a mass m_h is hung and allowed to fall from rest through a height h acquiring a speed of v.



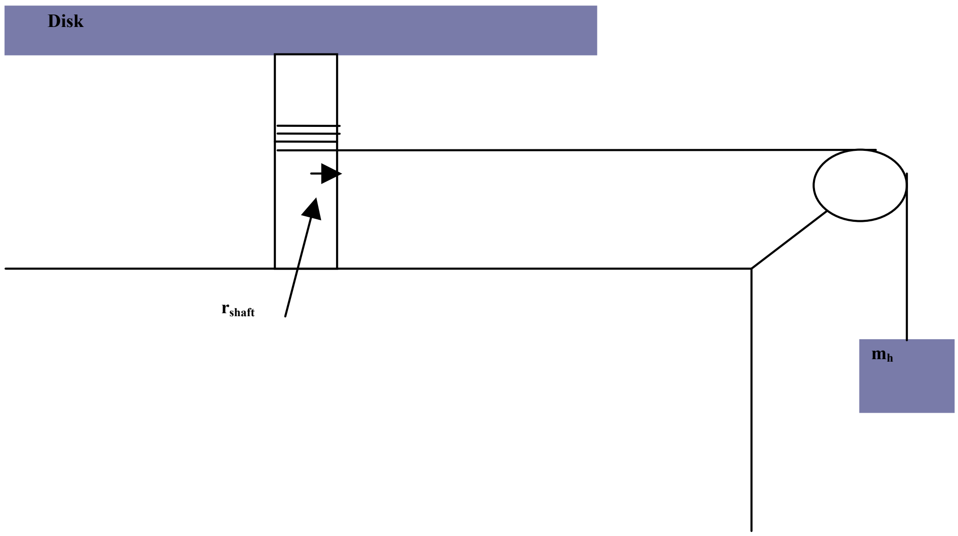
a. Assuming that the system is the shaft with the disk mounted on it, the hanging mass, and the Earth apply the energy principle (conservation of energy), and derive an expression for the moment of inertia of the ring in terms of the variables given in the problem.

$$\Delta E = 0 = \Delta KE_R + \Delta KE_T + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2} m_h v_h^2 - 0\right) + \left(\tfrac{1}{2} I_d \omega_d^2 - 0\right) + \left(0 - m_h g h\right)$$

$$0 = \tfrac{1}{2} m_h v_h^2 + \tfrac{1}{2} I_d \left(\frac{v_h}{r_{sh}}\right)^2 - m_h g h \rightarrow I_d = \frac{2\left(m_h g h - \tfrac{1}{2} m_h v_h^2\right) r_{sh}^2}{v_h^2}$$

$$\therefore I_d = \left[\frac{2gh}{v_h^2} - 1\right] m_h r_{sh}^2$$

b. Using the same diagram, shown below for reference, use ideas of forces and torques to derive another expression for the moment of inertia of the disk.



$$\sum\tau:\ \ \tau_{sh} = I_d\alpha = r_{sh}F_T \rightarrow I_d = \frac{r_{sh}F}{\alpha} = \frac{r_{sh}^2 F_T}{a}$$

$$\sum F_y:\ \ F_T - m_h g = -m_h a \rightarrow F_T = m_h g - m_h a$$

$$I_d = \frac{r_{sh}^2 F_T}{a} = \frac{r_{sh}^2}{a}\left(m_h g - m_h a\right)$$

$$\therefore I_d = \left[\frac{g}{a} - 1\right] m_h r_{sh}^2$$

Which, coincidentally is the same expression as in part a since $v_h^2 = 2gh$.

c. Suppose that the pulley were not massless, but has a mass m_p and a radius r_p. Due to the pulley not being massless, the moment of inertia of the disk would

1. increase because the mass of the pulley increases the moment of inertia of the disk.
2. increase because the energy needed to spin the disk is greater because of the pulley.
3. decrease because the energy needed to spin the disk is less because of the pulley
4. decrease because the mass of the pulley lowers the moment of inertia of the disk.
5. remain the same. (a moment of inertial is a property of the distribution of mass and how that distribution is rotated.)

Useful formulas:

Motion in the r = x, y or z-directions

$$r_f = r_0 + v_{0r}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{0r} + a_r t$$

$$v_{fr}^2 = v_{0r}^2 + 2a_r\Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$magnitude\ of\ a\,vector = \sqrt{v_x^2 + v_y^2}$$

$$direction\ of\ a\,vector \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\,m/s^2$ $\quad G = 6.67\times10^{-11}\,Nm^2/kg^2$

$N_A = 6.02\times10^{23}\,atoms/mole$ $\quad k_B = 1.38\times10^{-23}\,J/K$

$\sigma = 5.67\times10^{-8}\,W/m^2K^4$ $\quad v_{sound} = 343\,m/s$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$$

Heat

$$T_C = \tfrac{5}{9}\left[T_F - 32\right]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}(1 + \alpha\Delta T)$$

$$A_{new} = A_{old}(1 + 2\alpha\Delta T)$$

$$V_{new} = V_{old}(1 + \beta\Delta T) : \beta = 3\alpha$$

$$PV = Nk_BT$$

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$$

$$a_r = r\omega^2$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho g d$$

$$F_B = \rho g V$$

$$A_1 v_1 = A_2 v_2$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho g h_2$$

Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2 \rho v A^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$$