Physics 110

Spring 2008

Exam #2

May 12, 2008

Name____________________

Part	
Multiple Choice	/ 20
Problem #1	/ 32
Problem #2	/ 24
Problem #3	/ 24
Total	/ 100

In keeping with the Union College policy on academic honesty, it is assumed that you will neither accept nor provide unauthorized assistance in the completion of this work.

Part I: Free Response Problems

Please show all work in order to receive partial credit. If your solutions are illegible no credit will be given. Please use the back of the page if necessary, but number the problem you are working on.

1. We've examined the effect of forces on different bodies, for example a pine board. In addition we've applied a force to an object and defined *elastic* as being a property of a body that tends to return the body to its original shape after a force is removed. In general the stress is defined as the force F applied over an area A of a body, or $stress = \frac{F}{A}$. Under this force the object deforms and we define the fractional extent of the deformation, or change in a particular dimension of an object in the deformed state to that of the original dimension in the un-deformed state as the $strain = \frac{\Delta l}{l}$. For elastic materials there is a relation between the *stress* and the *strain*. This relation is called Young's Modulus, Y, where $Y = \frac{stress}{strain}$.

   a. Using the above definition of Young's Modulus and your knowledge of Hooke's Law, define an effective spring constant for an elastic material in terms of Y, A, and l.

   $$Y = \frac{stress}{strain} = \frac{Fl}{A\Delta l} \rightarrow F = \left(\frac{YA}{l}\right)\Delta l = k\Delta l \rightarrow k = \frac{YA}{l}$$

   b. Suppose that your *femur*, or leg bone, is an elastic material and were subjected to a compression force F, how much work is done compressing your femur by an amount Δl from its uncompressed length? Express your answer in terms of Young's Modulus.

   $$W = \Delta U_s = \tfrac{1}{2}ky_f^2 - \tfrac{1}{2}ky_i^2 = \tfrac{1}{2}kl^2 = \tfrac{1}{2}\left(\frac{YA}{l}\right)(\Delta l)^2$$

   c. Since you have two legs, the energy absorbed by the femurs in your legs during the compression is simply twice the value in *part b*. Suppose that one of your leg bones has a cross-sectional area of 4.3cm^2, that the leg bone is 43 cm in length and that Young's Modulus for bone is 1.4x10^6 N/cm^2. If each of your femurs were compressed in an accident by 5mm, how much energy would both femurs have absorbed?

   $$E = 2\times\tfrac{1}{2}\frac{YA}{l}(\Delta l)^2 = 1.46\times 10^6\,\tfrac{N}{cm^2}\times\left(\frac{4.3cm^2}{0.43m}\right)\times(0.005m)^2 = 350J$$

   d. Suppose that the accident you had was one in which you fell from a height H and landed straight legged on the ground so each femur was compressed by 5mm, what height H would you have fallen from, given that your mass is 50kg? Comment on the result you get.

   $$E = 350J = mgH \rightarrow H = \frac{350J}{50kg\times 9.8\frac{m}{s^2}} = 0.71m$$ You don't have to fall from a great height to break your legs if you land straight legged.

2. A karate expert strikes downward with her fist of mass $m_{fist} = 0.7kg$ breaking a $0.14kg$ board. The spring constant for the board is $4.1x10^4$ N/m and the board breaks at a deflection $d = 16mm$. You should note that this is not exactly the same situation as you encountered in lab. Here we are going to model the collision between your fist and the board as an inelastic collision and immediately after the collision your fist and the board ($v_{hand\ +\ board}$) will be moving with the same speed. The board will do work on your hand bringing it to rest after the board has been deflected by the distance d.

   a. Just before the board breaks what type of and how much energy is stored in the board?

      The board has done work on your hand brining it to rest and potential energy has been stored in the board. The $PE_{s,board} = \frac{1}{2}kx^2 = \frac{1}{2}\times 4.1\times 10^4 \frac{N}{m}\times(0.016m)^2 = 5.23J$

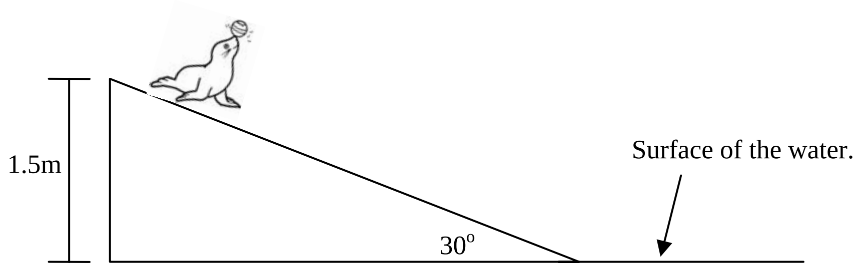
   b. Apply conservation of energy to the situation of your hand striking the board and determine how fast are *the board and your hand* ($v_{hand\ +\ board}$) are moving just after you strike the board.

$$\Delta U_s + \Delta KE = \left(\tfrac{1}{2}ky_f^2 - \tfrac{1}{2}ky_i^2\right) + \left(\tfrac{1}{2}m_{total}v_{f,hand+board}^2 - \tfrac{1}{2}m_{total}v_{i,hand+board}^2\right)$$
$$\rightarrow \tfrac{1}{2}ky_f^2 - \tfrac{1}{2}m_{total}v_{i,hand+board}^2 = 0$$
$$\therefore v_{hand+board} = \sqrt{\frac{2}{(m_{hand}+m_{board})}\times\left(\tfrac{1}{2}ky_f^2\right)} = \sqrt{\frac{2\times 5.23J}{0.7kg+0.14kg}} = 3.53\tfrac{m}{s}$$

   c. What is the minimum speed that your hand (v_{hand}) must be moving before it collides with the board so that you can break this karate board? (Hint: Apply conservation of momentum from just before you strike the board to just after you strike the board where your hand and board are moving at the same speed.)

$$p_{iy} = p_{fy} \rightarrow m_{hand}v_{hand} = (m_{hand}+m_{board})v_{hand+board}$$
$$\rightarrow v_{hand} = \frac{(m_{hand}+m_{board})v_{hand+board}}{m_{hand}} = \frac{(0.7kg+0.14kg)}{0.7kg}\times 3.53\tfrac{m}{s} = 4.24\tfrac{m}{s}$$

3. A 30-kg trained seal at an amusement park slides down a ramp into a pool of water subject to a constant frictional force. The ramp is inclined at 30° with respect to the horizontal and is 1.5 m above the water as shown below.



a. If there was no frictional force between the seal and the ramp, how fast would the seal enter the water?

$$\Delta U_g - \Delta KE = \left(mgy_f - mgy_i\right) + \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right)0$$
$$\rightarrow mgy_i = \tfrac{1}{2}mv_f^2 \rightarrow v_f = \sqrt{2gy_i} = \sqrt{2\times 9.8\tfrac{m}{s^2}\times 1.5m} = 5.4\tfrac{m}{s}$$

b. Now, suppose the frictional force acts on the seal and the seal now enters the water at 4.9 m/s. What are the potential energy of the seal at the top of the ramp (1.5m above the water's surface) and the kinetic energy of the seal at the bottom of the ramp?

$$U_{top} = mgy_i = 30kg \times 9.8\tfrac{m}{s^2} \times 1.5m = 441J$$
$$KE_{bottomo} = \tfrac{1}{2}mv_{fr}^2 = \tfrac{1}{2}\times 30kg \times \left(4.9\tfrac{m}{s}\right)^2 = 360.15J$$

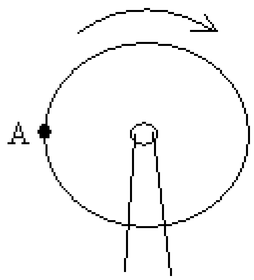
c. What is the coefficient of friction between the seal and the ramp?

$$\Delta U_g + \Delta KE = \left(U_{bottom} - U_{top}\right) + \left(KE_{bottom} - KE_{top}\right) = -\Delta E_{diss}$$
$$U_{top} = KE_{bottom} + \Delta E_{fr} = KE_{bottom} + \mu_k F_N d = KE + \mu_k mg\cos\theta \times \frac{h}{\sin\theta}$$
$$\rightarrow \mu_k = \frac{\left(U_{top} - KE_{bottom}\right)\sin\theta}{mgh\cos\theta} = \frac{\left(441J - 360.15J\right)\sin 30}{30kg \times 9.8\frac{m}{s^2}\cos 30 \times 1.5m} = 0.106$$

Part II: Multiple-Choice

Circle your best answer to each question. Any other marks will not be given credit. Each multiple-choice question is worth 2 points for a total of 20 points.

1. A child rides a Ferris wheel which goes around at constant speed. When the child is at point A in the diagram, what is the direction of the child's acceleration vector?

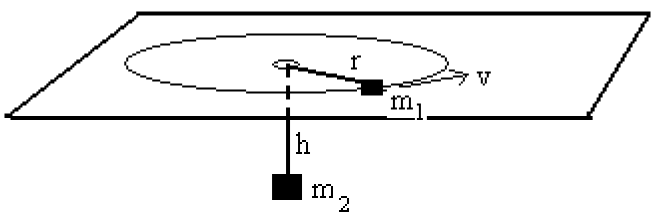


a. → b. ↑ c. ↓ d. ← e. none of these.

2. After colliding, two bodies move away with different velocities. In the space below, explain how you would determine if the collision was elastic or inelastic? (2 points)

We compare the kinetic energies before and after the collision. If the change is zero then the collision is elastic otherwise it is inelastic.

3. A block of mass m_1 on a frictionless horizontal tabletop is attached to a string which goes through a hole in the table to another block of mass m_2 which hangs at rest on the end of the string a distance h below the tabletop. The block on the tabletop moves in a circle of radius r and at speed v.



Which of the following statements must be true?

a. $\frac{1}{2} m_1 v^2 = m_2 g h$

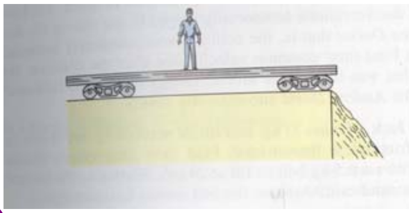
b. $m_1 \dfrac{v^2}{r} = m_2 g$

c. $I = m_1 r^2 + m_2 h^2$

d. None of these

4. A very light object (*a*) and a very heavy (*b*) object are sliding down along a frictionless ramp inclined at an angle θ with respect to the horizontal and at the bottom of the ramp they continue to slide along a frictionless horizontal surface. If each mass starts from the same height *h*, which of the following is true when the each of the objects reaches the bottom of the ramp?
   a. $v_a > v_b$ b. $v_b > v_a$ c. $v_a = v_b$ d. cannot tell from the information given.

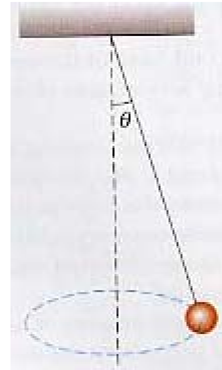
5. Super spy James Bond find himself caught in a trap set by *SPECTRE* in which Bond finds himself at the center of a railway car that has been placed at the edge of a cliff. Which way should Bond walk to minimize the danger of falling off of the edge of the cliff?



   a. To the left. b. To the right. c. There is no way to minimize the danger.

6. Suppose that the momentum of a particle is *p* and this particle has a kinetic energy *KE*. If the momentum of the particle doubles, the kinetic energy
   a. decreases by a factor of 2.
   b. increases by a factor of 2.
   c. increases by a factor of 4.
   d. cannot be determined since there is no relationship between momentum and kinetic energy.

Questions 7 & 8 refer to the conical pendulum shown on the right in which a bob of mass *m* is spun at an angle θ and the bob traces out a circle of radius *R* in the horizontal plane.



7. From Newton's 2nd law, the tension force in the string is

a. $F_T = mg$ b. $F_T = \dfrac{mg}{\cos\theta}$ c. $F_T = \dfrac{mg}{\sin\theta}$

d. $F_T = mg\cos\theta$

8. The speed of the bob in the horizontal plane is

a. $v = Rg\sin\theta$ b. $v = Rg\cos\theta$ c. $v = \sqrt{Rg\sin\theta}$ d. $v = \sqrt{Rg\tan\theta}$

9. The position as a function of time for a 1kg mass oscillating on a spring placed on a frictionless horizontal spring is given by $x(t) = 0.1\cos(2t)$ for the position measured in meters and time in seconds. The period of oscillation for the mass-spring system is

a. π seconds.

b. 2π seconds

c. $\sqrt{\pi}$ seconds

d. unable to be determined since *k* the spring constant is unknown.

10. If it takes 4J of work to stretch a spring by 10cm from its unstretched length, how much extra work does it take to stretch the same spring by an additional 10cm?

a. 4J b. 8J c. 12 J d.16 J

Useful formulas:

Motion in the x, y or z-directions

$r_f = r_0 + v_{0r}t + \frac{1}{2}a_r t^2$

$v_{fr} = v_{0r} + a_r t$

$v_{fr}^2 = v_{0r}^2 + 2a_r \Delta r$

Uniform Circular Motion

$a_r = \frac{v^2}{r}$

$F_r = ma_r = m\frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

$Quadratic\ equation: ax^2 + bx + c = 0,$

$whose\ solutions\ are\ given\ by: x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$magnitude\ of\ a\ vector = \sqrt{v_x^2 + v_y^2}$

$direction\ of\ a\ vector \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\,{}^{m}\!/\!_{s^2} \qquad G = 6.67\times10^{-11}\,{}^{Nm^2}\!/\!_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}\!/\!_{mole} \qquad k_B = 1.38\times10^{-23}\,{}^{J}\!/\!_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}\!/\!_{m^2K^4} \qquad v_{sound} = 343\,{}^{m}\!/\!_{s}$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCos\,\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta KE + \Delta U_g + \Delta U_S = 0$

$\Delta KE + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T): \ \beta = 3\alpha$

$PV = Nk_BT$

$\frac{3}{2}k_BT = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$\Delta s = r\Delta\theta: \ v = r\omega: \ a_t = r\alpha$

$a_r = r\omega^2$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm v_{max}\left(\sqrt{1 - \frac{x^2}{A^2}}\right)$

$v_{max} = \omega A$

$a_{max} = \omega^2 A$

$v = f\lambda$

$v = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$