# Physics 110

# Spring 2008

# Exam #3

# May 28, 2008

## Name_____

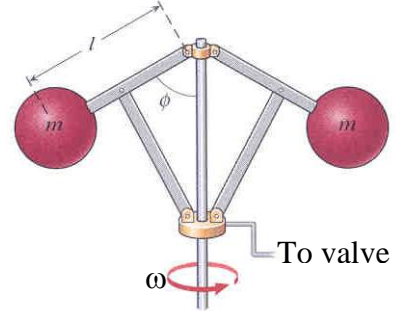| Part | |
|------|------|
| Multiple Choice | / 20 |
| Problem #1 | / 32 |
| Problem #2 | / 32 |
| Problem #3 | / 16 |
| Total | / 100 |

*In keeping with the Union College policy on academic honesty, it is assumed that you will neither accept nor provide unauthorized assistance in the completion of this work.*

**Part I: Free Response Problems**

*Please show all work in order to receive partial credit. If your solutions are illegible no credit will be given. Please use the back of the page if necessary, but number the problem you are working on.*

1. A fly-ball governor is a device used to prevent sudden speed changes in a steam engine. As the shaft rotates more rapidly, the balls rise, lifting the collar. The collar connects to a valve control, so that when it is lifted it reduces the steam pressure that is causing the speeding up of the shaft.

    a. Draw a carefully labeled free-body diagram and from it derive expressions for the net force in the x- and y-directions. (Hint: Assume that the force exerted on each ball by the rod of length $l$ is purely a tension force and that the connecting structures are very light compared to the balls.)

$$\sum F_x: \quad F_T \sin\phi = \frac{mv^2}{R}: \quad R = l\sin\theta; \quad v = R\omega \rightarrow \sum F_x: \quad F_T = ml\omega^2$$

$$\sum F_y: \quad F_T \cos\phi = mg \rightarrow \cos\phi = \frac{mg}{F_T} = \frac{g}{l\omega^2}$$

    b. Show that the relationship between the angle $\phi$ and the shafts' angular velocity $\omega$ is given by $\cos\phi = \frac{g}{l\omega^2}$. Does this relationship indicate that the angle $\phi$ is increasing or decreasing with $\omega$? Explain your reasoning.

    As $\omega$ increases, $\cos\phi$ decreases, so this indicates that as $\phi$ increases $\omega$ increases.

    c. What is the period of revolution of one of the balls?

$$T = \frac{2\pi R}{v} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l\cos\phi}{g}}$$

    d. What is the rotational kinetic energy of the fly-ball governor? (Hint: The moment of inertia of a point mass is $mR^2$.)

$$KE = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(2ml^2 \sin^2\phi\right)\left(\frac{g}{l\cos\phi}\right) = \frac{mgl\sin^2\phi}{\cos\phi} = mgl\left(\frac{1-\cos^2\phi}{\cos\phi}\right)$$

2. A garden water sprinkler is shown below. There are three veins (of radius $R = 200mm$) that distributes the water as the sprinkler rotates counterclockwise about the hub $H$. The sprinkler is connected to a water supply through a hose that is ¾" in diameter (where $1$" $= 2.54$ cm) and flows into the hose at a rate of $14.5$ m/s.

a. What is the flow rate out of the water supply?
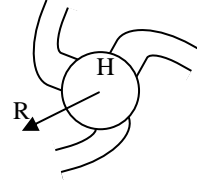


$$\text{flow rate} = A_{hose}v_{hose} = \pi\left(\tfrac{3}{8}" \times \tfrac{0.0254m}{1"}\right)^2 \times 14.5\tfrac{m}{s} = 4.13\times10^{-3}\,\tfrac{m^3}{s}$$

b. If the cross sectional area of a vein is $30$ $mm^2$, what is the speed of the exiting water from the vein?

$$\text{flow rate} = 4.13\times10^{-3}\,\tfrac{m^3}{s} = 3A_{vein}v_{vein} = 3\times\left(30mm^2 \times \tfrac{1m^2}{(1000mm)^2}\right)v_{vein}$$

$$\therefore v_{vein} = 45.6\tfrac{m}{s}$$

c. The exiting water causes the sprinkler to rotate and this causes a change the angular momentum of the sprinkler. What is the change in the angular momentum of the sprinkler? (Hint: Treat the sprinkler veins as thin rods spun about their ends where the moment of inertia of a single rod is given as $I = \tfrac{1}{3}mR^2$ and the mass of each rod is $50g$.)

$$I_{total} = 3\times\tfrac{1}{3}mR^2 = mR^2 = 0.05kg \times (0.2m)^2 = 0.002kgm^2$$

$$\omega = \frac{v}{R} = \frac{45.6\tfrac{m}{s}}{0.2m} = 229.5\tfrac{rad}{s}$$

$$\Delta L = I\omega_f - I\omega_i = I\omega_f = 0.002kgm^2 \times 229.5\tfrac{rad}{s} = 0.459\tfrac{kgm^2}{s}$$

d. If it takes $2$ s for the sprinkler to attain a constant final rotational speed, what magnitude of torque was produced about the hub $H$?

$$\tau = I\alpha = I\frac{\Delta\omega}{\Delta t} = \frac{\Delta L}{\Delta t} = \frac{0.459\tfrac{kgm^2}{s}}{2s} = 0.23Nm$$

3. A block of mass *4kg* is suspended by a rope that passes over a pulley of mass *2kg* and radius *5cm*. The rope is connected to a spring whose stiffness constant is *80 N/m*.
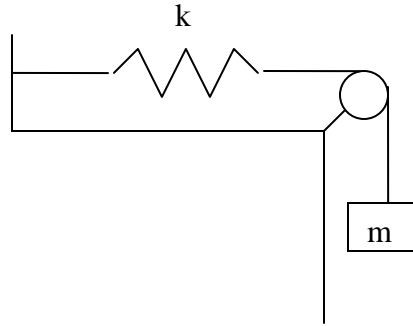


a. Using energy methods, what is the maximum extension of the spring, if the block is released from rest?

$$\Delta U_g + \Delta U_s + \Delta KE_T + \Delta KE_R = 0$$

$$(0 - mgy) + \left(\tfrac{1}{2}ky^2 - 0\right) + (0 - 0) + (0 - 0) = 0$$

$$\text{min} \rightarrow y = 0$$

$$\text{max} \rightarrow \frac{2mg}{k} = \frac{2 \times 4kg \times 9.8\frac{m}{s^2}}{80\frac{N}{m}} = 0.98m$$

b. What is the speed of the block after it has fallen *20cm*? (Hint: Treat the pulley as a disk with moment of inertia $I = \tfrac{1}{2}mR^2$.)

$$\Delta U_g + \Delta U_s + \Delta KE_T + \Delta KE_R = 0$$

$$(0 - mgy) + \left(\tfrac{1}{2}ky^2 - 0\right) + \left(\tfrac{1}{2}mv^2 - 0\right) + \left(\tfrac{1}{2}I\omega^2 - 0\right) = 0$$

$$v = R_p\omega \quad \& \quad I = \tfrac{1}{2}m_p R_p^2$$

$$\therefore v = \sqrt{\frac{mgy - \tfrac{1}{2}ky^2}{\tfrac{1}{2}m + \tfrac{1}{4}m_p}} = \sqrt{\frac{4kg \times 9.8\frac{m}{s^2} \times 0.2m - \tfrac{1}{2} \times 80\frac{N}{m} \times (0.2m)^2}{\dfrac{4kg}{2} + \dfrac{2kg}{4}}} = 1.58\frac{m}{s}$$

**Part II: Multiple-Choice**
   *Circle your best answer to each question. Any other marks will not be given credit.*
   *Each multiple-choice question is worth 2 points for a total of 20 points.*

1.  The table below gives pressure measured at various depths below the surface of a
   liquid in a container. A second liquid, whose density is *twice* that of the first liquid, is
   poured into a second container. Similar pressure measurements are taken for the
   second liquid at various depths below the surface
   of the second liquid. What is the pressure at a
   depth of 10cm for the second liquid?

   | Depth (cm) | Pressure (N/m$^2$) |
   |---|---|
   | 5 | 250 |
   | 10 | 450 |
   | 15 | 650 |

   a.  250 N/m$^2$          b.  450 N/m$^2$          c. 850 N/m$^2$          d. 1650 N/m$^2$

2.  A solid body can be in rotational equilibrium only when
   a.    it has zero angular momentum.
   b.    it is in free fall.
   c.    its external torques sum to zero.
   d.    its external forces sum to zero.

3.  The figure below shows three small spheres that rotate about a vertical axis. The
   perpendicular distance between the axis and the center of each sphere is given.
   Which of the following must be true if the moments of inertia?

   1m ———○ 36 kg

   2m ———○ 9 kg

   3m ———○ 4 kg

   Rotation
   axis
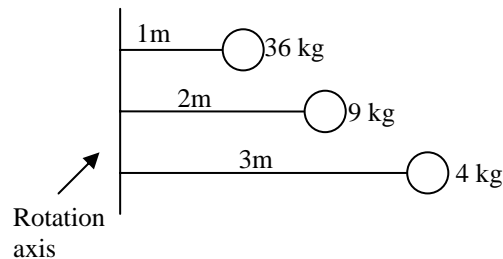
   a.  $I_1 = I_2 = I_3$          b.  $I_1 < I_2 < I_3$          c.  $I_1 = I_2 < I_3$          d.  $I_1 > I_2 = I_3$

4. When a downward force is applied at a point a distance 0.6m to the left of a fulcrum (a pivot), equilibrium is achieved by placing a mass of 10 kg at a point 0.4 m to the right of the fulcrum. What is the magnitude of the downward force?
    a. 1.5 N
    b. 6.5 N
    c. 9.8 N
    d. 15 N

5. It takes a good deal of effort to make homemade ice cream  If the torque required to turn the handle on the ice cream maker is 5 Nm, how much work is expended on each complete revolution of the handle?
    a. 3.14 J          b. 31.4 J                    c. 1800 J                    d. 0.8 J

6.  A skater pulls in her arms, decreasing her moment of inertia by a factor of two, and doubling her angular speed. Her final rotational kinetic energy is given as
    a. $KE_f = KE_i$
    b. $KE_f = 2KE_i$
    c. $KE_f = 4KE_i$
    d. $KE_f = \frac{1}{4}KE_i$

Questions 7 - 9 refer to passage below.

Three balls with the same volume of $1x10^{-6}\ m^3$ are in an open tank of water that has a density $\rho_{water} = 1000\ kg/m^3$. The balls are in the water at different levels as shown below. Ball 1 floats in the water with a part of it above the surface. Ball 2 is suspended in the water, and Ball 3 rests on the bottom of the tank.
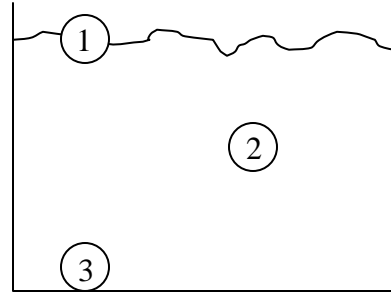


7. The densities of the balls, $\rho_1$, $\rho_2$ and $\rho_3$ are related by which of the following?
   a. $\rho_1 < \rho_2 < \rho_3$
   b. $\rho_1 < \rho_2 = \rho_3$
   c. $\rho_1 = \rho_2 < \rho_3$
   d. $\rho_1 = \rho_2 > \rho_3$

8. Ball 2 is in the water *20cm* above ball 3, what is the approximate difference in pressure between the 2 balls?
   a. 200 N/m²      b. 500 N/m²      c. 2000 N/m²      d. 5000 N/m²

9. Assuming that the side of the tank is punctured *5m* below the top of the water, and that air pressure is $1x10^5\ N/m^2$, what is the speed of the exiting water from the hole?
   a. 10 m/s          b. 12 m/s          c. 14 m/s          d. 17 m/s

10. An object starts from rest and rotates counterclockwise with constant angular acceleration and reaches an angular speed of *12 rad/s* in *3 s*. What is the angular acceleration of the wheel?

   a. 36 rad/s²          b. 2 rad/s²          c. 18 rad/s²          d. 4 rad/s²

**Useful formulas:**

**Motion in the x, y or z-directions**

$$r_f = r_0 + v_{0r}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{0r} + a_r t$$

$$v_{fr}^2 = v_{0r}^2 + 2a_r \Delta r$$

**Uniform Circular Motion**

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

**Geometry /Algebra**

| Circles | Triangles | Spheres |
|---|---|---|
| $C = 2\pi r$ | $A = \tfrac{1}{2}bh$ | $A = 4\pi r^2$ |
| $A = \pi r^2$ | | $V = \tfrac{4}{3}\pi r^3$ |

$$Quadratic\ equation: ax^2 + bx + c = 0,$$

$$whose\ solutions\ are\ given\ by: x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Vectors**

$$magnitude\ of\ a\ vector = \sqrt{v_x^2 + v_y^2}$$

$$direction\ of\ a\ vector \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

**Useful Constants**

$$g = 9.8\ m/s^2 \qquad G = 6.67\times10^{-11}\ Nm^2/kg^2$$

$$N_A = 6.02\times10^{23}\ atoms/mole \qquad k_B = 1.38\times10^{-23}\ J/K$$

$$\sigma = 5.67\times10^{-8}\ W/m^2K^4 \qquad v_{sound} = 343\ m/s$$

**Linear Momentum/Forces**

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

**Work/Energy**

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_s = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\,\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta KE + \Delta U_g + \Delta U_s = 0$$

$$\Delta KE + \Delta U_g + \Delta U_s = -\Delta E_{diss}$$

**Heat**

$$T_C = \tfrac{5}{9}[T_F - 32]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}(1 + \alpha\Delta T)$$

$$A_{new} = A_{old}(1 + 2\alpha\Delta T)$$

$$V_{new} = V_{old}(1 + \beta\Delta T): \ \beta = 3\alpha$$

$$PV = Nk_B T$$

$$\tfrac{3}{2}k_B T = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

**Rotational Motion**

$$\theta_f = \theta_i + \omega_i t \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2_f = \omega^2_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$\Delta s = r\Delta\theta: v = r\omega: a_t = r\alpha$$

$$a_r = r\omega^2$$

**Fluids**

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho gd$$

$$F_B = \rho gV$$

$$P_1 + \tfrac{1}{2}\rho v^2_1 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v^2_2 + \rho g h_2$$

**Simple Harmonic Motion/Waves**

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm v_{max}\left(\sqrt{1 - \frac{x^2}{A^2}}\right)$$

$$v_{max} = \omega A$$

$$a_{max} = \omega^2 A$$

$$v = f\lambda$$

$$v = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$