Fluids at Rest

Example 1:

The gas tank in most passenger cars is approximately $20\ gallons$ of gasoline, where $1gallon = 3.8\times10^{-3}m^3$.

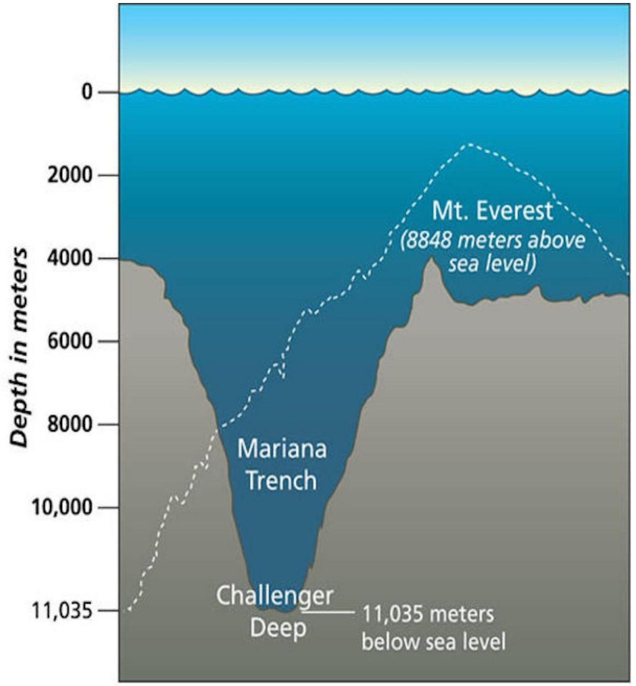
- If the density of gasoline is $\rho_{gas} = 680\frac{kg}{m^3}$ and a typical gas tank measures $0.5m\times0.9m\times d$, what is the depth d of the gasoline in the tank in the car if the mass of the gasoline 50kg?

- How deep d' is the gas tank itself, if it can hold $20\ gallons$?

Example 2:

The deepest point in the oceans of the world is called Challenger Deep and is in the pacific ocean ($\rho_{seawater} = 1025\frac{kg}{m^3}$) at the southern most end of the Mariana trench. The depth of Challenger Deep is about $33{,}000ft \sim 11000m$.

- What pressure would be recorded at this depth?



https://www.pinterest.com/pin/141793088246977612/

Fluids at Rest

Example 3:

A person of mass $70kg$ stands at rest on a scale.

- What does the scale read if we ignore the buoyant force on the person?
- If a person can be modeled as a cylinder of height $1.5m$ and radius $0.13m$, what would the scale read? The density of air is $\rho_{air} = 1.3\frac{kg}{m^3}$.
- Assume you didn't know the volume of the person? Could you still calculate what the scale would read and if so, what would the scale read?

Example 4

Suppose that you had a balloon ($m_b = 90kg$) that is spherical and roughly $10m$ in diameter and is filled with helium ($\rho_{He} = 0.179\frac{kg}{m^3}$).

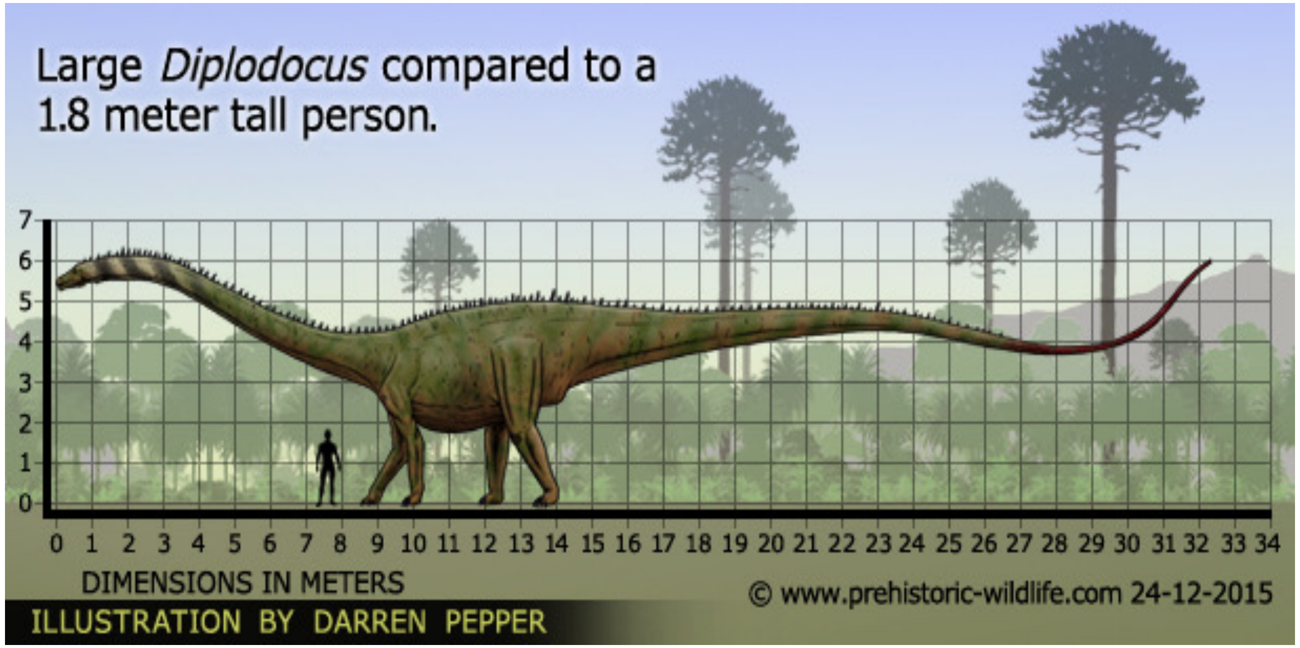
- What is the buoyant force on the balloon from the air ($\rho_{air} = 1.29\frac{kg}{m^3}$)?
- What is the net force on the balloon and what is the acceleration of the balloon?
- What maximum mass of a payload could the balloon support if it were to just leave the ground and hover?

Fluids at Rest

Example 5:

The dinosaur *Diplodocus* was enormous, with a long neck and tail and a mass that was large enough to test its leg strength. According to some scientists, *Diplodocus* waded in water, perhaps in water that was up to its head, so that it could use the water to help lighten the load on its legs. To check this idea, let's take the density of *Diplodocus* to be 90% of that of water ($\rho_{water} = 1000\frac{kg}{m^3}$) and assume that the mass of *Diplodocus* was about $1.85\times10^4 kg$ ($\sim$41,000$lbs\sim$21$tons$).

- If the dinosaur was submerged with 80% of its body under water, what would be the dinosaurs apparent weight? That is, what fraction of the dinosaur's actual weight would be supported?
- When it is 80% submerged its lungs would be about $4m$ below the surface of the water. At this depth, what would be the difference in pressure between the water and the dinosaur's lungs?
- If, for the dinosaur to breath, the maximum pressure difference needs to be $8000\,\frac{N}{m^2}$ or less, could Diplodocus have breathed submerged in the water?



http://www.prehistoric-wildlife.com/species/d/diplodocus.html

Fluids at Rest

Example 6:

Royal Caribbean has some of the largest cruise ships in the world. Consider the Oasis class cruise ship the *Oasis of the Seas* docked at a port. The ship has a weight of $226838 tons$ ($\sim 2\times 10^9 N$), is 1184 feet ($\sim 395m$) long, 208 feet ($\sim 69m$) wide, 213 feet ($\sim 71m$) tall and has a maximum draft of $30 feet$ ($\sim 10m$), where the draft is how deep the ship can comfortably sink into the water, or how much of the ship is below the water. This ship can accommodate 8461 passengers (6296) and crew (2165).

- Before any passengers and crew get on the ship, what is the draft of the ship?

- Suppose that the ship scheduled to depart and that it is fully laden. How much deeper in the water will the ship sink if all passengers and crew get on the ship for a voyage?

https://en.wikipedia.org/wiki/Oasis_of_the_Seas#/media/File:Oasis_of_the_Seas.jpg

Oasis of the Seas information: https://www.tripsavvy.com/oasis-of-the-seas-royal-caribbean-992876

Fluids in Motion

Example 7:

Consider a dam, like the O'Shaughnessy Dam in California, shown on the right. The dam holds back a reservoir of water.

Let the reservoir have a cross-sectional area A_{top} and someone opens a flood gate on the face of the dam at a depth d below the surface of the water in the reservoir. The flood gate has an area A_{fg} defined by its radius $r_{fg} = 1.7m$ ($\sim 5ft$).

- What is the speed of the water flowing out of the floodgate on the side of the dam if $d = 10m$?

https://en.wikipedia.org/wiki/O%27Shaughnessy_Dam_(California)

Fluids in Motion

Example 8:

Consider the Airbus A350 airplane shown below. What is the aerodynamic lift produced if the wings have a total surface area $A_{total\,wings} = 442m^2$ and the velocity of the airflow across the top and bottom of the wing are $250\frac{m}{s}$ ($\sim 560\frac{mi}{hr}$) and $240\frac{m}{s}$ ($\sim 540\frac{mi}{hr}$) respectively? The mass of the plane is $2.68\times10^5 kg$ and the density of air $\rho_{air} = 1.3\frac{kg}{m^3}$.

https://upgradedpoints.com/travel/airlines/air-france-us-routes-plane-types/

Fluids in Motion

Example 9:

Suppose that a 25-gauge syringe is held horizontally and that the body has a cross-sectional area $A_{body} = 7.9\times 10^{-5} m^2$ (corresponding to a radius of $r_{body} = 0.5cm$) while the needle has a cross-sectional area of $A_{needle} = 7.9\times10^{-7} m^2$ (corresponding to a radius of $r_{needle} = 0.5mm$). Suppose that a force of $F_a = 20N$ is applied to the plunger on the body side of the syringe and that the fluid inside has a density of $\rho = 1025\frac{kg}{m^3}$. In the absence of the applied force, the pressure on all sides of the syringe is $P_{air} = 1.01\times10^5\frac{N}{m^2}$.

- What is the speed of the fluid as it leaves the needle's tip?

Fluids in Motion

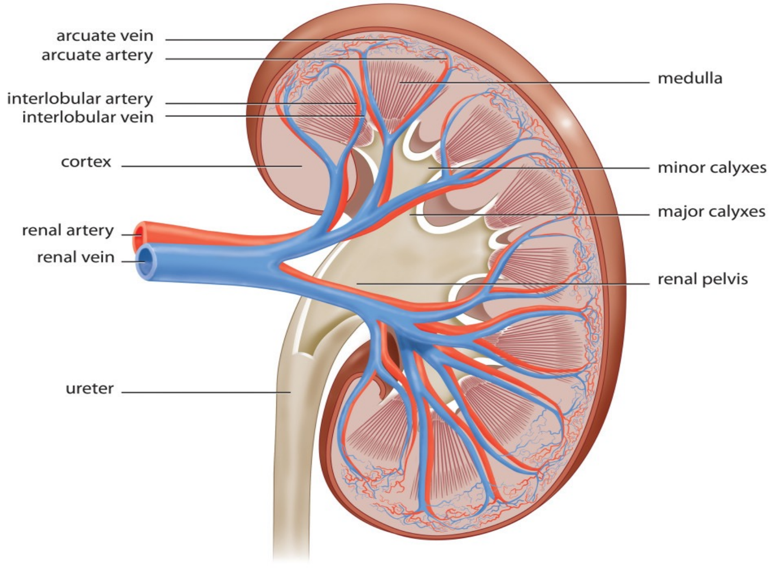
Example 10:

The kidneys require a high, stable pressure to filter waste out of the blood. Blood enters the kidney through the renal artery and is distributed into millions of microscopic afferent arterioles (these are blood vessels that deliver blood into the filtering unit of the kidney, the nephron in the renal pyramids.) Because these arterioles are much narrower, the velocity and pressure of the blood change dramatically, which directly impacts the kidney's ability to filter toxins.

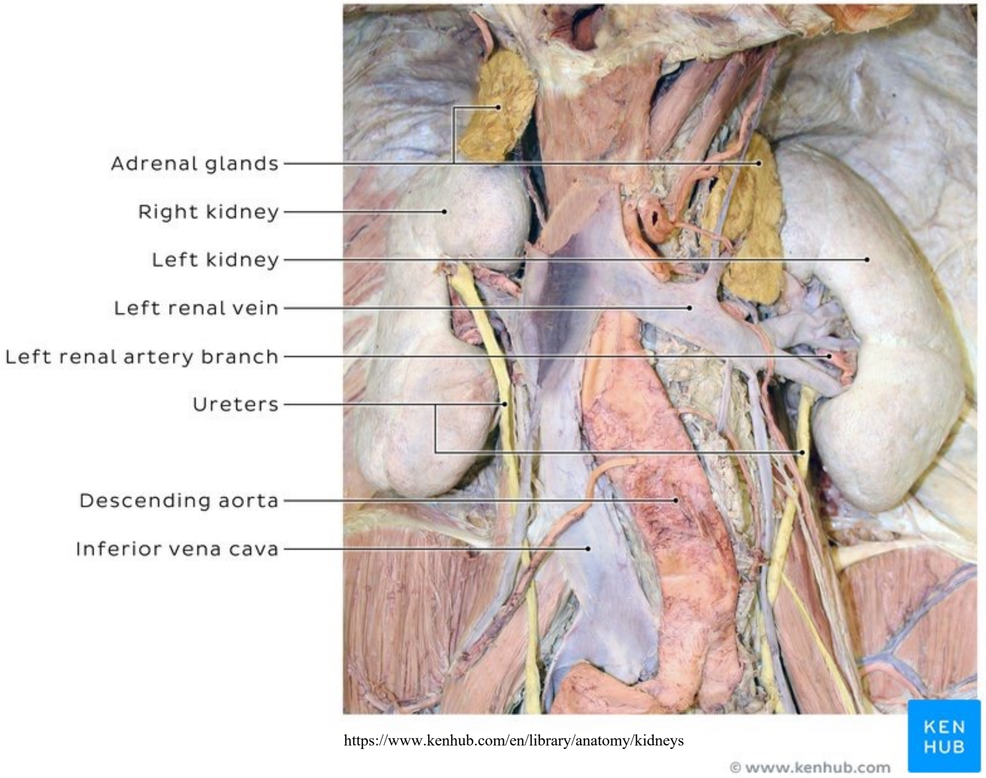
Suppose the renal artery ($r_{RA} = 2.5mm$) supplies blood to one million afferent arterioles to filter the blood. What is the speed of the blood in an arteriole ($r_{AA} = 0.025mm$) if the speed in the renal artery is $v_{RA} = 0.4\frac{m}{s}$?

What is atmospheric pressure in $mmHg$ if the density of mercury is $\rho_{Hg} = 13534\frac{kg}{m^3}$?

What is the pressure (in $mmHg$) inside an arteriole if the pressure in the renal artery is $P_{RA} = 1.33\times10^4\frac{N}{m^2} = 100mmHg$? Assume blood has a density $\rho_{blood} = 1050\frac{kg}{m^3}$.



https://interactivebiology.com/3254/the-anatomy-of-the-

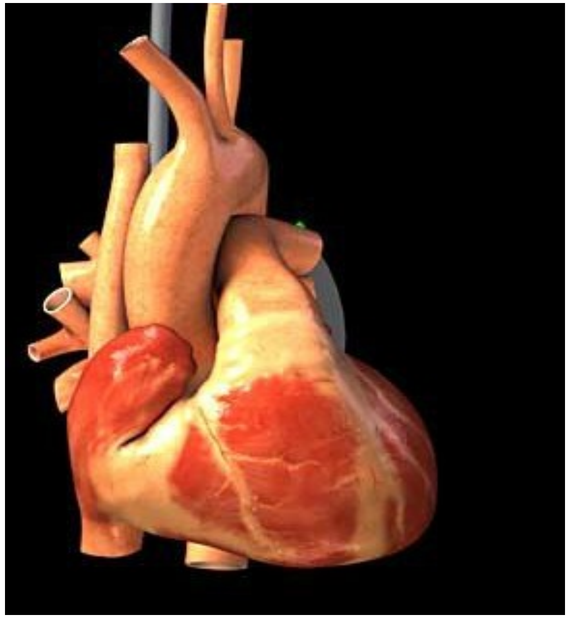


https://www.kenhub.com/en/library/anatomy/kidneys

The Physics of the Body

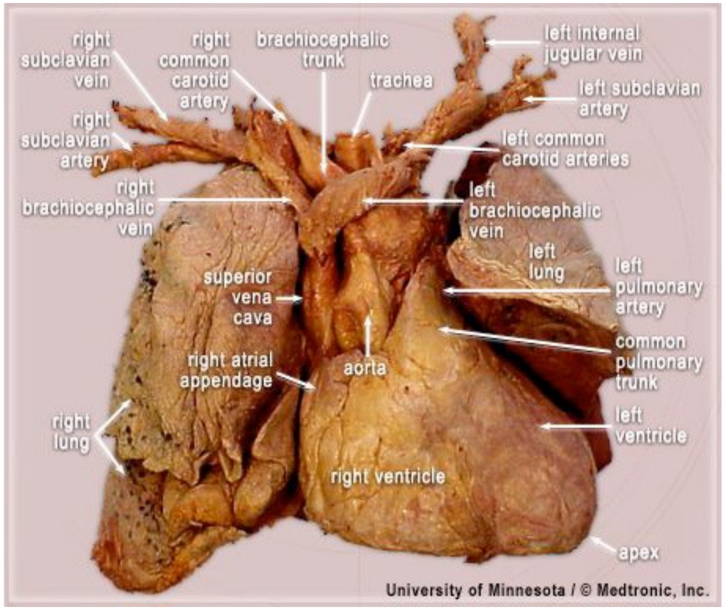
- The Human Heart

The human heart is a dual suction/pressure pump that pulls in blood from the major systems of the body and pumps blood back out to those major systems through a closed loop.



A schematic on the left of the human heart and a cadaver dissection of the human heart and lungs on the right showing part of the circulatory system.

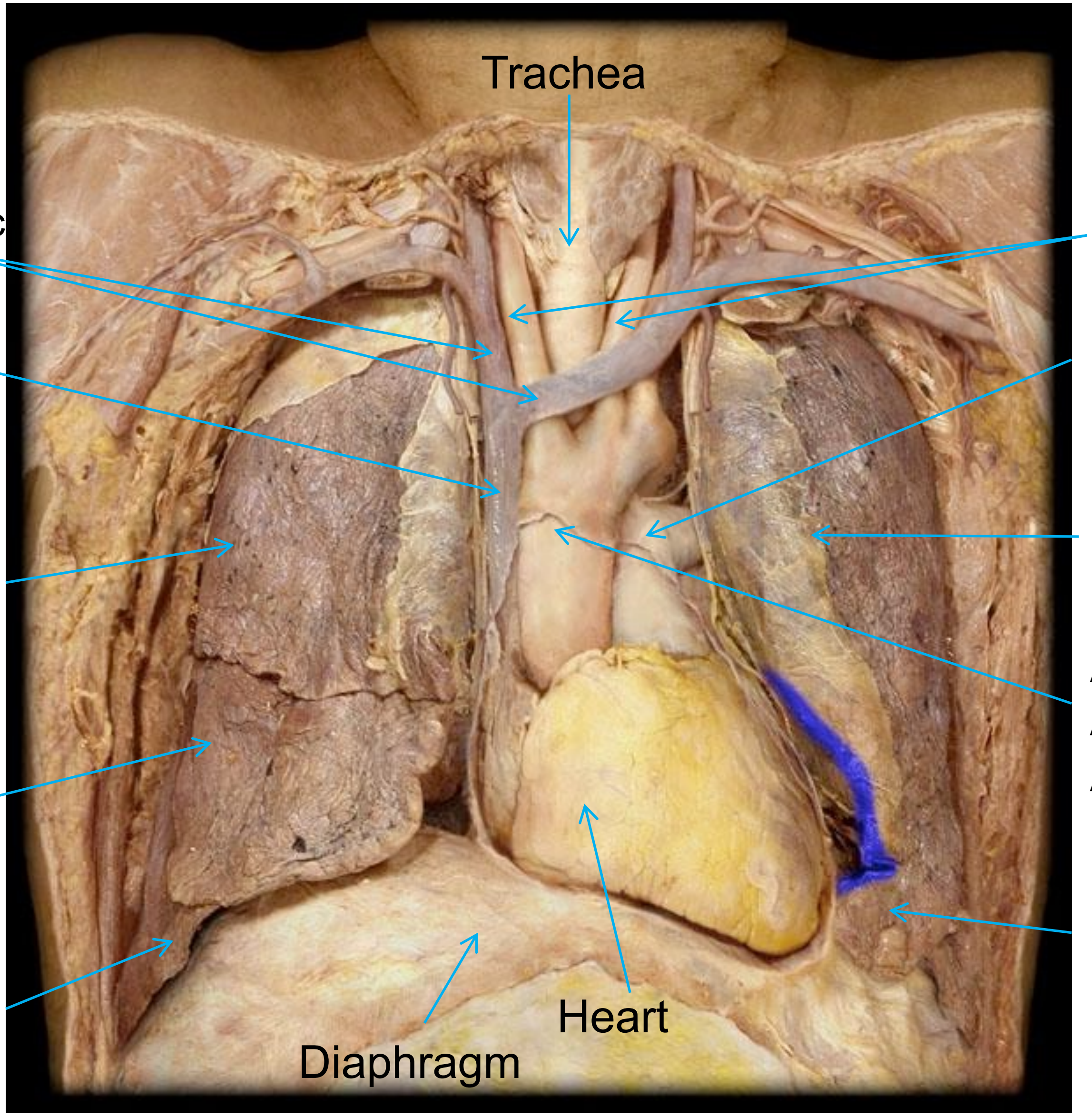
The atria of the heart provide suction to pull blood in from the systems of the body while the ventricles provide pumping to push the blood around to the systems of the body.



The function of the heart is to pump blood through the network of arteries and veins called the cardiovascular system.

Arteries carry oxygenated blood from the heart to the major systems of the body and veins carry oxygen poor blood back to the heart.

The Human Heart

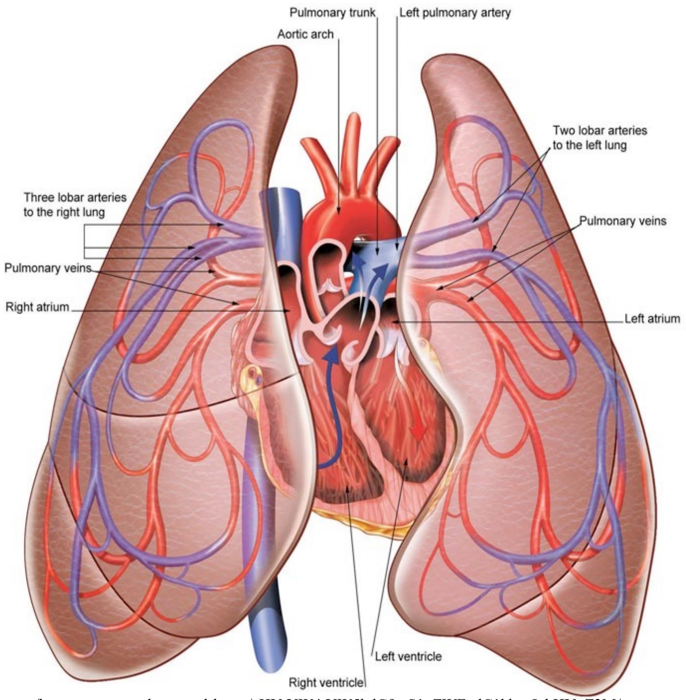


Anatomical view of a cadaver dissection of the thoracic (or chest) cavity.

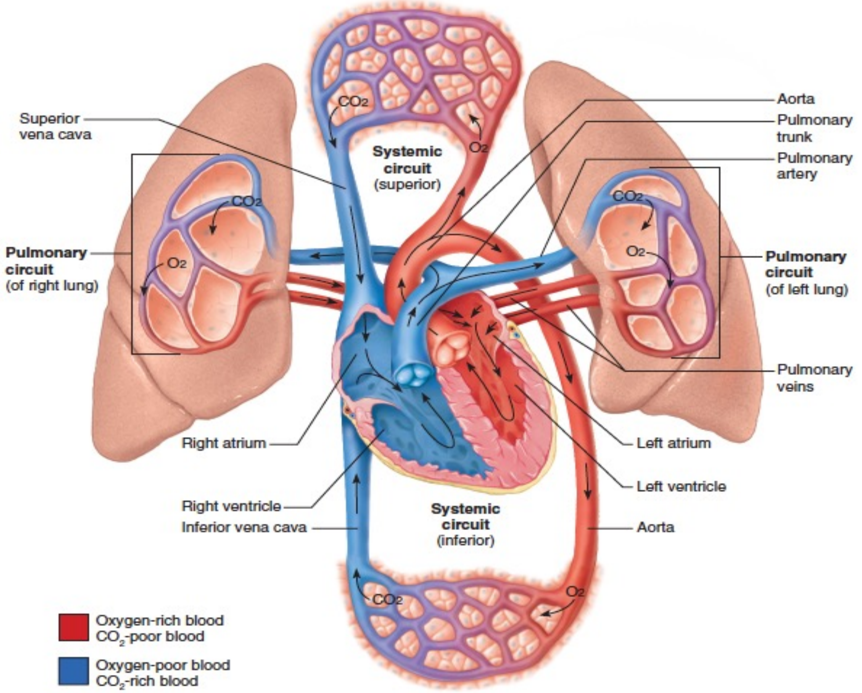
This shows the major organs in the chest and upper abdomen of the human body.

The Human Heart

- Cartoon diagrams showing the major anatomical locations of the heart and lungs showing the relation to major arteries/veins.



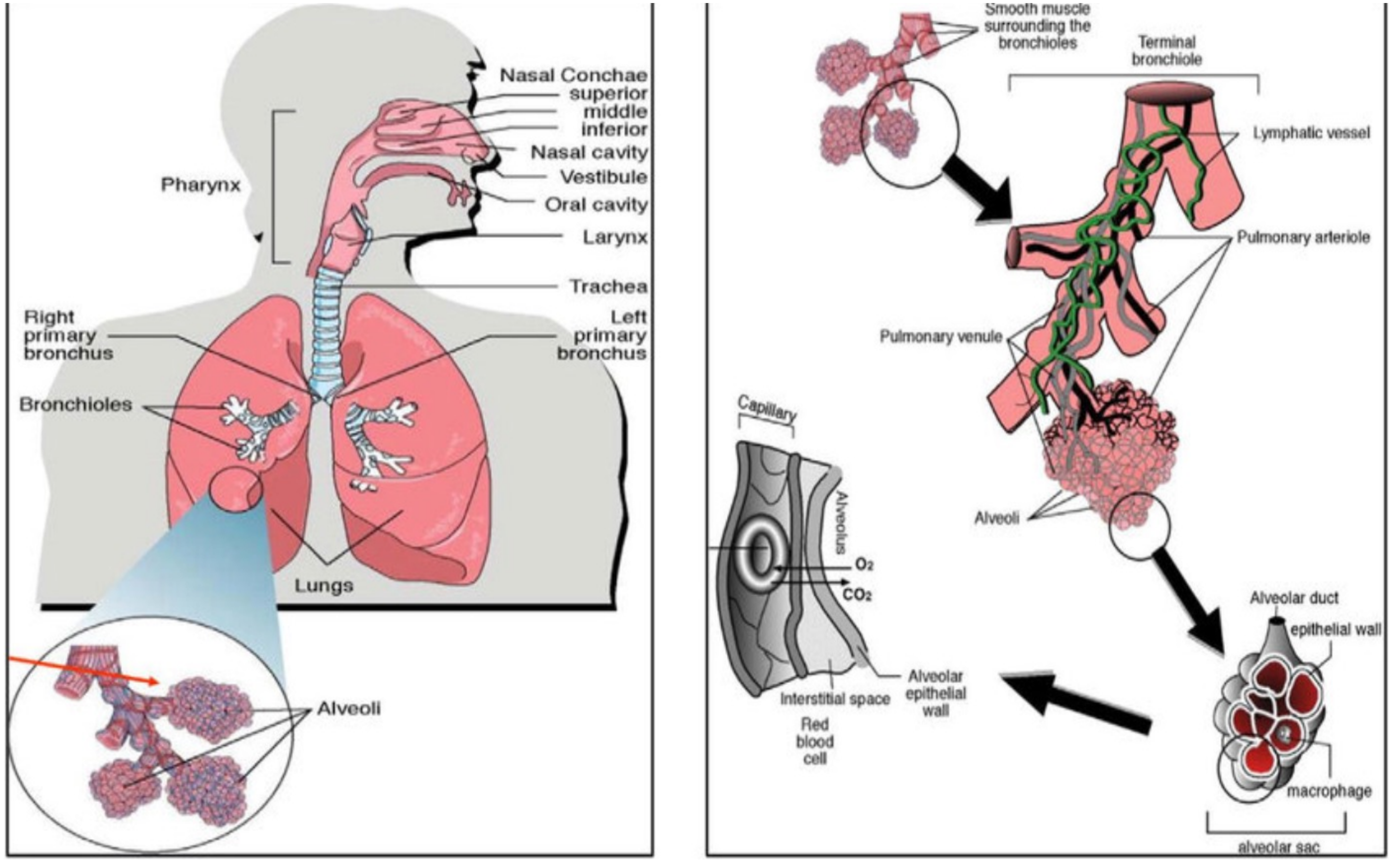
http://www.stylepinner.com/human-anatomy-heart-and-lungs/aHVtYW4tYW5hdG9teS1oZWFydC1hbmQtbHVuZ3M/



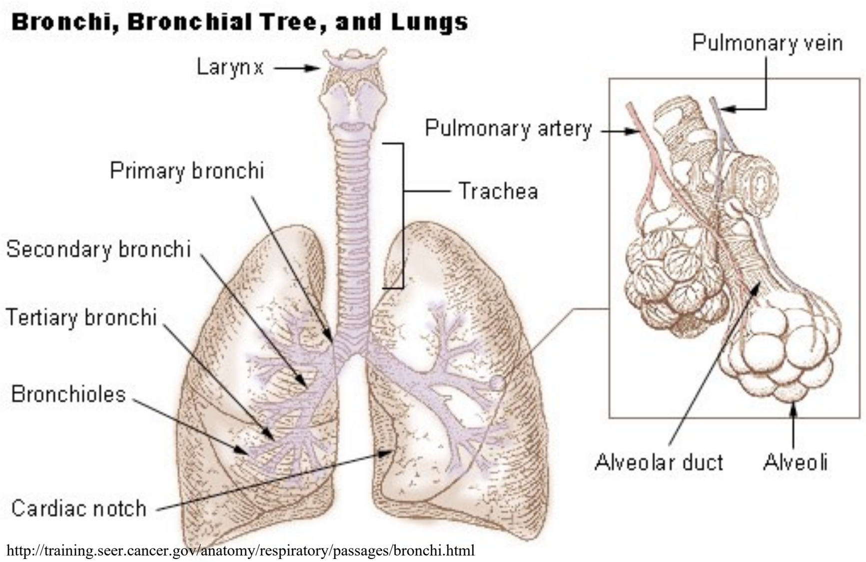
https%3A%2F%2Fgrayson.instructure.com%2Fcourses%2F63%2Ffiles%2F745964%2Fdownload%3Fwrap%3D1&bvm

The heart and lungs form a closed loop system of which forms the basis for the circulatory system.

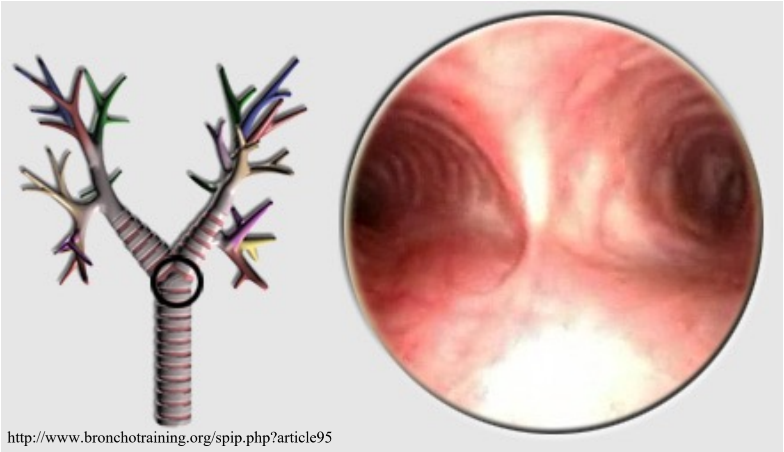
Anatomy of the Lungs



Chen, J., et al., A review of biomass burning: Emissions and impacts on air quality, health and climate in China, Sci Total Environ (2016), http://dx.doi.org/10.1016/j.scitotenv.2016.11.025



Anatomy of the Bronchi/Lungs



Normal Main Right and Left Bronchi

The Human Heart

Blood flows from the systems of the body into the right atrium where it is pushed into the right ventricle.

The blood that enters the right side of the heart from the superior (above) and inferior (below) vena cava is oxygen poor.

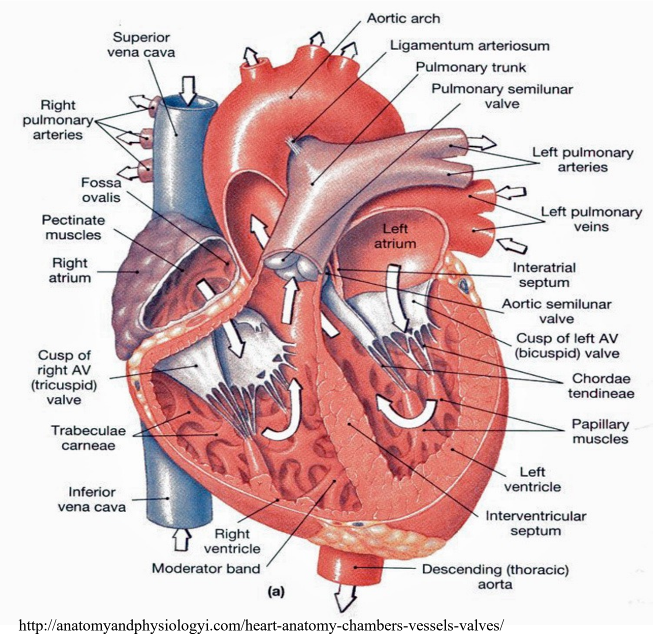
The blood is pumped from the right ventricle through the left and right pulmonary arteries to the lungs where carbon dioxide is exchanged for oxygen in the blood.

The oxygen rich blood is pulled into the left atria from the pulmonary veins and into the left ventricle.

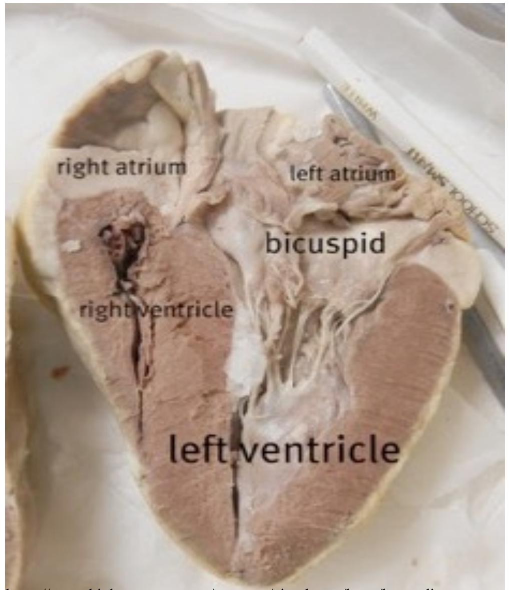
The left ventricle pumps the blood out through the aorta which ascends and splits into the carotid arteries into the head, neck and face and descends into the femoral arteries in the legs.

The blood flows from the arteries to the veins by passing through capillary beds of the major organs.

Valves in between the atria and ventricles control the blood flow so that it flows in only one direction (no regurgitation from the ventricles back into the atria.)



http://anatomyandphysiologyi.com/heart-anatomy-chambers-vessels-valves/



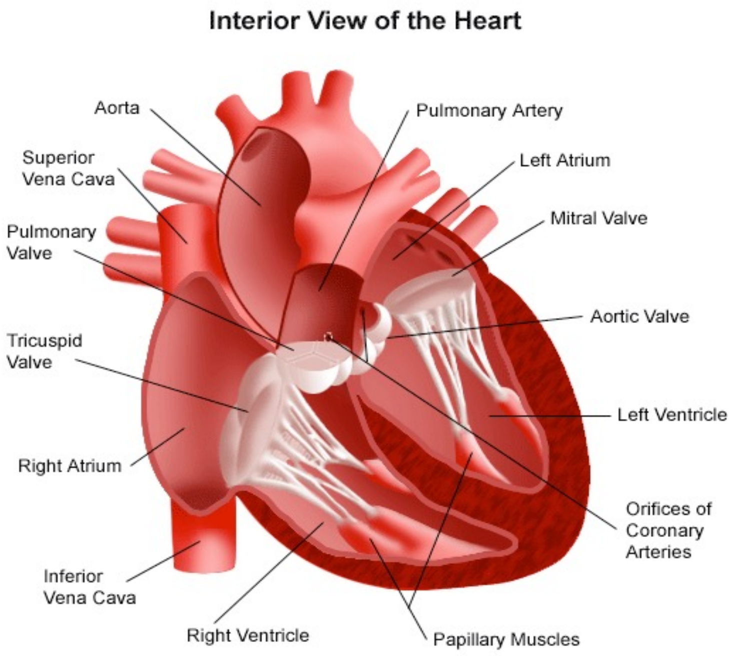
https://www.biologycorner.com/anatomy/circulatory/heart/heart_dissection.html

The Human Heart

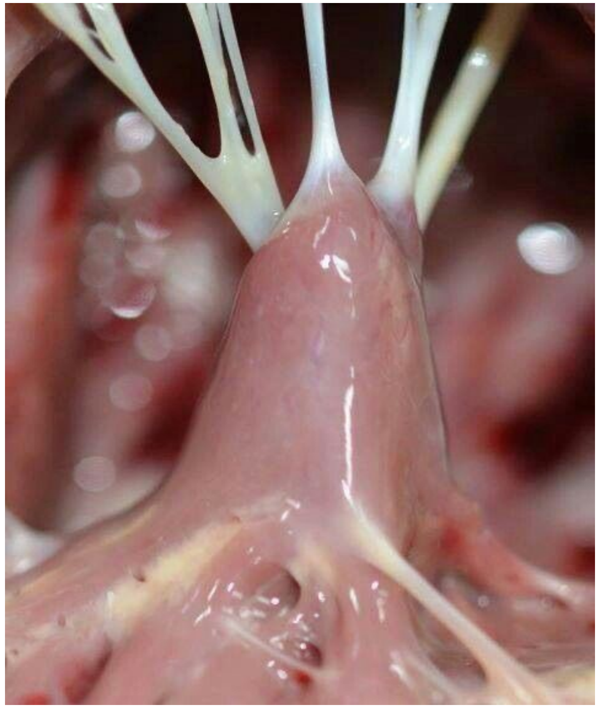
The *Chordae Tendineae* (or heartstrings) are strong, fibrous strings attached to the leaflets (or cusps) of the heart on the ventricular side; i.e., the lower chamber.

These strings originate from small mounds of muscle tissue, the *papillary muscles*, which project inward from the walls of the ventricle.

The heartstrings anchor the valves and aid in their opening/closing.



http://vector.childrenshospital.org/2012/10/new-valves-for-babies-that-can-grow-with-them/

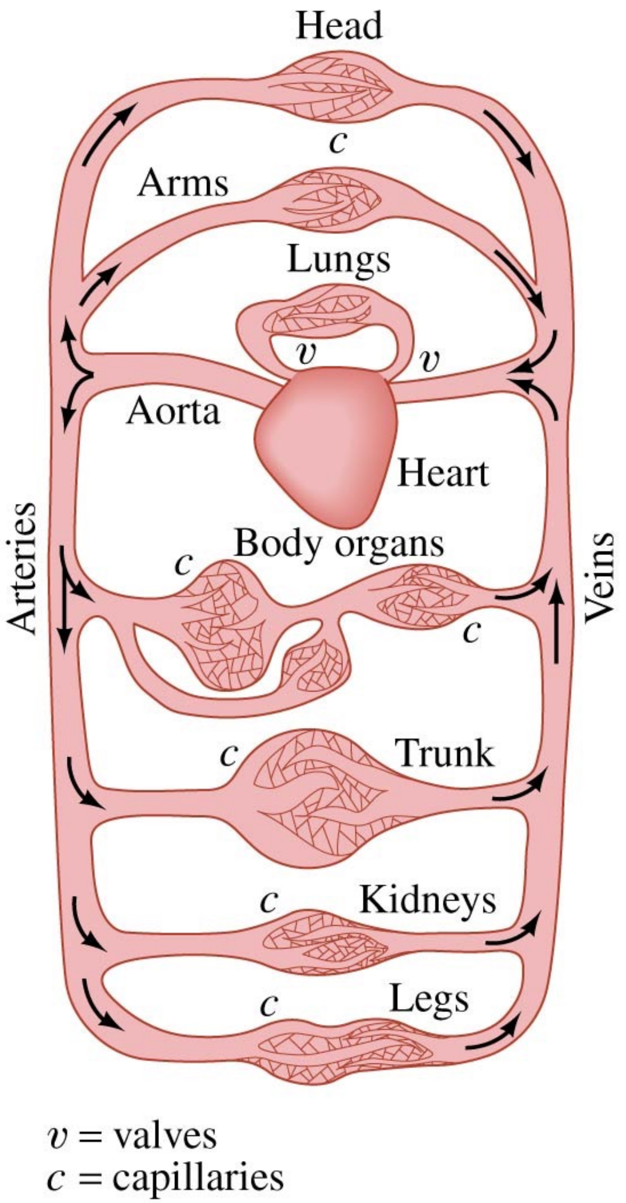


The Human Heart

Example 11:

Suppose that the aorta has an average radius of about $r_{aorta} = 1.25cm$ and the typical average blood velocity $11\frac{cm}{s}$ with an average density of $\rho_{blood} = 1050\frac{kg}{m^3}$. The human circulatory system is a closed loop system, so the flow rate of blood out of the heart has to be the same as the flow rate of blood coming back to the heart.

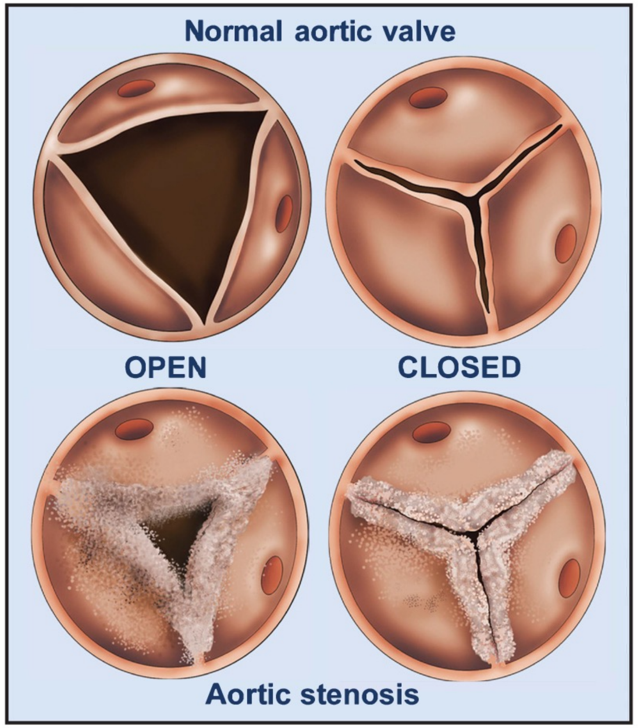
- What is the total flow rate of blood through the aorta?
- What is the average blood velocity in the major arteries if the total cross-sectional area of the major arteries is $20cm^2$?
- On the assumption that all the blood in the circulatory system goes through the capillaries, what is the total cross-sectional area of the capillaries if the average velocity of the blood in the capillaries is $0.03\frac{cm}{s}$?
- If a typical capillary has a cross-sectional area of $3\times10^{-7}cm^2$, about how many capillaries are there in the human body?
- If a capillary has an average length of $l = 0.75mm$ what is the average time that a red blood cell spends in a capillary?



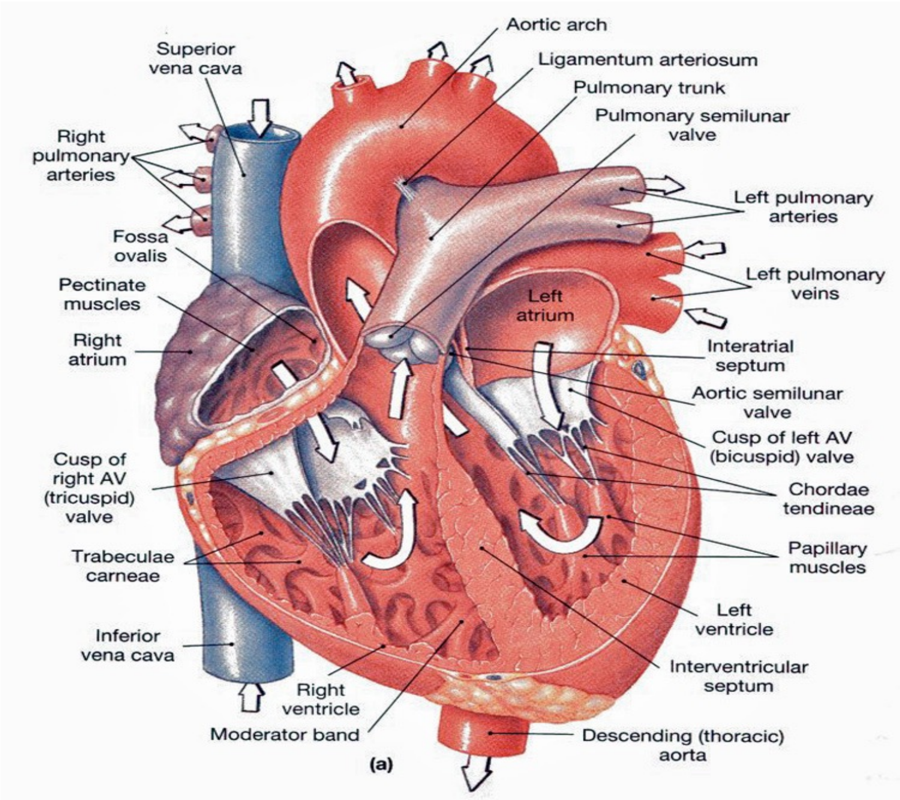
Fluids in Motion

Example 12:

A 68-year-old patient presents with shortness of breath and chest pain. A cardiologist suspects aortic valve stenosis, a condition where the heart's aortic valve becomes narrowed, forcing the left ventricle to pump harder to eject blood into the aorta. The aortic valve leads from the left ventricle to the aorta and acts as a one-way valve so that blood only flows in one direction (out to the body) and not back into the heart. Age related calcium deposits on the leaflets are the main cause of the stenosis in adults and these cause the leaflets to harden and narrow (the stenosis).



https://www.leedsth.nhs.uk/patients/resources/aortic-stenosis/



http://anatomyandphysiologyi.com/heart-anatomy-chambers-vessels-valves/

Fluids in Motion

Example 12:

The doctor orders a Doppler echocardiogram to measure the velocity of the blood. The Doppler echocardiogram is a non-invasive procedure that used ultrasound to measure the flow of fluid in the body. The ultrasound technician records the following data:

The velocity of the blood in the left ventricle just before the aortic valve $v_{LV} = 1\frac{m}{s}$.

The velocity of the blood as it comes through the aortic valve $v_{AV} = 5\frac{m}{s}$.

In cardiology, a pressure drop greater than $40mmHg$ indicates *severe* stenosis requiring surgical intervention. Would this patient be a candidate for valve surgery? Assume that the patient is lying horizontally on a table and that blood has a density $\rho_{blood} = 1050\frac{kg}{m^3}$.

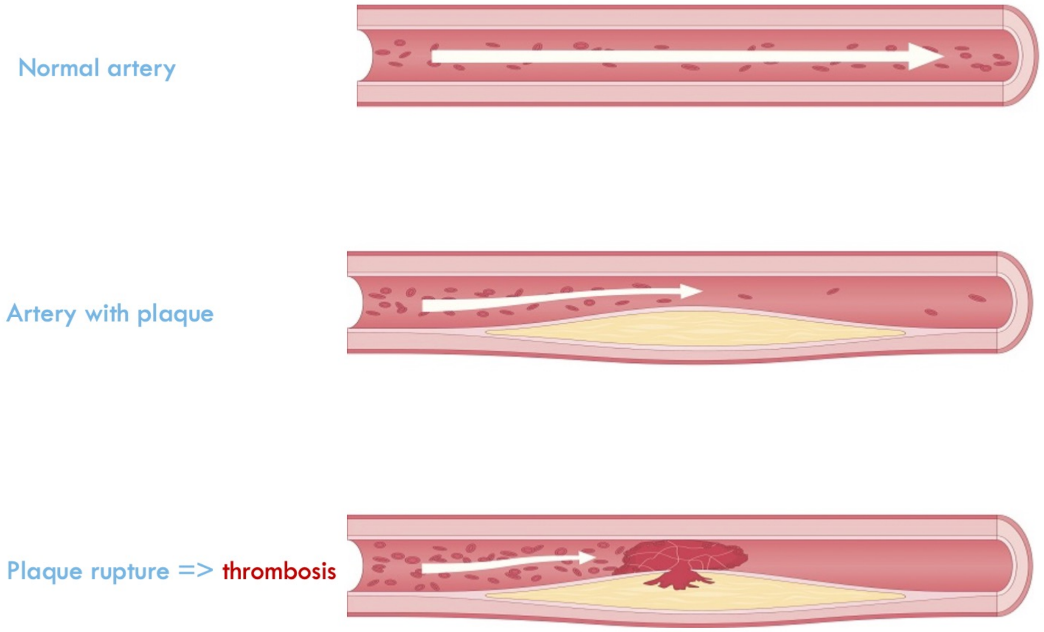
In a clinical setting physicians often use the $\Delta P = 4v^2$ "rule" to do the math without having to work out the actual calculation. What is this "rule" and where does it come from? Using this "rule", what would be the patients pressure drop?

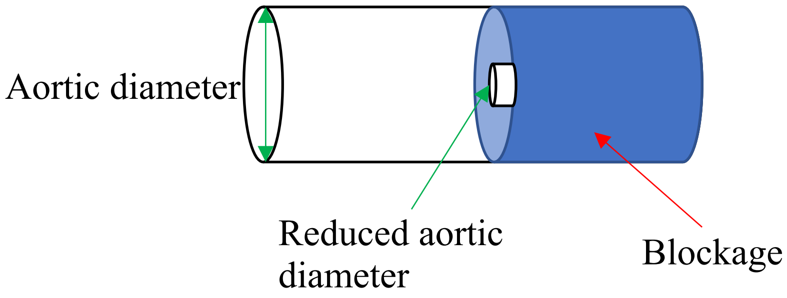
The Human Heart

Example 13:

The human heart can be modeled as a mechanical pump. The arteries carry oxygenated blood ($\rho_{blood} = 1050\frac{kg}{m^3}$) away from the heart to the major systems of the body. The aorta is a very large artery ($r_{aorta} = 1.25cm$) and arteries (such as the femoral or the carotid) that branch off of the aorta are smaller. A typical flow rate for blood out of the heart is $3\times10^{-3}\frac{m^3}{min}$ and the heart does work (with every beat) at a rate of $1\text{W} = 1\frac{J}{s}$. Suppose we were looking at the carotid ($r_{carotid} = 3\ mm$) for evidence of disease and a condition associated with arteries in the human body is *Atherosclerosis*.

Atherosclerosis is a disease in which plaque builds up inside your arteries. Arteries are blood vessels that carry oxygen-rich blood from your heart and other parts of your body. Plaque is made up of fat, cholesterol, calcium, and other substances found in the blood. Over time, plaque hardens and narrows your arteries. This limits the flow of oxygen-rich blood to your organs and other parts of your body. Atherosclerosis can lead to serious problems, including heart attack, stroke, or even death.





The Human Heart

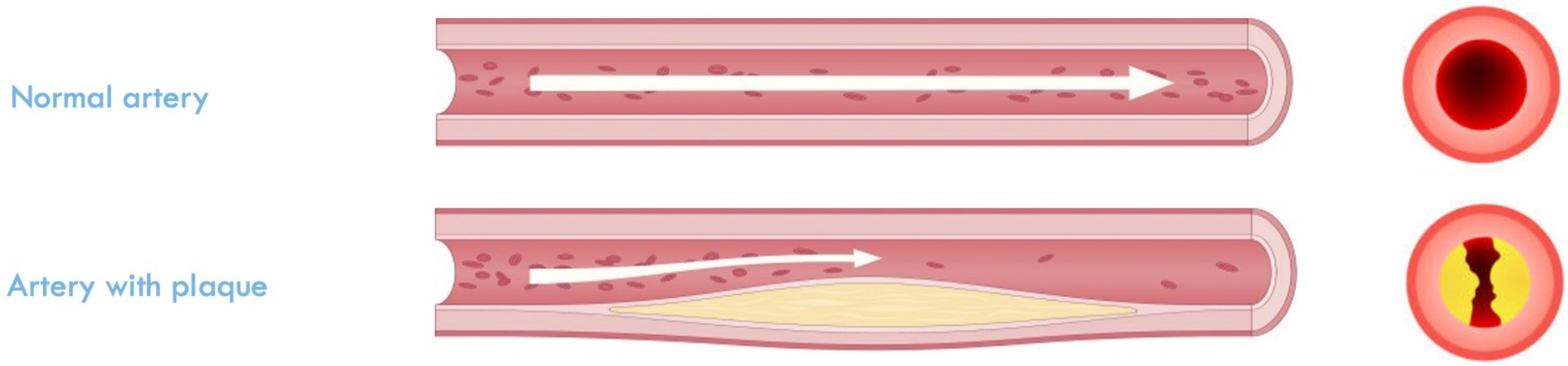
Example 12:

- What is the expression for the energy per unit volume for blood flowing in the carotid?

- What is the expression for the power produced by the heart in terms of the flow rate of blood out of the heart and the energy per unit volume of blood flowing in the carotid?

- What is the radius of the opening of the blockage in the carotid?

- What percent of the carotid is blocked? This is called the *percent stenosis* and anyone having around 70% or greater blockage is probably a candidate for surgery.



The Human Heart – Hemodynamics

The *heart rate* (HR) is the number of heartbeats per minute when performing normal tasks. What is the average resting heart rate of an adult human?

Systole is the contraction phase of the heart when blood is pumped out of the left ventricle into the aorta and from the right ventricle into the pulmonary arteries. This is the high-pressure side of the heart and the blood pressure that you'd measure is the highest pressure in the cardiac cycle and it is the shortest phase in the cycle.

Diastole - is the relaxation phase of the heart when the heart is filled with blood in the right atria from the SVC and IVC and into the left atria from the pulmonary veins. This is the low-pressure side of the heart and the blood pressure that you'd measure is the lowest pressure in the cardiac cycle. It is also the longest phase in the cycle.

Cardiac Cycle encompasses both systole and diastole, as well as the filling and emptying of the atria and ventricles. The cycle begins with diastole, followed by atrial systole (atria contract), then ventricular systole (ventricles contract), and finally, ventricular diastole (ventricles relax).

Blood pressure is the pressure measured in the arteries during systole and diastole, and we report the number as the ratio of systolic pressure to the diastolic pressure.

What is the ”normal” blood pressure in an adult human in $mmHg$? The numbers are the ratio of the *systolic pressure* (active squeezing) to the *diastolic pressure* (active resting).

The Human Heart – Hemodynamics

The *mean arterial pressure* (MAP) is defined as the average blood pressure over one cardiac cycle and is reported in terms of the systolic and diastolic pressures by:

$$MAP = \frac{1}{3}(systolic\ pressure) + \frac{2}{3}(diastolic\ pressure)$$

What is the MAP (in $mmHg$) for the blood pressure we picked?

The *stroke volume* (SV) is the volume of blood pumped by the heart (the left ventricle) in one given cardiac cycle.

It is the difference between the blood in the left ventricle at the end of diastole (EDV) and the blood in the left ventricle at the end of systole (ESV). This is on average between $50\frac{mL}{beat} - 100\frac{ml}{beat}$, so let's assume $50\frac{mL}{beat}$.

For an adult human, what is the average cardiac output (CO) in $\frac{L}{min}$? In $\frac{m^3}{min}$? How does this compare to the flow rate we've been using in these problems?

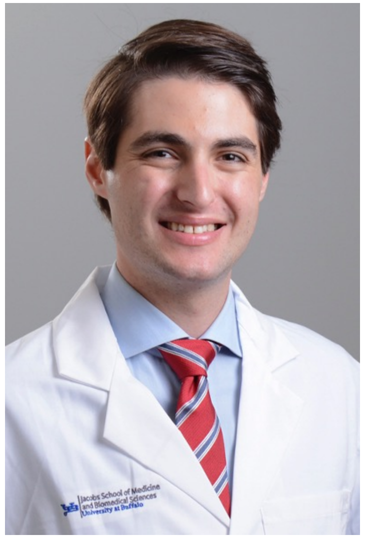
The Human Heart – Hemodynamics

As the blood flows through the system of the body (the capillaries in particular) there is a resistance to the flowing of blood (blood has a mass and hence an inertia).

This resistance is called the *total peripheral resistance* (TPR) or the *systemic vascular resistance*.

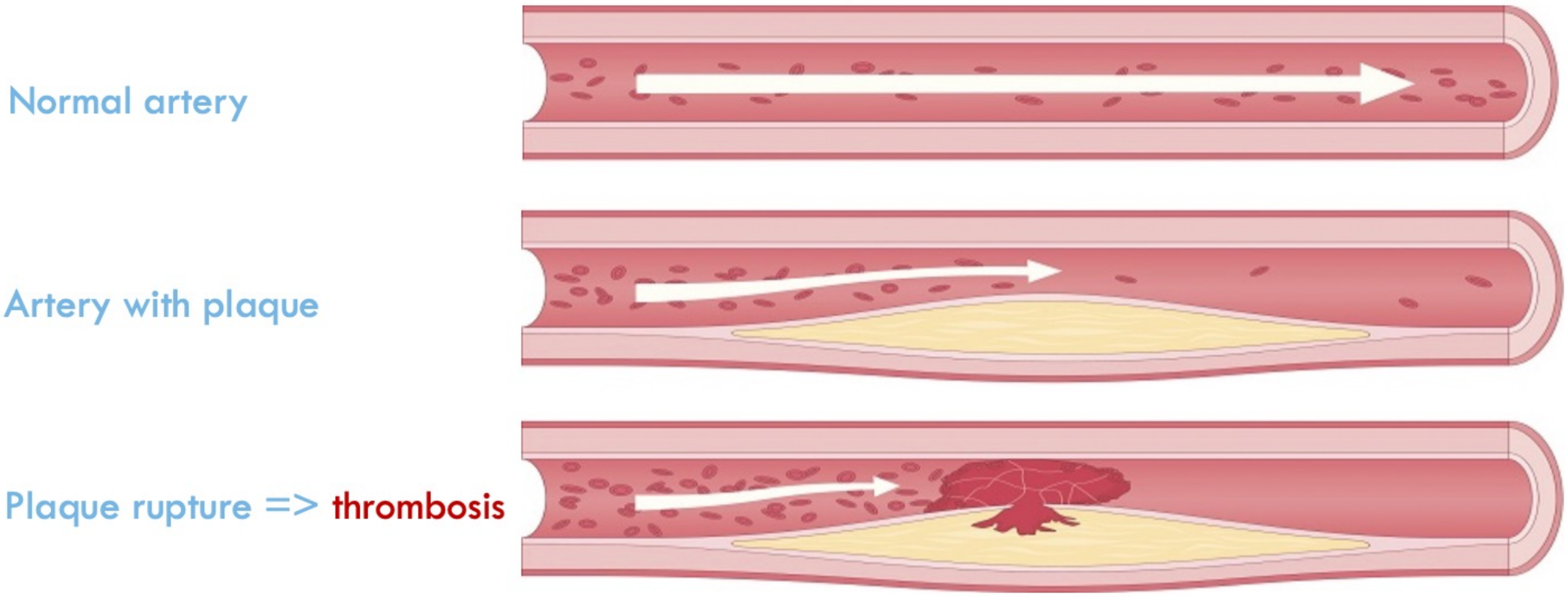
What is the average TPR for an adult human and how does this compare to $100M\Omega = 100\times10^6\frac{Ns}{m^5}$? This is called *Ohm's Law of Cardiology*!

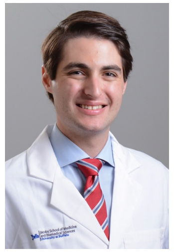
I have to give a big thank you to Dr. Mathew Kaye, MD for all of the cardiology problems and data.



Dr. Kaye is a 2016 Union college graduate (in History), a 2020 graduate of the University of Buffalo, Jacobs School of Medicine and Biomedical Sciences, and is a cardiology fellow at the Virginia Commonwealth University.

The Human Heart





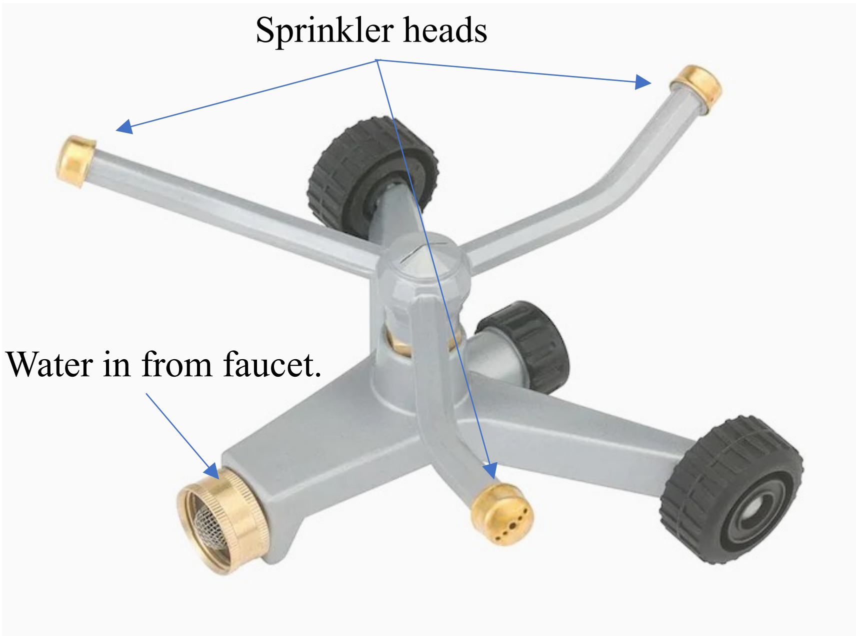
Mathew Kaye, MD
Jacobs School of Medicine and Biomedical Sciences at the University at Buffalo, Union College 2016

Fluids in Motion

Example 8:

A lawn sprinkler is being fed by a hose with a diameter of $0.019m$ ($\sim\frac{3}{4}$”). The flow rate for the water in the hose from a faucet at the house is $6.31\times10^{-5}\frac{m^3}{s}$ and the water ultimately flows out of three cylindrical sprinkler heads each with a diameter of $0.0064m$ ($\sim\frac{1}{4}$”). In the figure, ignore the “gold” looking ends and assume the water exits through one circular opening at the end of each sprinkler head.

- What is the speed of the water as it enters the sprinkler from the hose?
- What is the speed of the water as it exits from one of the sprinkler heads?
- The outflow from the sprinkler creates a torque about the center of the sprinkler and this distributes water to the lawn. If the water is turned on and the sprinkler spins at a final rate of $\omega = 61\frac{rev}{min}$ after a time of $t = 3s$, what is the radius of the sprinkler?



https://www.lowes.com/pd/Gilmour-Whirling-Heavy-Duty-692-sq-ft-Rotating-Sled-Lawn-Sprinkler/1000738722?cm_mmc=shp-_-c-_-prd-_-sol-_-google-_-lia-_--_-watering-_-1000738722-_-0&store_code=1740&placeholder=null&gclid=EAIaIQobChMIlL7M277F6QIVDNvACh1P5gIDEAQYBiABEgLUCvD_BwE&gclsrc=aw.ds