Lab 2: Projectile Motion & Errors

Name: ____________________________

Lab Partner(s): ____________________________

Honor Code Statement: I affirm that I have carried out my academic endeavors with full academic honesty. ____

Please neatly answer all of the questions in the lab packet. Make sure you attach any graphs generated, Excel files you produced, and any calculations/derivations you did. You can hand write the calculations/derivations or type them. This lab packet is due one week from the completion of the lab.

Introduction

In the last lab we looked at motion in 1-dimension with constant acceleration. In this lab we will study the motion of small metal balls fired from a spring-powered projectile launcher in both 1- and 2-dimenstions under the action of gravity (a constant acceleration). You will use two different methods to determine the velocity of the ball as it leaves the launcher, and use that velocity, together with the equations governing projectile motion, to predict the range of a projectile fired horizontally ($\theta = 0^0$) from the table top and to determine the launch angle of a projectile fired from the table top for a launch angle $0^0 < \theta < 90^0$.

In the last lab we made many measurements of quantities like position and time and in the sciences, you will routinely make measurements of various quantities for any particular experiment. To make measurements we use devices, such as a ruler to measure distance, a watch to measure time, or a thermometer to measure temperature. Each device has a limitation in how accurate it can measure the given quantity. This limitation is called the uncertainty of the device. For example, in lab 1 you used a meter stick to measure the lengths and heights. The meter stick has a scale on it and maybe the smallest division on the scale is $1mm$. Thus, the best you can probably measure something to, using a meter stick, is $1mm$. Therefore, the uncertainty in the measurement of the device, a meter stick, is $1mm$. We will begin to determine ways to calculate uncertainties for many repeated measurements of a single quantity and see how those uncertainties affect our final answer. Along the way, you will also investigate some new techniques of error analysis, and the differences between systematic and random errors.

Apparatus

The apparatus for this lab consists of a spring-powered projectile launcher (shown in Figure 1) which will fire a small ball into the air. The goal of the lab is to determine the velocity of the ball leaving the launcher, and use that velocity to predict the flight of the ball when the launcher is used to fire the ball across the room.

For the first part of the experiment, the launcher will be fired straight up into the air. Place the launcher on the floor, and make sure the protractor reads 90° measured from the horizontal, and that the screws holding the launcher to the mount are tightened down. Make sure that the launcher is not aimed at a light (launchers which are mistakenly fired on the "high" setting will fire a ball into the ceiling hard enough to shatter light bulbs and make a mess), set the launcher to the "medium" setting (the second notch down), and fire the ball by tugging sharply on the launch cord. When firing the launcher, be sure to hold it down with your free hand. For the second part of the laboratory experiment we will determine the angle at which the ball was fired by measuring parameters associated with the projectile's flight.

Figure 1: Pasco projectile launcher. https://www.pasco.com/products/lab-apparatus/mechanics/projectiles/projectile-launcher

Procedure

Part 1: Muzzle Velocity from Time of Flight

a. One method you can use to find the velocity of a ball leaving the launcher is to measure the time it takes for the ball to reach its maximum height. Fire the launcher straight up, and use a stopwatch to measure the time between the firing of the launcher and the highest point the ball reaches. Each member of your group should repeat the measurement a total of ten times. Enter the time of flight for each of your measurements in Excel and determine the average and standard deviation σ of the repeated measurements. To have Excel calculate the standard deviation of a set of measurements, in a cell type "=StDev(cell range)" without the quotes and fill in the cell range for the data you want to determine the standard deviation for. The standard deviation of a set of repeated measurements of a quantity tells us how far an individual measurement is from the average of that measured quantity or how spread out the data is.

$t_{avg} =$

$\sigma =$

b. For the uncertainty in the time measurement Δt, we will use two times the standard error of the mean ($\sigma_{sem} = \frac{\sigma}{\sqrt{N}}$) given by

$$\Delta t = 2\sigma_{sem} = 2\left(\frac{\sigma}{\sqrt{N}}\right)$$

where *N* is the number of measurements made. The standard error of the mean tells me how much the average of a sample is likely to fluctuate from the true population average. It gets smaller as your sample size grows because larger samples give you a more precise average. As the number of trials increases the uncertainty in the average value of the measured quantity decreases and we approach the true value of that quantity. The more measurements we make the better and more closely we'll get to the true value. We evaluate two times the standard error of the mean as this a 95% confidence interval about the mean. That is, any measurement we make the range of values that likely contains the true, unknown population average will fall in this range 95 out of 100 times we make the measurement. The confidence interval tells you how much uncertainty is tied to that estimate. Calculate the uncertainty in the time and record the time-of-flight as $t_{avg} \pm \Delta t$ using the correct number of significant digits.

$\Delta t =$

$t = t_{avg} \pm \Delta t =$

Part 2: Muzzle Velocity from Maximum Height

a. A second method of determining the initial velocity is to instead measure the maximum height y reached by the ball. You may need to stand on the table in order to read the height. Take a few practice shots to determine the best way to read the ball's height using the meter stick. Each member of your group should repeat the measurement a total of ten times. Enter the height the ball reaches for each of your measurements in Excel and determine the average and standard deviation σ of the repeated measurements.

$y_{avg} =$

$\sigma =$

b. For the uncertainty in the height measurement Δy we will use two times the standard error of the mean given by

$$\Delta y = 2\sigma_{sem} = 2\left(\frac{\sigma}{\sqrt{N}}\right)$$

where N is the number of measurements. Calculate the uncertainty and record the average value of y and the uncertainty as $y_{avg} \pm \Delta y$ below with the proper number of significant digits.

$\Delta y =$

$y = y_{avg} \pm \Delta y =$

Part 3: Projectile Motion – Horizontal Launch and Range

a. Clamp the projectile launcher to the edge of your lab table. Make sure that the launcher is pointing in a direction such that the projectile can travel a horizontal distance of about $2m$ without hitting anything or anyone. Set the projectile launcher to angle of $\theta = 0^0$representing the projectile launched horizontally parallel to the ground. Measure the height h of the muzzle from the floor and record it below along with an estimated uncertainty as $y \pm \Delta y$ with the proper number of significant digits. The muzzle height is the height from the floor to where the projectile leaves the launcher.

$h = y \pm \Delta y =$

b. Load the ball and fire it horizontally across the floor a few times to see about where it lands. Then tape a piece of carbon paper and a piece of paper back-to-back on the floor about where the ball lands. Then fire the ball horizontally from the launcher for a total of 6 times and measure the horizontal distances D that the ball lands from the launcher. Determine an average value of the horizontal distance $D_{avg,expt}$.

$D_{avg,expt} =$

c. Determine the uncertainty ΔD in the average horizontal distance that the projectile goes by using 2 times the standard error of the mean. Be sure to use the proper number of significant digits. Anytime you fire the ball horizontally, the ball should land within a circle of radius ΔD about the mean horizontal distance.

$\Delta D_{expt} = 2\sigma_{sem} = 2\left(\frac{\sigma}{\sqrt{N}}\right) =$

d. What is the experimental horizontal distance D' traveled by the ball and its uncertainty?

$D_{expt} = D_{avg,expt} \pm \Delta D_{expt} =$

Part 4: Projectile Motion – Random Launch Angle

a. Clamp the projectile launcher to the edge of your lab table. Make sure that the launcher is pointing in a direction such that the projectile can travel a horizontal distance of about $2m$ without hitting anything or anyone. Have your instructor set the projectile launcher to a random angle between $0 \leq \theta_{launch} \leq 90^0$. Record the angle below along with an estimate of the uncertainty in the angle from the gauge on the apparatus.

$\theta_{launch} = \theta_{launch,theo} \pm \Delta\theta_{launch,theo} =$

b. Measure the height h of the muzzle from the floor and record it below along with an uncertainty as $y \pm \Delta y$ with the proper number of significant digits. The muzzle height is the height from the floor to where the projectile leaves the launcher.

$h = y \pm \Delta y =$

c. Load the ball and fire it at this unknown angle θ a few times to see where the ball lands. Tape a piece of carbon and regular paper back-to-back at the location the ball lands and then fire the ball 6 times and determine the average horizontal distance and its uncertainty (using 2 times the standard error of the mean) in the same manner as you did for the horizontal range. Measure the horizontal distance D' the ball lands from the launcher and record it as $D'_{avg,expt} \pm \Delta D'_{expt}$ with the proper number of significant digits.

$\Delta D'_{expt} = 2\sigma_{sem} = 2\left(\frac{\sigma}{\sqrt{N}}\right) =$

$D' = D'_{avg,expt} \pm \Delta D'_{expt} =$

Analysis, Post-Lab Questions, and Discussion

Name:

Lab Partner(s):

Due Date: Tuesday, September 29, 2026 at the beginning of lab.

Honor Code Statement:

Part 1: Muzzle Velocity from Time of Flight

1. What were the average time of flight of your projectile and the uncertainty in the time of flight? Show the calculation for the uncertainty in the time of flight of the projectile.

   $t = t_{avg} \pm \Delta t =$

   $\Delta t = 2\sigma_{sem} = 2\left(\frac{\sigma}{\sqrt{N}}\right) =$

2. Using an appropriate equation for projectile motion derive an expression for the average velocity $v_{avg,1}$ of the projectile that involves the time of flight. Then evaluate the average initial velocity of the ball when it left the launcher using the numbers you have.

   $v_{avg,1} =$

3. Derive an expression for the uncertainty in the velocity Δv_1 from the uncertainty in the time measurement and then evaluate the uncertainty using the numbers you have. Record the final answer in the form $v_{avg,1} \pm \Delta v_1$ in the space below with the proper number of significant digits.

   $\Delta v_1 =$

   $v_1 = v_{avg,1} \pm \Delta v_1$

4. What are some of the sources of the uncertainty in the velocity calculation that you made and why? What do these sources do to the value of the speed determined? Explain your answers.

Part 2: Muzzle Velocity from Maximum Height

1. What was the average maximum height of your projectile and the uncertainty in the maximum height? Show the calculation for the uncertainty in the maximum height of the projectile.

   $y_{max} = y_{max,avg} \pm \Delta y =$

   $\Delta y = 2\sigma_{sem} = 2\left(\frac{\sigma}{\sqrt{N}}\right) =$

2. Using an appropriate equation for projectile motion derive an expression for the average velocity $v_{avg,2}$ of the projectile that involves the maximum height. Then evaluate the average initial velocity of the ball when it left the launcher using the numbers you have.

   $v_{avg,2} =$

3. Derive an expression for the uncertainty in the velocity Δv_2 from the uncertainty in the maximum height measurement and then evaluate the uncertainty using the numbers you have. Record the final answer in the form $v_{avg,2} \pm \Delta v_2$ in the space below with the proper number of significant digits.

   $\Delta v_2 =$

   $v_2 = v_{avg,2} \pm \Delta v_2$

4. What are some of the sources of the uncertainty in the velocity calculation that you made and why? What do these sources do to the value of the speed determined? Explain your answers.

5. Of the two methods to determine the initial launch velocity of the ball which do you think is a more accurate value, the value determined from the time to maximum height v_1 or the value determined from the maximum height v_2? For the method you chose as the more actuate one explain why you think that was more accurate and what made it more accurate. Or, another way, why is the other method less accurate and what make is less accurate?

Part 3: Projectile Motion – Horizontal Launch and Range

1. Using your best value of the launch speed of the ball and the height the ball fell through h, derive an expression for the average horizontal distance that the projectile should travel when launched horizontally. Evaluate your result for the numbers you have.

   $D_{avg} =$

2. We want to determine an experimental expression for the horizontal distance D that the ball will go when fired horizontally off of the table including the uncertainties in this distance ΔD. In order to determine the uncertainty in the distance you calculated in question #2, we're going to use the following method, called the *propagation of uncertainties*. Suppose that we have a quantity f which depends on two variables, say x and y. That f is a function of both x and y, or $f(x,y)$. The uncertainty in f depends on the uncertainties in x (which we'll call Δx) and on the uncertainties in y (which we'll call Δy). To determine the uncertainty in f (which we'll call Δf) we need to evaluate the following:

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \Delta x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \Delta y^2} \quad eqn\ 1,$$

where, $\frac{\partial f}{\partial x}$ is how the function f varies with x (which means take the derivative of f with respect to x (treating all other variables (y) as constants). This expression could be negative, so we square it to make it positive and then dimensionally we multiply by the uncertainty in x squared, Δx^2. In an analogous manor, $\frac{\partial f}{\partial y}$ is how the function f varies with y (which means take the derivative of f with respect to y (treating all other variables (x) as constants). This expression could be negative, so we square it to make it positive and then dimensionally we multiply by the uncertainty in y squared, Δy^2. We call this process, *adding the uncertainties in quadrature*.

Using your best value of the launch speed of the ball derive an expression for the uncertainty in the horizontal distance traveled by the projectile, ΔD. Then evaluate the uncertainty using the numbers you have and report the theoretical horizontal displacement of the projectile as $D_{theo} = D_{avg,theo} \pm \Delta D_{theo}$. This represents the theoretical average distance $D_{avg,theo}$ plus or minus a circle of radius ΔD_{theo} about this average value that the ball should go when fired horizontally.

$\Delta D_{theo} =$

$D_{theo} = D_{avg,theo} \pm \Delta D_{theo} =$

This page is blank and can be used for the derivation asked for in question #2.

3. What was the measured horizontal distance (and its uncertainty) traveled by the projectile when fired horizontally? How does this value compare the expected horizontal distance calculated in question #3? Do the values agree within experimental uncertainty? If they do not, explain why you think they do not. Calculate a percent difference between the two average values of the horizontal distance traveled.

   $D_{theo} = D_{avg,theo} \pm \Delta D_{theo} =$

   $D_{expt} = D_{avg,expt} \pm \Delta D_{expt} =$

Part 4: Projectile Motion – Random Launch Angle

1. Using your best value of the launch speed of the ball, the average horizontal distance $D'_{avg,expt}$ and the height the ball fell through h, derive an expression for angle at which the projectile was launched, θ_{launch}. Then evaluate your result for the numbers you have.

   $\theta_{launch} =$

2. Ideally, we should determine and uncertainty in the number for θ_{launch}. To determine the uncertainty, we need to use $eqn1$. Notice that in the equation you determined in question #1 above there are two angles that you obtained by solving a quadratic. The solutions both involve a square root with multiple terms under that square root. Determining the uncertainty in θ_{launch} by $eqn1$ is a rather long and very tedious set of derivatives to take and then to evaluate. Rather than calculating the uncertainty in θ_{launch} and seeing if the theoretical and experimental values compare within uncertainties, let's just do this. How does your calculated value ($\theta_{launch,expt}$) compare to the theoretical value with its uncertainty ($\theta_{launch,theo} \pm \Delta\theta_{launch,theo}$) that your instructor set? Does $\theta_{launch,expt}$ fall within the range $\theta_{theo} \pm \Delta\theta_{theo}$ or outside of this range? Calculate a percent difference between $\theta_{launch,theo}$ and $\theta_{launch,expt}$ and comment on any similarities or discrepancies between the two values.