Lab 3: A Study of Air Resistance and an Introduction to Error Analysis

Name: ______________________________

Lab Partner(s): ______________________________

Honor Code Statement: I affirm that I have carried out my academic endeavors with full academic honesty. ____

Introduction

In labs 1 & 2 we made measurements of the motion of an object, a cart in lab 1 and a projectile in lab 2, ignoring the object's interaction with the air. We know, though, that the force of air resistance can be significant, as a sheet of paper falls more slowly than a metal ball. Or perhaps when you were riding in a car with your hand out of the window you notice that when the palm of your hand is parallel to the road your hand slides easily through the wind but when the palm of your hand is perpendicular to the road you encounter a strong pushback on your palm. This is due to air resistance as your hand is pushed through the air and we can quantify this pushback by something called the *drag coefficient* of an object. In this lab, you will experimentally determine the strength of the force of air resistance on a falling body. One interesting (and useful) consequence of air resistance is the concept of *terminal speed*. As a body falls, at some point, it stops accelerating and since the acceleration vanishes, the object's velocity does not change and the object falls at a constant speed from that point forward. Since this is the largest speed the object acquires, we call this the object's terminal speed.

In the last two labs we made many measurements of quantities like position and time and in the sciences, you will routinely make measurements of various quantities for any particular experiment. To make measurements we use devices, such as a ruler to measure distance, a watch to measure time, or a thermometer to measure temperature. Each device has a limitation in how accurate it can measure the given quantity. This limitation is called the uncertainty of the device. For example, in labs 1 & 2 you used a meter stick to measure the lengths and heights. The meter stick has a scale on it and maybe the smallest division on the scale is $1mm$. Thus, the best you can probably measure something to, using a meter stick, is $1mm$. Therefore, the uncertainty in the measurement of the device, a meter stick, is $1mm$. Different devices have different uncertainties and you need to know what the uncertainty in these measuring devices are. Why? Any quantity that you are determining is affected by the uncertainties in the measuring devices used. No device is perfect. The result of an experimentally determined quantity is a best guess plus or minus some amount. It tells a reader how good the measurements are and how well I can trust your results. In this lab, we'll start looking at how our measurement uncertainties affect our final results by *propagating the uncertainties* through calculations.

Procedure

Part 1: General Observations

1. Take a flat-bottom coffee filter, stand on the table top, hold the coffee filter as high up as you can, right-side up, as it would be in a coffee maker. Let go of the coffee filter, letting it drop to the floor, and watch. Does it reach terminal speed? Explain why or why not. If it does, how quickly does it reach terminal speed?

2. Now drop a pack of 4 coffee filters. Does the pack reach terminal speed? Explain why or why not.

3. Drop one coffee filter and pack of 4 at the same time. Do they reach the ground at the same time? Do they reach the same terminal speed? Explain.

4. Considering the answer to #3, what must the magnitude of the force of air resistance equal in both cases. (There is a measurement that you'll need to make to complete the calculation.)

5. Are the magnitudes of the air resistance forces the same?

6. The very instant after you let go of the coffee filter, when its speed is zero, what is the force of air resistance?

7. Considering the answer to #6, does the force of air resistance depend on the body's speed? Or mass? Explain why it does or why it does not.

8. Why might the strengths of air resistance forces in the two cases be different? What parameter(s) differ in the two cases that could have an effect on the force of air resistance? *Hint: Consider your own experience with air resistance. Imagine sticking your hand out of the window in a moving car and consider when the force on your hand is larger?*

Part 2: The Experiment

1. Open Excel and in column #1 label $m_1(kg)$ through $m_8(kg)$. Select a single coffee filter and measure its mass and enter it in Excel in the 2nd column 2nd row corresponding to $m_1(kg)$.

   $m_1 =$

2. Measure the masses of several different packs of coffee filters and enter the data in the respective rows in column 2 for $m_2(kg)$ through $m_8(kg)$. In the 3rd column determine the mass of a single filter by typing "=B2/1" without the quotes in cell C1 and hit enter. This will determine the mass of a single filter form the stack of 2 filters. Repeat this process for the remaining stacks determine the average mass of a single filter in column #3. Do not use the fill down command in this step as the calculation will be in correct. Instead in each subsequent cell in column #3 change the cell so that it reads "=Bn/n", where n is the cell number and also the number of coffee filters.

   $m_2 =$ $\rightarrow m_1 =$

   $m_3 =$ $\rightarrow m_1 =$

   $m_4 =$ $\rightarrow m_1 =$

   $m_5 =$ $\rightarrow m_1 =$

   $m_6 =$ $\rightarrow m_1 =$

   $m_7 =$ $\rightarrow m_1 =$

   $m_8 =$ $\rightarrow m_1 =$

3. Determine the average mass of a single coffee filter, $m_{1,average}$ in Excel by typing "=Average(C1:C8)". From the measurements you made in question #2, what is the uncertainty in the mass Δm_1 of a single coffee filter (with its units) and what is the mass of a single coffee filter $(m_{1\,filter})$ along with its uncertainty (and units)? Hint: You made a series of measurements. How do you find the uncertainty from many repeated measurements? If you don't remember, go back to lab #2 and look. You will need the "StDev()" command.

   $\Delta m_1 =$

   $m_{1\,filter} = m_{1,average} \pm \Delta m_1 =$

4. In Excel make columns labeled $t_1(s)$ through $t_8(s)$ and a column labeled $v\left(\frac{m}{s}\right)$. Start by dropping the single filter from the ceiling to the floor and measure the time it takes t_1 to fall from 2 meters to the ground. Repeat this step 5 times and average the results and determine the uncertainty Δt_1 as you did in question #3.

   $t_1 \pm \Delta t_1 =$

5. Stack 2 coffee filters (one inside the other), and repeat the drop process (taking five measurements) to get an average drop time for the stack of two coffee filters $t_2$and determine the uncertainty in this average Δt_2.

   $t_2 \pm \Delta t_2 =$

6. Do the same for remaining stacks of filters.

   $t_3 \pm \Delta t_3 =$

   $t_4 \pm \Delta t_4 =$

   $t_5 \pm \Delta t_5 =$

   $t_6 \pm \Delta t_6 =$

   $t_7 \pm \Delta t_7 =$

   $t_8 \pm \Delta t_8 =$

The goal is to determine, empirically, how the force of air resistance depends on the velocity of the falling object. Since most laws of physics involve power laws, we start with the assumption that the air force goes as

$$F_{air} = Dv^p \quad eqn\ 1$$

where p is the "power" index of the speed in the drag force of air resistance equation and D is some overall constant. The goal in this lab is to determine the value of p. If the air force is in fact a power law relationship, then a plot of $\log F$ versus $\log v$ should be a linear relationship as we saw in lab #1.

$$\log F_{air} = \log D + p \log v \quad eqn\ 2$$

Note that this is the equation of a straight line: $y = mx + b$, where the slope (m) equals the power index (p) of the velocity and $b = \log D$ is the y-intercept. So, if the force of air resistance is indeed a power-law, the plot of $\log F_{air}$ versus $\log v$ should be a straight line ***and*** the slope of that straight line equals the power index of the velocity, p. This is what we're going to experimentally determine.

7. Use Excel to determine the terminal velocity of each of the stacks of coffee filters using the times of fall for each set of coffee filters. Then create a column called $\log v$ and calculate $\log v$. Here, we want to calculate the base-10 log of the terminal velocity, v. To do this, in the 2nd cell of the $\log v$ column, type "=Log10(cell containing v value)" without the quotes and then click on the cell that has the first value of v. Drag down the column to calculate all of the $\log v$ values for each value of the terminal speed v.

8. Again, using Excel, label a column and calculate a value of F_{air} for each run and then for each value of F_{air} calculate $\log F_{air}$ using the method of question #7.

9. Insert a scatter plot chart (look back at lab #1 if you forgot how) and right-click on the chart to select the data in the $\log F_{air}$ and $\log v$ columns to construct a plot of $\log F_{air}$ (on the y-axis) versus $\log v$ (on the x-axis). If the data fit a straight line, right-click on the data points and fit a trendline to the data along with the equation of the fit. If your data do not fit a straight line, ask your instructor for help.

Analysis, Post-Lab Questions, and Discussion

Name:

Lab Partner(s):

Due Date: Tuesday, October 6, 2026 at the beginning of lab.

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1. When you first let go of a single coffee filter and watched it fall to the floor, did it reach terminal speed? Explain why it did or why it did not. How do you know it reached a terminal speed or not?

2. If the coffee filter does reach a terminal speed, about how quickly did it reach terminal speed?

3. When you dropped a pack of 4 coffee filters, did the 4-pack reach terminal speed? Explain why or why not and how you know if it did or did not.

4. For the case that you dropped one coffee filter along with a pack of 4 at the same time, did they reach the ground at the same time? From the answer to this, do they reach the same terminal speed? Explain why they do or why they do not.

5. If the single coffee filter and the 4-pack do not reach the same terminal speed, what would account for the difference in the terminal speeds. If they do reach the same terminal speed, what accounts for them reaching the same terminal speed.

6. From a carefully labeled free-body diagram, what is the expression for Newton's 2^{nd} law when you first release the coffee filters from rest?

7. What is the expression for the magnitude of the air force F_{air} when the coffee filters reach terminal speed? That is, what is the expression for F_{air} when the filters no longer accelerate downward?

8. Assuming that the filters reach terminal speed very quickly what is the expression for the terminal speed of the filters? Hint: Use one of your equations of motion to answer this question.

9. From your plot of $\log F_{air}$ versus $\log v$ do the data fit a straight line? If so, what is the relationship between F_{air} and v?

10. From your plot of $\log F_{air}$ versus $\log v$, assuming that the data do follow a linear relationship what is the expression for the fit to the data? Express the fit in terms of the quantities that are actually plotted. What are the values for the slope and intercept of the line with appropriate units?

    fit:

    $slope: m =$

    $intercept: b =$

11. We want to know what the power p of the velocity term is along with its uncertainty. To determine the uncertainty in the power p of the velocity term, perform a linear regression on the data. To do this, in Excel, click on the *Data* menu and on the right-hand side you should see "*Data Analysis*". Click on *Data Analysis* and choose "*Regression*". In the Regression pop-up window, select the data of $\log F_{air}$ (as the *Input Y Range*) and $\log v$ (as the *Input X Range*) and click "ok". This will perform a linear regression analysis on the data. Since the data are linear, Excel fit the data with a straight line. Imagine doing the fit by hand. You could envision putting several straight lines through the data and this would slightly change the slope and intercept of the line. Thus, there is some uncertainty in the slope and y-intercept values. Linear regression is a method for determining how far each data point is from the linear fit and therefore gives an *uncertainty* in the slope and y-intercept values. In the new chart that appears, the values given are the slope (and immediately to the right its uncertainty) and *y*-intercept (and immediately to the right its uncertainty), in the lower left corner. What is the uncertainty in the slope Δp and intercept Δb with their units?

$\Delta p =$

$\Delta b =$

We want to determine an experimental expression for the air resistance force $F_{air,expt}$ including the uncertainties. From $eqn1$ we have that $F_{air,expt} = Dv^p$ from which we can write $F_{air,expt} = (D \pm \Delta D)v^{(p \pm \Delta p)}$ to include the uncertainties. In the expression for the air resistance force $F_{air,expt}$ you can easily do the power of the velocity p and its uncertainty Δp. How do you do handle the y-intercept and its uncertainty since they are logs? To do this, we're going to use the following method. Suppose that a quantity f depends on two variables x and y. That is $f(x, y)$. The uncertainty in f depends on the uncertainties in x (which we'll call Δx) and on the uncertainties in y (which we'll call Δy). To determine the uncertainty in f (which we'll call Δf) we need to evaluate the following:

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \Delta x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \Delta y^2} \quad eqn\ 3,$$

where, $\frac{\partial f}{\partial x}$ is how the function f varies with x (which means take the derivative of f with respect to x (treating all other variables (y) as constants). This expression could be negative, so we square it to make it positive and then dimensionally we multiply by the uncertainty in x squared, Δx^2. In an analogous manor, $\frac{\partial f}{\partial y}$ is how the function f varies with y (which means take the derivative of f with respect to y (treating all other variables (x) as constants). This expression could be negative, so we square it to make it positive and then dimensionally we multiply by the uncertainty in y squared, Δy^2. We call this process, adding the uncertainties in quadrature. To calculate the uncertainty in D, the constant term, we'll use $eqn3$. From $eqn2$ we see that $b = \log D$ so that $D = 10^b$. The uncertainty in D from $eqn3$ is $\Delta D = \sqrt{\left(\frac{\partial D}{\partial b}\right)^2 \Delta b^2} = \frac{\partial D}{\partial b}\Delta b$. We have Δb from performing the linear regression on the data. We need to determine $\frac{\partial D}{\partial b}$. This is a little trickier. This is not the power rule from calculus for the derivative (where the base is variable and the power is constant), but rather the exponential rule (where the base is constant and the exponent varies). Let's write the following in terms of the natural log:

$$D = 10^b = e^{\ln 10^b} = e^{b \cdot \ln 10} \quad eqn4.$$

Now we can determine $\frac{\partial D}{\partial b}$ by taking the derivative in $eqn4$.

$$\frac{\partial D}{\partial b} = \frac{\partial}{db}\left(e^{b \cdot \ln 10}\right) = (\ln 10)e^b = 2.3e^b \quad eqn5.$$

Therefore, the uncertainty in the constant term, ΔD is:

$$\Delta D = \sqrt{\left(\frac{\partial D}{\partial b}\right)^2 \Delta b^2} = \frac{\partial D}{\partial b}\Delta b = 2.3e^b \Delta b \quad eqn6.$$

Using $eqn6$, evaluate ΔD with its units and write the expression for the air resistance force $F_{air,expt}$.

$\Delta D =$

$F_{air,expt} = (D \pm \Delta D)v^{(p \pm \Delta p)}$

12. What are some limitations on the air resistance force expression $F_{air,expt}$ that you acquired? Explain.

13. The theoretical air resistance force expression is found to obey

$$F_{air} = \frac{1}{2} C \rho A v^2 \quad eqn\ 7$$

where ρ is the density of air, taken to be $\rho = 1.3\frac{kg}{m^3}$, A is the cross-sectional area of the object presented to the air, and C is a constant called the drag coefficient. This is called *quadratic drag* and large sized objects obey this law when they are moving at medium-to-high speeds through a relatively dense fluid like air. Determine the cross-sectional area A of a coffee filter along with its uncertainty ΔA. You will need to use $eqn3$ to determine ΔA and show the derivation of ΔA below.

$A_{expression} =$ $A_{value} =$

$\Delta A_{expression} =$ $\Delta A_{value} =$

14. Using $eqn1$ and $eqn7$ for F_{air} and the results from the fit from the plot of $\log F_{air}$ versus $\log v$, determine the drag coefficient C of a coffee filter, with its units. Show your calculation below.

    $C =$

15. What is the uncertainty ΔC in the value of drag coefficient C (with appropriate unites)? Show the work below or on a sperate sheet of paper. Then express the experimental drag coefficient as $C_{expt} = C \pm \Delta C$. Hint: Evaluate ΔC using the $eqn3$.

    $\Delta C =$

    $C_{expt} =$

16. How reasonable of a value is the value for the drag coefficient you obtained? Does this value of or expression for drag coefficient work for any or all objects dropped in the presence of air? Explain why or why not.