Physics 110
Lab 4
Hooke's Law and Work Done in Breaking a Board

Theory:

Spring Force:

Hooke's Law describes the force applied by a spring when the length of the spring is stretched or compressed away from its equilibrium position. The ***equilibrium-restoring force*** of the spring is given by

$$\vec{F}_s = -k\,\vec{d}$$

where k is the spring constant and $\vec{d}$ is the displacement of the end of the spring relative to its relaxed position. (Note, this only applies if $\vec{d}$ is along the physical direction of the spring). A strong and stiffer spring has a larger value of k.

Hooke's Law also describe any force that brings a system back to an equilibrium when it is near the equilibrium.

Work:

The work done by a constant force F moving a body a distance d is given by

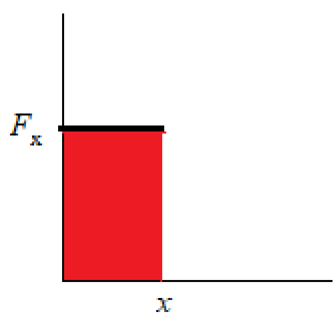
$$W = \vec{F}\cdot\vec{d}$$

If the body moves in the x-direction a distance x then the work by done by the force is

$$W = F_x\, x,$$

where F_x is the component of the force in the x-direction.

On a graph with F_x on the y-axis and x on the x-axis, the work done is equal to the height times the width of the rectangle under the line showing the force F_x occuring over the distance x. For one-dimensional motion, the work can be said to be equal to the "area under the Force vs. distance" curve.



Work by a Variable Force

The total work done by a variable force can be considered as a sum of the work done in each small step, Δx, i.e.

$$W = \sum F\Delta x = F(x_1)\,(x_2 - x_1) + F(x_2)\,(x_3 - x_2) + \cdots$$

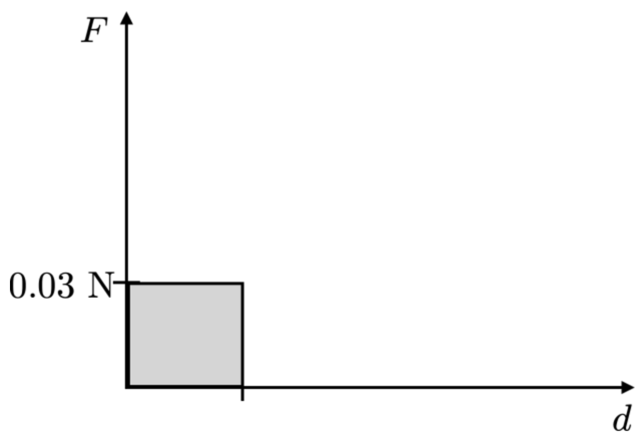
Exercises:

1. Sketch a plot of the magnitude of the spring force as a function of the displacement of the end of the spring.

2. Imagine a spring with $k = 30\ N/m$ and an external force of magnitude 0.03 N pulls on the spring.

(a) By how much does the spring stretch to come to a new equilibrium?

(b) The graph below represents the stretching of the spring due to this external force. Add a label showing the *x* distance that the spring moves.



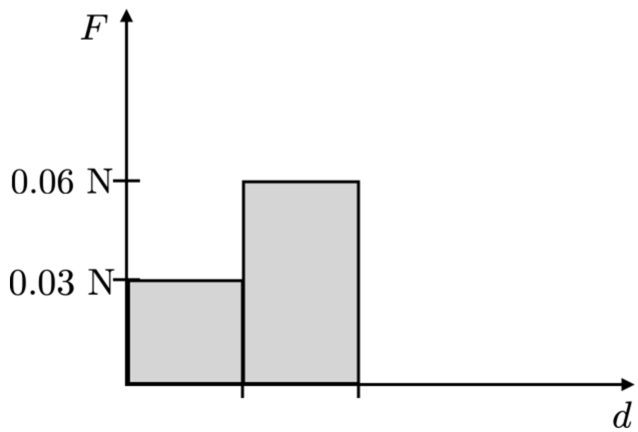
How much work is done?

Is this work done by the spring or on the spring?

3. Imagine this spring is further stretched by an additional force of 0.03 N so that the total force now becomes 0.06 N.

(a) By how much does the spring stretch to come to a new equilibrium now?

(b) Here is a graph showing the total stretching of the spring. Add labels showing the *x* distances that the spring moves.



How much work is done during this second stretching?

What is the total work done in stretching the spring from its relaxed position?

4. Imagine that this spring is then stretched further by 0.03 N so the total force is now 0.09 N. How far does the spring stretch to its new equilibrium?

How much work is done during this third stretching?

What is the total work done in stretching the spring from its relaxed position?

5. Write out the sum equation for the total work calculated in question 4.

6. How does the sum equation in question 5 relate to the graph in question 4?

Experiment:

Part I: Springs

In this part of the lab, you will observe how well Hooke's Law describes the restoring force of a spring, obtain a representative plot of F vs. x, and determine the spring constant for two springs.

1. Open Excel or Google Sheets to record your data.
2. Hang a spring from the hook. Measure the position of the end of the spring and record this as the spring's *relaxed position*.
3. Hang a small mass on the end of the spring, allow the mass to come to rest, and measure and record the new position of the end of the spring (*not the end of the mass*). Since the mass is stationary, the spring force must exactly cancel the gravity of the mass. Use this to calculate and record the spring force.
4. Hang a slightly larger mass on the spring, measure the new position of the end of the spring, and calculate the force of the spring.
5. Repeat for a number of masses (use at least five masses, total).

Analysis

7. Make a plot of the spring force vs. the stretch of the spring. Does your spring obey Hooke's Law? What aspects of the plot supports your claim? Do your curves fit proportionalities (a linear relation through the origin)?
7. Fit a straight line to each curve and use the fits to calculate the spring constant. Should you demand that the straight line go through the origin?
8. Do a "regression" analysis to get the uncertainties in the fit parameters. Does a zero y-intercept agree with your fits?
9. Record your spring constant, k, with uncertainties.

Part II: Restoring Force of Wood and the Work Done in Breaking a Board

When a force is applied to a wood board, the board must exert a force in return to hold itself together. But there is a limit; if the external force does enough work on the board, it will break.

1. Measure and record in a net data table, the mass of five different bricks and the average. Calculate the standard error in your inferred mass of an individual brick.
2. Measure and record the mass of the apparatus' tray.
3. In the data table create columns for:

   Number of bricks;

   gauge reading in mm;

   total deflection of the board in m (i.e., gauge reading divided by 1000);

   change in deflection in m (i.e., $x_2 - x_1$);

   total mass in kg;

   total force in N;

   and work done on the board.

4. Place a piece of wood on the cross bars, place the gauge's needle at the center of the board and carefully hang the tray on the board.
5. With one person lifting up the bar connected to the tray, another person turns on the gauge and presses the "zero" button. Then, carefully place the rod back onto the board and read the gauge again to obtain a reading with the weight of the tray hanging on the board.
6. In the appropriate columns, enter the equations to calculate the total force hanging on the board due to the tray and the deflection of board from the relaxed position.
7. Carefully add one brick onto the middle of the tray and enter '1' for number of bricks. In the total force cell, enter the equation to calculate the total force, which includes the number of bricks as well as the tray. Read and record the new gauge reading in the appropriate box.
8. Continue adding bricks, and entering the data until the board breaks. (Note: be careful to keep your feet away from the area below tray, in case the board breaks while you're there.)

Analysis

9. Plot F_{board} vs. total displacement. Judging from the shape of the curve, does the wood board's restoring force obey Hooke's Law? Is Hooke's Law appropriate for modeling structural forces?
10. Use Excel to determine the spring constant, with uncertainty, for the board?
11. In the column for work done on the board enter an equation that correctly calculates the amount of work done *in each individual step*. Note that the force, F, with that many bricks, moves the center of the board only a small step Δx, so $\boldsymbol{\Delta W = F\,\Delta x}$.
12. Have Excel sum the work column to calculate the *total* work done in breaking the board.
13. Work should also equal the area under the curve of F vs. x. Calculate the area under the curve. Do you get the same answer as in step 12?

Further Application of Results.

1. Consider breaking the board by dropping a 1-kg object onto the board. As the object falls, gravity does work to give it kinetic energy. When the object hits the board, the force of the board does negative work on the object to bring it to a stop. Calculate the minimum height from which a 1-kg mass must be dropped for it to break the board.

2. Estimate the mass of a fist, with uncertainty, and calculate the speed that it must move in order to break a board. Do you think this can be accomplished?

Worksheet for Board Breaking Experiment

Name: ________________________ Partners: ______________________________________

Specify uncertainties and appropriate units for all quantities.

1) Cradle mass _____________________±__________________

2) Avg brick mass _____________________±__________________

3) Number of bricks that broke the board ______±______

4) Spring constant of board ___________________±__________________

5) Total work done on board by summing over the steps_______________

6) Total Work done on board by calculating area under the curve ______________

7) Height for 1 kg mass to break board ___________________± _______________________

Calculation (including final formula in terms of variables):

8) Mass of your hand _________________±____________________________

9) Speed necessary to break board with fist ____________________±____________________
Calculation (including final equation in terms of variables).