

Lab 5: Hooke's Law: Springs and Karate Boards

Name: _____

Lab Partner(s): _____

Honor Code Statement: I affirm that I have carried out my academic endeavors with full academic honesty. _____

Please neatly answer all of the questions in the lab packet. Make sure you attach any graphs generated, Excel files you produced, and any calculations/derivations you did. This lab packet is due one week from the completion of the lab.

Introduction

Elastic materials are a class of materials that deform under a force and when the force is removed, the material returns to its original shape. Springs provide a simple but accurate model of how structural forces respond to deformations and are a great example of an elastic material. The expression for the spring force is given by Hooke's Law, $\text{stress} = E \cdot \text{strain}$, where the stress $\sigma = \frac{F}{A}$, for a material of cross-sectional area A . The strain of the material under the force F , is given by $\frac{\Delta L}{L}$, where ΔL is the change in length of the system of length L , and E is the elastic (or Young's) modulus of the material. Rearranging the above we can cast Hooke's law in a more useful form, $F = -kx$, where $x = \Delta L$, the stretch or compression from equilibrium, $k = \frac{EA}{L}$, is the stiffness or spring constant, and the negative sign is due to the spring force being a restoring force, tending to restore the system to its equilibrium position. In the first part of the lab, we will investigate the relationship between the restoring spring force and the stretch of a spring. In the second part of the lab, we will look at a seemingly unspring like system, a pine karate board.

Procedure

Part I: Springs

1. Using Excel, make the following columns: mass, spring stretch x , and the magnitude of the spring force.
2. Hang a spring from the hook. Measure the position of the end of the spring and record this as the spring's *relaxed position*, L_0 .
3. Hang a small mass on the end of the spring, allow the mass to come to rest, and measure and record the new position of the end of the spring (*not the end of the mass*), L . Determine the stretch of the spring, $x = \Delta L = L - L_0$.
5. Hang 6 larger masses, one at a time, on the spring, and measure the new position of the end of the spring L , and calculate x .
6. If there are multiple springs, repeat this procedure for the others.

Analysis:

1. Attach your plots for each spring to this handout. For each spring, plot the spring force vs. the stretch of the spring. Does your spring obey Hooke's Law? What aspects of the plot supports your claim? Consider the expression of Hooke's law and whether your plot fits a proportionality (a linear relation through the origin).
2. Fit a straight line to your curve (and write the equation of the curve fit for F below) and use the fit to calculate the spring stiffness parameter k . Should you demand that the straight line go through the origin? Explain.

$$F =$$

$$k =$$

3. Perform a linear regression analysis to get the uncertainties in the slope (Δm) and the intercept (Δb). Does a zero y-intercept agree with your fit? What do the uncertainties in the slope and intercept mean? Explain.

$$\Delta m =$$

$$\Delta b =$$

4. What is the stiffness of your spring with uncertainty? Does your value seem reasonable or not? Explain.

$$k_{spring} = k \pm \Delta k$$

5. From the plot of the force versus the stretch of the spring, calculate the work done on the spring(s) by the weights. Hint: The work done is the area under the curve of the force vs. stretch plot.
6. Using the maximum stretch of the spring from your data and the plot, how does the work done that you calculated in question 5 compare to $U = \frac{1}{2}ky^2$. Explain how well they do or do not compare.

Part II: Restoring Force of Wooden Pine Karate Board

When a force is applied to a wood board, the board must exert a force in return to hold itself together. But there is a limit; if the external force does enough work on the board, it will break.

1. Measure and record in a *new Excel table* the mass of five different bricks and calculate the average. Calculate the standard error Δm in your inferred mass of an individual brick. Look back at lab #1 if you forgot how to calculate the standard error. Show the calculation below.

$$m_{avg} =$$

$$\Delta m =$$

2. Measure the mass of the apparatus' tray and its uncertainty and record these in the Excel table and below.

$$m_{tray} =$$

$$\Delta m_{tray} =$$

3. Carefully place the piece of wood on the cross bars and place the gauge's needle at the center of the board. Read the initial setting of the gauge, L_0 . Record this number in Excel and label as the relaxed position for the board. (Record your estimated uncertainty, ΔL_0 , in this number also.). Note: In some of the dial scales you can set the distance L_0 to be zero. Then all of the measurements will be the stretch. Ask your instructor if yours does this or not.

$$L_0 =$$

$$\Delta L_0 =$$

4. Input the equations into Excel to calculate the total force hanging on the board due to the tray and the deflection of board from the relaxed position, $|L - L_0|$. The gauge's reading will *decrease* as the deflection of the board increases and if you have a dial gauge, the needle may go around a few times. You need to keep careful track of the gauge readings.
5. Carefully hang the tray on the board and then record the new gauge reading, L . Excel should calculate the stretch from equilibrium as you enter the data.
6. Now, carefully and methodically add one brick at a time onto the support starting in the middle. Then put one on the right of this brick, then on the left, repeating this pattern until you have a row of 5 bricks on the support. Read and record the new gauge setting L in the appropriate box after each brick.
7. Continue adding bricks, and entering the data until the board breaks. (Note: be careful to keep your feet and fingers away from the area below tray, in case the board breaks while you're there.)

Analysis

1. Plot the weight added versus the displacement of the board from equilibrium. What is the relationship between the weight added and the displacement of the board from equilibrium? Does the wood board's restoring force obey Hooke's Law? Is Hooke's Law appropriate for modeling structural forces and is the board an example of an elastic material? Explain.
2. From your plot, what is the stiffness parameter k_{board} , with uncertainty Δk_{board} , for the board? Perform a linear regression to determine the uncertainty.

$$k = k_{board} \pm \Delta k_{board} =$$

3. Derive an expression for the work done by the force of gravity with the addition of each brick, W_i ? In the column for work done on the board enter the equation derived that correctly calculates the amount of work done in each individual step.

$$W_i =$$

4. How much *total* work done in breaking the board from Excel.

$$W_{total} = \sum W_i =$$

5. Since $W = \int \vec{F} \cdot d\vec{r}$, or $\int F dy$ for the one-dimensional case here, the work done should also equal the area under the curve of the force vs. distance. Note the shape of this plot and calculate the area under the curve. Do you get the same answer as in step 4? Explain why or why not.

$$W_{total} = \int \vec{F} \cdot d\vec{r} = \int F dy =$$

6. For the total work done, you need to determine an uncertainty in the work done on the board. Like the last few labs, you now need to perform a “propagation of uncertainty” calculation. Your calculation of uncertainty in the work depends on the mass of each brick, the number of bricks, and the deflections of the board. You should already have uncertainties in the mass of each brick and the deflection of the board. You now need to propagate these 3 uncertainties to get an uncertainty in the total work. To propagate the uncertainties, we add the uncertainties in quadrature according to:

$$\delta W = \sqrt{\left(\frac{\partial W}{\partial m}\right)^2 \Delta m^2 + \left(\frac{\partial W}{\partial N}\right)^2 \Delta N^2 + \left(\frac{\partial W}{\partial y}\right)^2 \Delta y^2}.$$

In the expression above each term has an expression of the form $\frac{\partial W}{\partial z}$. These terms represent the partial derivative of the function (W) taken with respect to a given variable (say mass m) with the other variables held constant (number of bricks N and deflection y). These terms could be negative; hence we square them to make them positive. Also, we need the uncertainty in the work done, so dimensionally, we need to multiply each square partial derivative term by an appropriate dimensional term in either mass Δm , number of bricks ΔN , or deflection Δy .

Derive an expression (and then evaluate it) for the uncertainty in the work done and enter it below.

$\delta W =$

7. The work that the weight of the bricks did on the board did not change the board's kinetic energy. Where did this work done, or energy, go? Explain.
8. Consider breaking the board by dropping a hard object onto the board, such as your hand. As the object falls, gravity does work to give it kinetic energy. When the object hits the board, the force from the board does work on the object, your hand, to bring it to rest, and in the process deflecting the board by an amount Δy at which point the board breaks. Using the Work-Kinetic energy theorem calculate the minimum height from which a 1-kg mass must be dropped in order for it to gain enough kinetic energy to break the board. If your instructor wants you to, use propagation of uncertainty to get an uncertainty in this number.

$$h_{min} = h \pm \Delta h =$$

9. Determine and explain a method to calculate the mass of a human fist, with its estimated uncertainty. Starting from the definition of work, calculate the speed that your hand must moving just before it strikes the board in order to break the board. Again, if your instructor wants, propagate your uncertainties again. Do you think this speed is reasonable and can be accomplished by you? Explain.

$$v_{min} = v \pm \Delta v =$$