

Lab 6: Rotational Dynamics

Name: _____

Lab Partner(s): _____

Honor Code Statement: I affirm that I have carried out my academic endeavors with full academic honesty. _____

Please neatly answer all of the questions in the lab packet. Make sure you attach any graphs generated, Excel files you produced, and any calculations/derivations you did. This lab packet is due one week from the completion of the lab.

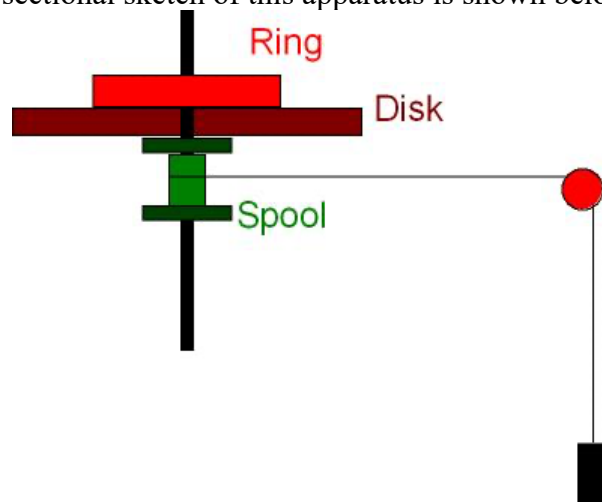
Introduction:

In this laboratory experiment we will investigate several aspects of rotational dynamics by examining torque and rotational energy considerations. In the first part of our experiment we determine the value of the rotational inertia of a solid ring and compare it with the value that we calculate using a theoretically derived expression. In the second part of the experiment we determine the speed of a hanging mass connected to our rotational apparatus by applying conservation of energy relationships for spinning objects.

The primary purpose of this lab is to become more familiar with rotational dynamics and to examine how the rotational and translational variables are related to each other. As a result, we will spend most of our attention on theoretical derivations of the relationships that we will use.

Apparatus:

The experimental apparatus that we use in this experiment is manufactured by Pasco Scientific Company (www.pasco.com). A cross sectional sketch of this apparatus is shown below:



A hanging mass, m_h , is connected to the rotational apparatus by a string that passes over a Smart Pulley. The pulley is connected to our Capstone data collection system and allows us to record the speed, v , of the falling mass m_h (i.e. the tangential speed of the rim of the pulley) as a function of time. The other end of the string is wound around a spool, of radius r_{shaft} , that is coaxial with the disk of the rotational apparatus. So, as the mass m_h falls, it causes the disk to spin. A ring of radius R can be placed on the disk in a groove so that it also is coaxial with the apparatus.

Theory/Pre-Lab Exercises: Derive expressions for the moment of inertia, on a separate sheet(s) of paper in each of the following cases. Attach the derivations to this handout.

1. **Torque and acceleration analysis** (consider the mass of the pulley to be negligibly small):

Analyzing the forces and torques in the system, derive an expression on a separate sheet of paper and write the results below for the moment of inertia I of the system in terms of the hanging mass m_h , the acceleration a of the hanging mass m_h , the acceleration due to gravity g and the radius of the shaft r_{shaft} .

The expression that you determine will be called the moment of inertia by the torque balance method. Write your expression for $I_{TB,expression}$ below.

$$I_{TB,expression} =$$

2. **Energy Considerations:**

Analyzing the energy changes in the system, derive an expression on a separate sheet of paper and write the results below for the moment of inertia I of the system in terms of the hanging mass m_h , the speed v of the hanging mass m_h after it has fallen a distance h , the height h , the acceleration due to gravity g , and the radius of the shaft r_{shaft} .

The expression that you determine will be called the moment of inertia by the energy balance method. Write your expression for $I_{EB,expression}$ below.

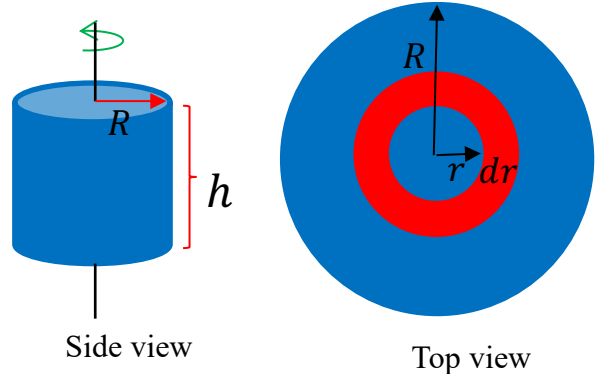
$$I_{EB,expression} =$$

3. The Definition of Moment of Inertia

Moments of inertia are often experimentally measured, as most distributions of mass do not possess a unique symmetry to be able to use the definition of moment of inertia below. The definition for the moment of inertia is defined as:

$$I = \int r^2 dm$$

where, r is the perpendicular distance from the axis of rotation to the piece of mass, dm and then we integrate over the distribution of mass. As an example of this consider the solid circular cylinder of mass M , radius R and height h shown on the right spun about an axis through its center.



The cylinder has a constant density ρ and thus its mass is given by $\rho = \frac{m}{V} \rightarrow m = \rho V = \rho \pi R^2 h$.

To determine the moment of inertia of this cylinder (or disk if h is small) about an axis through its center, let's break the cylinder up into little rings of radius dr , each located at distance r from the axis of rotation as shown above. Each ring has a height h and this corresponds to a mass dm of a ring given by $\frac{dm}{dr} = \frac{d}{dr}(\rho \pi r^2 h) = 2\pi r \rho h$, where the perpendicular distance from the axis of rotation to the ring of mass dm is r .

The moment of inertia integral is:

$$I = \int_0^R r^2 dm = 2\pi \rho h \int_0^R r^3 dr = 2\pi \rho h \left[\frac{R^4}{4} - \frac{0^4}{4} \right] = \frac{\pi \rho h R^4}{2}$$

Removing the density from this expression, we get

$$I = \left(\frac{M}{\pi R^2 h} \right) \left(\frac{\pi h R^4}{2} \right) = \frac{1}{2} M R^2.$$

As, we've said in class, all moments of inertia look like $I = c m r^2$ where c is a constant that tells me something about the distribution of mass and how it's rotated. Here, for the cylinder $c = \frac{1}{2}$.

Perform an analysis similar to the one given above and derive an expression on a separate sheet of paper and write the results below for the moment of inertia of a ring of mass M , height h , and radii R_{inner} and R_{outer} . Write your expressions for $I_{T_{theo,expression}}$ below.

$$I_{TB} = I_{T_{theo,expression}} =$$

Procedure and analysis:

1. First make sure that the Smart Pulley is at the right height so that the string winding around the spool is perfectly horizontal (why?). Attach a disk to the shaft it is not already attached.
2. Record the height h through which the mass m_h fell and its uncertainty.

$$y = h \pm \Delta h =$$

3. Keep a tension in the string and rotate the disk manually to wind the string around the spool. Again, take care for the string to remain horizontal throughout the winding process.
4. Open Capstone and choose from hardware setup Smart Pulley and then pick Graph (velocity versus time)
5. Start with a $50g$ mass and hang it and each subsequent mass m_h on the string and release them from rest. Start taking data using Capstone just before you release each mass and stop collecting data just as the hanging mass m_h is about to just hit the floor.
6. Fit a line to the velocity versus time data in Capstone and record the acceleration in Table 1 for m_h .
7. Determine the maximum speed v of the hanging mass and record this in Table 2.
8. Repeat steps 5-7 for all the other hanging masses for a minimum of 10 in steps of $50g$ until you reach $500g$.
9. Now, place the ring on top of the disk and, and for the same set of hanging masses m_h , repeat steps 5 - 7 above to determine the new acceleration and speed of m_h . Record your data in Tables 1 & 2.
10. Using a pair of Vernier calipers, measure the diameter, $2r$, of the spool and its uncertainty. Calculate r and its uncertainty and record it in the space provided.

$$r_{shaft} = r \pm \Delta r =$$

11. Using the two equations you derived from considering torques and energy balances, determine the rotational inertia of the system, for the disk only and then for the disk and ring, for each hanging mass m_h . Subtract these values and calculate the rotational inertia of the ring for each trial of the hanging mass m_h for each method.
12. Measure the mass, M , of the ring, its inner and outer radii, R_{inner} and R_{outer} , and calculate its rotational inertia using the expression you derived for the theoretical moment of inertia. Record these values in Table 3.
13. For the piece of art located in the room, determine its rotational inertia one time using a hanging mass of $0.250kg$.

Table 1

m(kg)	a () disk only	I () disk only	a () disk and ring	I () disk and ring	I () ring
0.050					
0.100					
0.150					
0.200					
0.250					
0.300					
0.350					
0.400					
0.450					
0.500					

You should have ten results for the moment of inertia of the ring listed in Table 1. Calculate the average value of the moment of inertia of the ring and calculate the uncertainty in the moment of inertia of the ring from standard error. Do the standard error calculation in excel and print out the results and attach them to the lab.

$$I_{R,TB} = I_{ring,avg} \pm \Delta I_{ring,avg} =$$

Table 2

m(kg)	h (m)	v () disk only	I () disk only	v () disk and ring	I () disk and ring	I () Ring
0.050						
0.100						
0.150						
0.200						
0.250						
0.300						
0.350						
0.400						
0.450						
0.500						

You should have ten results for the moment of inertia of the ring listed in Table 2. Calculate the average value of the moment of inertia of the ring and calculate the uncertainty in the moment of inertia of the ring using the standard error. Do this calculation in excel and attach the results to the lab.

$$I_{R,EB} = I_{ring,avg} \pm \Delta I_{ring,avg} =$$

Radius of Spool: $r \pm \Delta r =$ _____

Mass of ring: $M \pm \Delta M =$ _____

Inner radius of ring: $R_{inner} \pm \Delta R_{inner} =$ _____

Outer radius of ring: $R_{outer} \pm \Delta R_{outer} =$ _____

From table 1, enter your value of $I_{TB,ring} = I_{ring,avg} \pm \Delta I_{ring,avg}$ in the torque balance method in table 3.

From table 2, enter your value of $I_{EB,ring} = I_{ring,avg} \pm \Delta I_{ring,avg}$ in the energy balance method in table 3.

From the definition of moment of inertia, derive an expression for the moment of inertia of a ring and then evaluate your expression (with uncertainty). Enter this value into the theory section in table 3 showing the results of the calculations by attaching your work to the lab.

Determine the moment of inertia (with uncertainty) for the piece of art and enter it in table 3, under Art.

Table 3

Method	$I_{ring} \pm \Delta I_{ring}$ ()
Torque balance	
Theory	
Energy balance	
Art	

A written paragraph discussion of the results and answers to the following questions.

Which method gives a better way to determine the moment of inertia of an object and why?

Of the method that you do not believe gives a better method of determining the moment of inertia, what explicitly is the reason or variable or measurement that makes it fail.

What change would make it a better method and one that is comparable to the one you felt was the best method to determine I.

How confident are you in your value for the moment of inertia for the piece of art? Why are you or why are you not confident in your measurement?

In the problems we've done so far, we've assumed masses add. Thus, for example $m_{total} = m_1 + m_2$. What can you say about moments of inertia? Are they too additive? Justify your answer.