

Lab 7 - Clogged Arteries and Cardiovascular Pressure

INTRODUCTION

Blood flow. In humans, blood flows from the heart into the aorta, from which it passes into the major arteries, which in turn branch into a myriad of tiny capillaries. The blood returns to the heart via the veins. The radius of the aorta is about 1.2cm. A typical capillary has a radius of about 4×10^{-4} cm

Peripheral arterial disease (PAD) is a nearly pandemic condition that has the potential to cause loss of limb or even loss of life, affecting 4% to 12% of people aged 55 to 70 years and 15-20% of people aged over 70 years. Peripheral arterial disease manifests as insufficient tissue perfusion (flow) caused by a significant narrowing of arteries distal to the arch of the aorta, most often due to plaque in the arteries (atherosclerosis.) (Stephens, 2017).

In PAD, plaque (made up of fat, cholesterol, calcium, fibrous tissue, and other substances in the blood) builds up in the arteries that carry blood to your head, organs and limbs. Over time, plaque can harden and narrow the arteries. This limits the flow of oxygen-rich blood to your organs and appendages. (“Peripheral Arterial Disease,” n.d.).

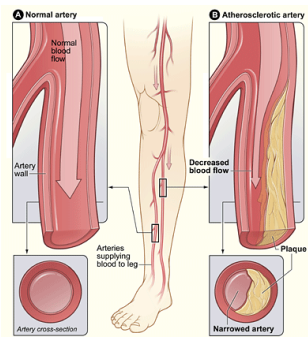


Figure 1: a normal artery with normal blood flow. The inset image shows a cross-section of the normal artery. Figure B shows an artery with plaque buildup that's partially blocking blood flow. The inset image shows a cross-section of the narrowed artery. <https://www.nhlbi.nih.gov/health/health-topics/topics/pad/> (“Peripheral Arterial Disease,” n.d.).

Peripheral artery disease is often caused by *atherosclerosis*. In atherosclerosis, fatty deposits (plaques) build up on *artery* walls and reduce blood flow. Although discussions of atherosclerosis usually focus on the heart, the *disease* can and usually does affect *arteries* throughout the body (“Peripheral artery disease,” 2018). Severe cases of PAD may manifest in plaque building up in an artery to the point that flow is blocked. A blocked artery can lead to gangrene and eventually, if not treated, it can lead to loss of limb.

PART I – Cardiovascular Pressure

The pressure due to the liquid at depth h is due to the weight of the column of liquid above it. If the fluid has a mass density ρ , the pressure at depth h is

$$P = \rho gh. \quad (1)$$

Determine the density of the fluid using:

1. Definition of density: $\rho = \frac{\text{mass}}{\text{volume}}$
2. The relationship between the pressure and the height of a column of fluid $P = \rho gh$.

Experiment:

- Measure the mass of several volumes of water
- Represent the graph mass versus volume
- Determine the density and its uncertainty from the graph

The density of the water using the definition of density is:

$\rho =$ _____

- Fill the beaker with water.
- Using the meter stick and pressure gauge, measure the pressure of the water at five different depths. (select absolute pressure as the hardware in PASCO Capstone)
- Make a graph with pressure on the y-axis and depth on the x.
- Determine the density and its uncertainty from the graph

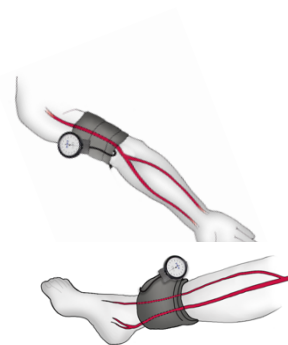
The density of the water using the equation for pressure at different heights is:

Put your knowledge into context...

A test called an Ankle-Brachial Index (ABI) helps diagnose PAD prior to gangrene development. The ankle-brachial index test is a quick, noninvasive way to check the risk of PAD by comparing the systolic blood pressure measured in an artery at the ankle to the systolic blood pressure measured at the brachial artery of the arm

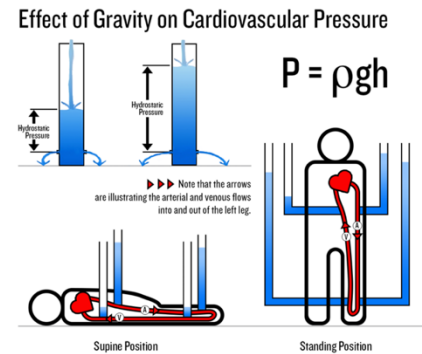
$$ABI = BP_{\text{ankle}} / BP_{\text{upperarm}}$$

In a healthy individual *who is supine*, a normal resting ankle-brachial index is ~ 1.0 . This means that the blood pressure reading at the ankle is approximately the same as the blood pressure reading at your arm and suggests that the patient does not have significant narrowing or blockage of blood flow.



Answer the following questions:

1. Can we measure the ABI index in the standing position? Explain why.
2. As the health of the patient in our PAD scenario decreased, we saw gangrene develop in her left foot indicating significant blockage of blood flow to the ankle. What would you expect the ABI ratio to do, increase/decrease? Explain why.



PART II – Arterial Clogs

As fluid flows through a tube of varying width, the total rate of flow of the fluid (amount of fluid passing any point in the tube per second) must stay constant. This can be expressed mathematically by the equation of continuity. For a fluid of mass density ρ , moving through a tube of cross-sectional area A at speed v , the **equation of continuity** states that the flow rate, Q in volume of fluid per second, is constant. For incompressible fluids, where the density is constant, flowing in a tube with varying width, this equation can be simplified to:

$$A_1 v_1 = A_2 v_2 = Q, \quad (2)$$

where A_1 and A_2 are the cross-sectional areas and v_1 and v_2 are the speeds of the blood flow in the two sections.

By the **Bernoulli principle**, for a horizontal fluid flow (so that the weight of the fluid doesn't change) the sum of the pressure and kinetic energy per volume of the fluid is constant. Comparing the two sections of a tube of varying width:

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2, \quad (3)$$

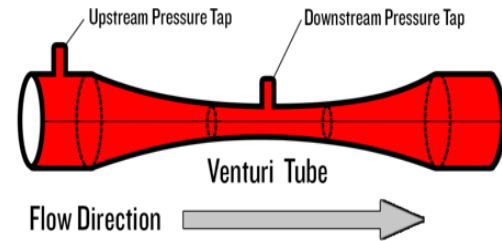
where we assume, as before, that the density is constant.

Theory:

By combining Equations 2 and 3, find an expression for the ratio of the cross-sectional areas in the two sections $\frac{A_1}{A_2}$ as a function of variables that we can measure: difference of pressure between both sections of the tube ($P_1 - P_2$), density ρ , Area of the larger section A_1 and the flow rate Q .

Experiment:

A Venturi Tube is a device for measuring the difference in pressure in a flow of a fluid in a tube with a short, narrow center section and widened, tapered ends. A PVC pipe, with venturi tube must be connected to a port at the bottom of a bucket, and the “general flow sensor” is connected to the pressure ports of the venturi tube. The exit of the venturi tube must empty into a large beaker. You will also need a stopwatch, or timer.



Procedure:

- With the valve in the PVC pipe closed, fill the bucket with water.
- Measure the Flow Rate Q (in m^3 per second) by measuring the time it takes to fill the beaker from the bucket. Do it several times to also provide the uncertainty of the measurement.
 $Q =$ _____
- Open Capstone, click on “hardware setup”:
- In the hardware setup window, under “Available wireless devices”, select the airlink with the code corresponding to that printed on your device.
- Click on “hardware setup” to make that window go away.
- Select “Sensor Data”.
- In the y-axis parameter of the graph, select “Differential Pressure”
- Click “Record”
- Read the differential pressure ($P_1 - P_2$) on the graph (use the “delta tool”) Repeat four more times also to provide the corresponding uncertainty

$P_1 - P_2 =$ _____

Analysis:

- Applying the equation, you derived from the Continuity and Bernoulli Equations to your data, determine the average of the ratio of the cross-sectional areas in the venturi tube $\frac{A_1}{A_2}$
- What is the actual ratio of the cross-sectional areas? Read the sectional areas of two sections of the venturi tube from the product description sheet: $A_1 =$ _____ ; $A_2 =$ _____

Does the ratio of areas as inferred from the difference in pressure agree with that according the difference in diameters?

Put your knowledge into context...

With a clogged artery, the area through which blood can flow is reduced. We have modeled the narrowing of vessels due to plaque in our patient scenario by studying flow in the Venturi Tube as we relate it to a lower limb Clogged Artery in PAD. We have obtained a measure of the change in the cross-sectional area, and therefore the amount of clogging, by measuring the change in pressure.

1. Do you expect the blood in the clogged section to move faster or slower than in the unclogged part? Explain why.

NOTE for instructor: Derivation of Equation $\frac{A_1}{A_2}$

By Equation 3:

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2.$$

Substituting from Equation 2:

$$\begin{aligned} P_1 - P_2 &= \frac{1}{2}\rho \left(\frac{A_1}{A_2} v_1\right)^2 - \frac{1}{2}\rho v_1^2 \\ &= \frac{1}{2}\rho v_1^2 \left(\frac{A_1^2}{A_2^2} - 1\right). \end{aligned}$$

Re-arranging to solve for the ratio A_1/A_2 :

$$\frac{A_1^2}{A_2^2} = (P_1 - P_2) \frac{2}{\rho v_1^2} + 1$$

Or

$$\frac{A_1}{A_2} = \sqrt{(P_1 - P_2) \frac{2}{\rho v_1^2} + 1}.$$

Replacing v_1 using $A_1 v_1 = Q$, (Equation 2):

$$\frac{A_1}{A_2} = \sqrt{(P_1 - P_2) \frac{2A_1^2}{\rho Q^2} + 1}.$$