Physics 110
Exam #2
May 8, 2006

Name_____

|	
Multiple-Choice	/20
Problem 1	/27
Problem 2	/23
Problem 3	/30

Total	/100

Part I: Free Response Problems

Please show all work in order to receive partial credit. If your solutions are illegible no credit will be given. Please use the back of the page if necessary, but number the problem you are working on. The numbers in parentheses following the question correspond to the point values for each part.

1. A ballistic pendulum is used to study the principles of momentum and energy. Suppose that a steel ball of mass $m = 50g$ traveling with an initial velocity V undergoes an inelastic collision with a stationary pendulum arm of length $R_{cm} = 30.5cm$ of mass $M = 250g$. After the collision the center of mass of the ball and pendulum arm rises from its lowest point thought a height Δh_{cm}, where it momentarily comes to rest at an angle $\theta = 27^{\circ}$ as shown below.

    a. Write an equation (no numbers) that governs the momentum of the ball and pendulum arm during the collision and solve this for the initial velocity of the ball. (6)

    $$mV = (m + M)V_{\text{after collision}} \rightarrow V = \frac{(m+M)}{m}V_{\text{after collision}}$$
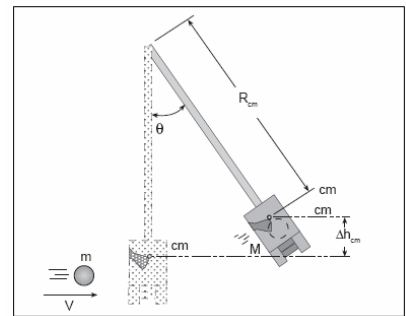
    b. After the collision, mechanical energy is conserved. Write an equation (no numbers) that shows conservation of mechanical energy immediately after the collision to the point where the pendulum arm and ball come to rest momentarily at the angle θ. Solve this equation for the velocity of the ball and pendulum arm after the collision. Express your answer in terms of R_{cm} and θ and you may ignore any rotational motion of the arm. (6)

    $$\tfrac{1}{2}(m + M)V_{\text{after collision}}^2 = (m + M)g\Delta h_{cm} = (m + M)g(R_{cm} - R_{cm}\cos\theta)$$
    $$V_{\text{after collision}} = \sqrt{2gR_{cm}(1 - \cos\theta)}$$

    c. Using the equations that you have written in parts *a* and *b* what are the expression for and the value of the initial velocity of the ball? (9)
    $$V = \frac{(m+M)}{m}V_{\text{after collision}} = \frac{(m+M)}{m}\sqrt{2gR_{cm}(1 - \cos\theta)}$$
    $$V = \frac{(50g + 250g)}{50g} \times \sqrt{2 \times 9.8\tfrac{m}{s^2} \times 0.305m \times (1 - \cos 27)} = 4.84\tfrac{m}{s}$$

 $$\therefore \vec{V} = 4.84\tfrac{m}{s} \text{ in the } + x \text{ - direction}$$

    d. What fraction of the initial kinetic energy of the ball has been lost to the collision? (6)
    $$\%_{lost} = \left[1 - \frac{KE_{after}}{KE_{initial}}\right] \times 100\% = \left[1 - \frac{\tfrac{1}{2}(m + M)V_{\text{after collision}}^2}{\tfrac{1}{2}mV^2}\right] \times 100\% = \left[1 - \frac{\tfrac{1}{2}(m + M)\left(\frac{m^2 V^2}{(m + M)^2}\right)}{\tfrac{1}{2}mV^2}\right] \times 100\% = \left[1 - \frac{m}{m + M}\right] \times 100\%$$

    $$\%_{lost} = \left[1 - \frac{m}{m + M}\right] \times 100\% = \left[1 - \frac{50g}{50g + 250g}\right] \times 100\% = 83\%$$
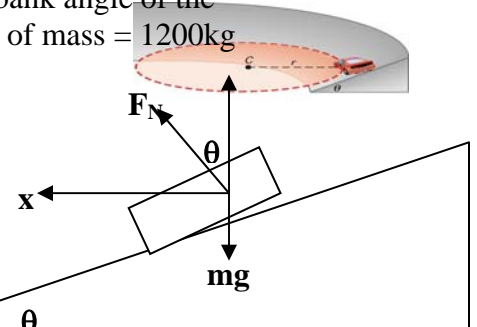
2. The problem below is designed to illustrate the reasons why engineers sometimes bank the roads around corners rather than making them flat.

a. Consider as a first example a flat road in which a 1200kg car rounding a corner that is 45m in radius. Suppose further that the coefficient of static friction between the tires and the road is 0.82 as shown below. Draw a carefully labeled free body diagram and from this what is the maximum speed that the car can have to negotiate the corner without skidding out to larger radii? (Hint: In problems involving friction we usually use kinetic friction. Here the bottom of the tire is momentarily at rest, so static friction must be used. The force of friction is still given in the usual way, except that we replace μ_k by μ_s.) (6)
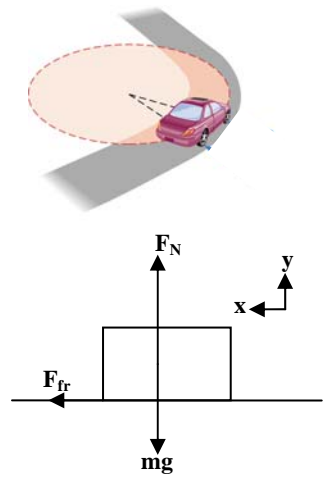
$$F_{fr} = \frac{mv^2}{R} \rightarrow \mu_s mg = \frac{mv^2}{R} \rightarrow v = \sqrt{\mu_s gR} = \sqrt{0.82 \times 9.8 \tfrac{m}{s^2} \times 45m} = 19 \tfrac{m}{s}$$

b. Now consider the case where the roadway is banked at an angle θ as shown below. If the roadway is banked at the proper angle then the car can negotiate the corner **without any assistance from friction** between the tires and the road. Draw a carefully labeled free body diagram and determine the bank angle of the roadway if the radius of the curved road is 85m and a car of mass = 1200kg approaches the curve at a velocity of 21 m/s. (11)

$$\sum F_x: \quad F_N \sin\theta = \frac{mv^2}{R}$$

$$\sum F_y: \quad F_N \cos\theta - mg = 0 \rightarrow F_N = \frac{mg}{\cos\theta}$$

$$\therefore mg\tan\theta = \frac{mv^2}{R} \rightarrow \theta = \tan^{-1}\left(\frac{v^2}{Rg}\right) = \tan^{-1}\left(\frac{(21\tfrac{m}{s})^2}{85m \times 9.8\tfrac{m}{s^2}}\right) = 27.9^o$$

c. Suppose now that the road is actually banked at 45°, what speed should the car have in order to make the turn without friction, assuming that the radius of the curved road is 85m again? If the bank angle increases, what conclusion can be drawn about the speed of the car around these curves? (6)

$$\tan\theta = \frac{v^2}{Rg} \rightarrow v = \sqrt{Rg\tan\theta} = \sqrt{85m \times 9.8\tfrac{m}{s^2} \times \tan 45} = 28.9\tfrac{m}{s} \text{ and increasing bank}$$

angle allows one to negotiate the curve at a faster rate.

3. Suppose that the Nott Memorial is topped with an approximately hemispherical dome. Suppose that the dome is frictionless when wet. Somehow an individual has balanced a pumpkin of mass 9.1kg at the top to the dome at an angle of $\theta_i = 0^o$ with respect to the vertical. Suppose that on a rainy night, a gust of wind starts the pumpkin sliding from rest and leaves the dome at an angle of $\theta_f = 48.2^o$ and that the dome of the Nott Memorial has a radius of 13.6 m and the base of the dome is located 17.6 m above the ground.

   a. Using energy methods, what is the speed of the pumpkin when it leaves the dome of the Nott? (6)

   $$mgh_i = mgh_f + \tfrac{1}{2}mv^2_{leaves\ roof}$$

   $$v^2_{leaves\ roof} = 2g(h_i - h_f) = 2g(R - R\cos\theta)$$

   $$\therefore v = \sqrt{2g(R - R\cos\theta)} = \sqrt{2 \times 9.8\tfrac{m}{s^2} \times (13.6m - 13.6m\cos48.2)} = 9.43\tfrac{m}{s}$$
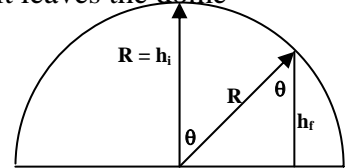
   b. Suppose that someone has placed a large spring, of stiffness k = 14477 N/m, at the exact location that the pumpkin should land from the base of the Nott. Using energy methods, how far does the spring compress, from equilibrium, when the pumpkin lands on the spring? (Ignore any changes in gravitational potential energy of the pumpkin when it hits the spring.) (6)

   $$mgh_i = mgh_f + \tfrac{1}{2}mv^2_{leaves\ roof} = \tfrac{1}{2}ky^2$$

   $$y = \sqrt{2\frac{mg}{k}h_i} = \sqrt{2 \times \frac{9.1kg \times 9.8\tfrac{m}{s^2}}{14477\tfrac{N}{m}} \times 13.6m + 17.6m} = 0.62m$$

   c. What is the period of the resulting simple harmonic motion if the pumpkin attaches itself to the spring and oscillates without friction? (2)

   $$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{9.1kg}{14477\tfrac{N}{m}}} = 0.16s$$

   d. What is the magnitude of the maximum velocity of the pumpkin on the spring and where does this maximum velocity occur? (5)

   $$v_{max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{14477\tfrac{N}{m}}{9.1kg}} \times 0.62m = 24.7\tfrac{m}{s} \text{ at the equilibrium position.}$$

   e. What is the magnitude of the maximum acceleration of the pumpkin on the spring and where does this maximum acceleration occur? (5)

   $$a_{max} = \frac{k}{m}A = \frac{14477\tfrac{N}{m}}{9.1kg} \times 0.62m = 986.3\tfrac{m}{s^2} \text{ at both endpoints.}$$

   f. Taking time t = 0 when the pumpkin strikes the spring, what are the position and speed as functions of time? (Hint: Don't forget units!) (6)

   $$x(t) = 0.62m\sin(39.3s^{-1}t)$$

   $$v(t) = 24.7\tfrac{m}{s}\cos(39.3s^{-1}t)$$

Part II: Multiple-Choice
    *Circle your best answer to each question. Any other marks will not be given
credit. Each multiple-choice question is worth 2 points for a total of 20 points.*

1. A railroad car of mass m traveling with speed v collides and sticks to an identical
   railroad car that is initially at rest. The ratio of the kinetic energy after the
   collision to that before the collision is
   a. $\frac{1}{2}$.   b. $\frac{1}{3}$.   c. $\frac{1}{4}$.   d. $\frac{1}{8}$.

2. A snowboarder coasts on a smooth track that rises from
   one level to another. If the boarder's initial speed is 4m/s,
   the snowboarder just makes it to the upper level and comes
   to rest. With a slightly greater initial speed of 5m/s, the
   snowboarder is still moving to the right on the upper level.
   The snowboarder's final speed in the second case is
   a. 1m/s.
   b. 2m/s.
   c. 3m/s.
   d. 4m/s.
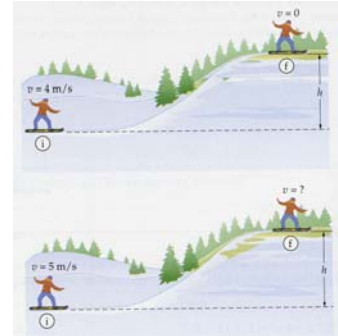
3. The top view of a box of mass m is shown on the right and
   this box is moved along the sides of a square of length d.
   The coefficient of kinetic friction that exists between the box
   and the floor is μ_k. As the box is moved around the entire
   square from points A to B to C to D back to A, the net work
   done by friction is
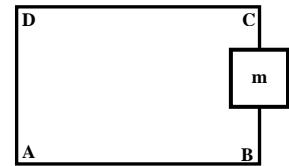
   a. zero.   b. $\mu_k mgd$ c. $-2\mu_k mgd$ d. $-4\mu_k mgd$

4. To accelerate a certain car from rest to a speed v requires the work W_1. The work
   needed to accelerate the same car from v to $2v$ is W_2. Which of the following is a
   correct statement?
   a. $W_2 = \frac{1}{2} W_1$ b. $W_2 = W_1$ c. $W_2 = 2W_1$ d. $W_2 = 3W_1$

5. A mallet is balances when suspended from its center of mass, as shown below
   left. If you were to cut the mallet into two pieces at its center of mass as shown
   below right, how do the masses of the two pieces compare?
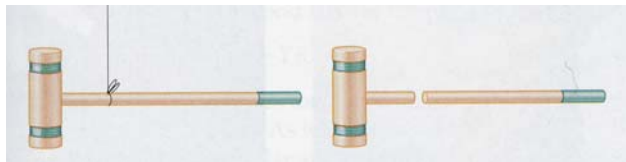
   a. The masses are equal.
   b. The piece with the head of the mallet has the greater mass.
   c. The piece with the head of the mallet has the smaller mass.
   d. There is not enough information given to make a comparison.

6. A solid sphere and a hollow sphere are both rotated at the same angular velocity, ω. If the moment of inertia of the solid sphere is $\frac{2}{5}MR^2$ while the hollow sphere is $\frac{2}{3}MR^2$, the ratio of the rotational kinetic energy of the solid sphere to the hollow sphere is

    a. $\frac{5}{3}$.   b. 1.   c. $\frac{3}{5}$.   d. $\frac{9}{25}$.

7. Two children ride on a merry-go-round with child 1 at a greater distance from the axis of rotation than child 2.
    a. The angular and linear speeds of child 1 are the same as that of child 2.
    b. The angular speeds of child 1 and child 2 are the same while the linear speed of child 1 is less than child 2.
    c. The angular speeds of child 1 and child 2 are the same while the linear speed of child 1 is greater than child 2.
    d. The angular speed of child 1 is greater than that of child 2 while the linear speeds of child 1 and child 2 are equal.

8. As blood enters the aorta from the heart, the body recoils in the other direction in order that linear momentum be conserved. The ballistocardiograph is a device used to calculate the speed of this blood. To do this, the patient lies on a large 'frictionless' table, and as the heart beats, the motions of the table are monitored. After each heart beat, about 30 g of blood is pumped to the body. If a 70 kg person is found to recoil at 0.3 mm/s, what is the exit speed of the blood? (Assume the blood in the aorta is initially stationary.)
    a. 0.7 mm/s   b. 700 mm/s   c. 700 cm/s   d. 7 m/s

9. A certain sports car of mass 1000kg can accelerate from rest to 60 miles per hour (~ 26.7 m/s) in 6s over a distance of ¼ mile (= 400 m). The power output of the car is approximately
    a. 30 W   b. 300 W   c. 3 kW   d. 300 kW

10. A pickup truck races a freight train to a crossing trying to beat the train. Suppose that the truck doesn't make it and has a collision with the train. The magnitude of the force exerted on the truck by the train during the collision is
    a. smaller than that which the truck exerts on the train.
    b. larger than that which the truck exerts on the train.
    c. the same as the truck exerts on the train.
    d. unable to be determined since the masses of the truck and train are unknown.

Useful formulas:

Motion in the x, y or z-directions

$$r_f = r_0 + v_{0r}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{0r} + a_r t$$

$$v_{fr}^2 = v_{0r}^2 + 2a_r \Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

$$T = \frac{1}{f}$$

Geometry /Algebra

Circles	Triangles	Spheres
$C = 2\pi r$	$A = \tfrac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \tfrac{4}{3}\pi r^3$

$$\text{Quadratic equation}: ax^2 + bx + c = 0,$$

$$\text{whose solutions are given by}: x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Vectors

$$\text{magnitude of a vector} = \sqrt{v_x^2 + v_y^2}$$

$$\text{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$$g = 9.8\,\text{m/s}^2 \qquad G = 6.67\times10^{-11}\,\text{Nm}^2/\text{kg}^2$$

$$N_A = 6.02\times10^{23}\,\text{atoms/mole} \qquad k_B = 1.38\times10^{-23}\,\text{J/K}$$

$$\sigma = 5.67\times10^{-8}\,\text{W/m}^2\text{K}^4 \qquad v_{sound} = 343\,\text{m/s}$$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_s = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\ \theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

Heat

$$T_C = \tfrac{5}{9}\left[T_F - 32\right]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}\left(1 + \alpha\Delta T\right)$$

$$A_{new} = A_{old}\left(1 + 2\alpha\Delta T\right)$$

$$V_{new} = V_{old}\left(1 + \beta\Delta T\right):\ \beta = 3\alpha$$

$$PV = Nk_B T$$

$$\tfrac{3}{2}k_B T = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$a_R = r\omega^2$$

$$\Delta s = r\Delta\theta;\ v = r\omega;\ a = r\alpha$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_{air} + \rho gh$$

$$F_B = \rho gV$$

$$P + \tfrac{1}{2}\rho v^2 + \rho gh = C$$

Simple Harmonic Motion/Waves

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\frac{2\pi}{T}t\right)$$

$$v(t) = \sqrt{\frac{k}{m}}A\cos\left(\frac{2\pi}{T}t\right)$$

$$a(t) = -\frac{k}{m}A\sin\left(\frac{2\pi}{T}t\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$