Name____

Physics 110 Quiz #2, September 27, 2013

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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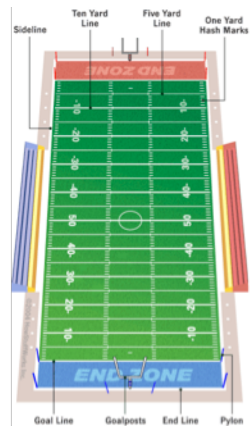
A field goal kicker in football is attempting to kick a $50yd(\sim 45.7m)$ field goal. The kicker kicks the ball always at a 53^o angle but is capable of varying the initial speed of the kicked football. The bottom bar of the goal post uprights is $3.1m$ above the ground and assumes that the football is kicked from the ground. (For a $50yd$ attempt, the football would actually be placed on the $40yd$ line, the end zone $10yd$ is deep. This information is not needed for the actual problem.)



http://entertainment.howstuffworks.com/football1.htm

1. What minimum initial speed (magnitude of the velocity) would the kicker have to kick the football with so that it just barely clears the bottom bar of the goal post uprights?

   For this part of the problem, there are two unknowns, the time it takes the football to reach the bottom bar of the goal posts and the initial velocity of the kick. We'll write the horizontal and vertical trajectory equations and eliminate the time dependence and determine the initial velocity. Once this is known we can determine the time to reach the bottom bar of the goalpost.

$$x_f = x_i + v_{ix}t = (v_i\cos\theta)t \rightarrow t = \frac{x_f}{v_i\cos\theta}$$

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2 = (v_i\cos\theta)\left(\frac{x_f}{v_i\cos\theta}\right) - \frac{g}{2}\left(\frac{x_f}{v_i\cos\theta}\right)^2 = (\tan\theta)x_f - \frac{gx_f^2}{2v_i^2\cos^2\theta}$$

$$\therefore v_i = \left[\sqrt{\frac{-2\cos^2\theta\left(y_f - (\tan\theta)x_f\right)}{gx_f^2}}\right]^{-1} = \left[\sqrt{\frac{-2\cos^2 53\left(3.1m - (\tan 53)45.7m\right)}{9.8\,\frac{m}{s^2}(45.7m)^2}}\right]^{-1} = 22.2\,\tfrac{m}{s}$$

2. What is the time of flight of the football from its kickoff point until it just barely clears the bottom bar of the uprights?

From the horizontal equation of motion we can determine the time it takes the ball to reach the bottom bar of the goalpost from its kicked location.

$$x_f = v_{ix}t = (v_i\cos\theta)t \rightarrow t = \frac{x_f}{v_i\cos\theta} = \frac{45.7m}{22.2\,\tfrac{m}{s}\cos 53} = 3.42s$$

3. What is the velocity of the football just as it passes the bottom bar of the?

$$v_{fx} = v_{ix} = v_i\cos\theta = 22.2\,\tfrac{m}{s}\cos 53 = 13.4\,\tfrac{m}{s}$$

$$v_{fy} = v_{iy} = a_y t = v_i\sin\theta = \left(22.2\,\tfrac{m}{s}\sin 53\right) - 9.8\,\tfrac{m}{s^2}\times 3.42s = -15.8\,\tfrac{m}{s}$$

$$\therefore \vec{v}_f = \sqrt{v_{fx}^2 + v_{fy}^2}\;@\tan\phi = \frac{v_{fy}}{v_{fx}} = \sqrt{\left(13.4\,\tfrac{m}{s}\right)^2 + \left(-15.8\,\tfrac{m}{s}\right)^2}\;@\,\phi = \tan^{-1}\left(\frac{-15.8}{13.4}\right) = 20.7\,\tfrac{m}{s}\,@\,\phi = -49.7^o$$

5. Suppose that the kicker were to kick the ball with an initial velocity that is about 10% greater than the value you found in *part 1*. Assuming that the distances involved do not change and that the kicker still kicks the ball at the 53^o angle, the ball

   a. will clear the uprights by passing above the bottom bar and a goal will be scored.
   b. will clear the uprights by passing below the bottom bar and a goal will not be scored.
   c. will never make it to the uprights and actually land in front of the uprights and a goal will not be scored.
   d. and it's motion cannot be determined since there is not enough information given.

So what we want to calculate is the final vertical height. If it is greater than 3.1m then it clears above the goal post, less than 3.1m under the goal post, and if the final vertical height is negative then it never made it to the goal post landing in front and short. To see this solution, consider the fact that the distances remain the same as does the launch angle. Since the initial velocity increases, from the horizontal motion, the time needed to reach the goalpost must decrease. Therefore in the vertical trajectory equation, the first term involving the velocity and the time, this piece remains constant. However, in the last piece this term decreases. Therefore the final height is a constant minus a smaller number than in part 1. Therefore the ball clears the goal posts by passing above the bar.

Useful formulas:

Motion in the r = x, y or z-directions

$r_f = r_0 + v_{0r}t + \frac{1}{2}a_r t^2$

$v_{fr} = v_{0r} + a_r t$

$v_{fr}^{\ 2} = v_{0r}^{\ 2} + 2a_r \Delta r$

Uniform Circular Motion

$a_r = \dfrac{v^2}{r}$

$F_r = ma_r = m\dfrac{v^2}{r}$

$v = \dfrac{2\pi r}{T}$

$F_G = G\dfrac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

$Quadratic\ equation: ax^2 + bx + c = 0,$

$whose\ solutions\ are\ given\ by:\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$magnitude\ of\ a\ vector = \sqrt{v_x^{\ 2} + v_y^{2}}$

$direction\ of\ a\ vector \rightarrow \phi = \tan^{-1}\left(\dfrac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\ m/s^2 \qquad G = 6.67\times10^{-11}\ Nm^2/kg^2$

$N_A = 6.02\times10^{23}\ atoms/mole \qquad k_B = 1.38\times10^{-23}\ J/K$

$\sigma = 5.67\times10^{-8}\ W/m^2K^4 \qquad v_{sound} = 343\ m/s$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T):\ \beta = 3\alpha$

$PV = Nk_B T$

$\frac{3}{2}k_B T = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \dfrac{\Delta Q}{\Delta t} = \dfrac{kA}{L}\Delta T$

$P_R = \dfrac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta:\ v = r\omega:\ a_t = r\alpha$

$a_r = r\omega^2$

Sound

Fluids

$\rho = \dfrac{M}{V}$

$P = \dfrac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1v_1 = A_2v_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \dfrac{2\pi}{T}$

$T_S = 2\pi\sqrt{\dfrac{m}{k}}$

$T_P = 2\pi\sqrt{\dfrac{l}{g}}$

$v = \pm\sqrt{\dfrac{k}{m}}A\left(1 - \dfrac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\dfrac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\dfrac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\dfrac{F_T}{\mu}}$

$f_n = nf_1 = n\dfrac{v}{2L}$

$I = 2\pi^2 f^2 \rho v A^2$

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L}; \; f_n = nf_1 = n\frac{v}{4L}$$