Name____
Physics 110 Quiz #3, October 7, 2016
Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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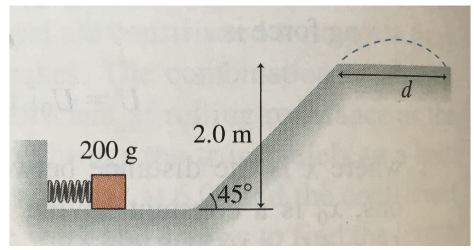
1. A block of mass $m = 0.2kg$ is compressed a distance $x = 0.15m$ against a spring of stiffness $k = 1000\frac{N}{m}$. The mass is released from rest and when the spring get's to its equilibrium position, the mass moves away from the spring toward the incline. The horizontal surfaces are frictionless, but between the block and the incline there is friction and the coefficient of kinetic friction between the block and the incline is $\mu_k = 0.2$. What is the speed of the block when it loses contact with the spring? Use energy methods to solve the problem.



$$\Delta E_{system} = \Delta K + \Delta U_g + \Delta U_s = 0$$
$$0 = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + 0 + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right)$$
$$0 = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}kx_i^2$$
$$v_f = \sqrt{\frac{k}{m}x_i^2} = \sqrt{\frac{1000\frac{N}{m}}{0.2kg}(0.15m)^2} = 10.6\tfrac{m}{s}$$

2. What is the speed of the block at the top of the incline? Use energy methods to solve the problem.

$$\Delta E_{system} = \Delta K + \Delta U_g + \Delta U_s = -W_{fr}$$
$$-F_{fr}l\cos\phi = \left(\tfrac{1}{2}mv_{top}^2 - \tfrac{1}{2}mv_{bottom}^2\right) + \left(mgy_f - mgy_i\right) + 0$$
$$-\mu_k mg\cos\theta l = \left(\tfrac{1}{2}mv_{top}^2 - \tfrac{1}{2}mv_{bottom}^2\right) + mgh$$
$$v_{top}^2 = v_{bottom}^2 - 2hg - \mu_k g\cos\theta l \rightarrow v_{top} = \sqrt{v_{bottom}^2 - 2hg - \mu_k g\cos\theta l}$$
$$v_{top} = \sqrt{\left(10.6\tfrac{m}{s}\right)^2 - \left(2\times 9.8\tfrac{m}{s^2}\times 2m\right) - \left(2\times 0.2\times 9.8\times\cos 45\times 2.8m\right)}$$
$$v_{top} = 8.1\tfrac{m}{s}$$

where the distance the block slides along the incline is calculated from
$$\sin\theta = \frac{2m}{l} \rightarrow l = \frac{2m}{\sin 45} = 2.8m.$$

3. At the top of the incline the block is launched into the air. To what maximum height above the top of the incline does the block reach?

The block is launched at a speed of $v_{top} = 8.1\frac{m}{s}$ directed at an angle of $\theta = 45^0$. Thus the components of the velocity are: $v_{top,x} = v_{top}\cos\theta = 8.1\frac{m}{s}\times\cos 45 = 5.73\frac{m}{s}$ and $v_{top,y} = v_{top}\sin\theta = 8.1\frac{m}{s}\times\sin 45 = 5.73\frac{m}{s}$.

The maximum height is when the vertical component of the velocity is zero. Thus

$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$$

$$0 = v_{iy}^2 - 2gy_{\max}$$

$$y_{\max} = \frac{v_{iy}^2}{2g} = \frac{\left(5.73\frac{m}{s}\right)^2}{2\times 9.8\frac{m}{s^2}} = 1.67m$$

4. From the point the block is launched into the air at the top of the incline, how far horizontally will it travel? That is, what is the horizontal distance d that the block covers?

The time of flight of the projectile is given by the vertical trajectory:

$$y_f = y_i + v_{iy}t + \tfrac{1}{2}a_y t^2$$

$$0 = 0 + \left(v_{iy} - \frac{g}{2}t\right)t$$

$$t = \begin{cases} 0 \\ \dfrac{2v_{iy}}{g} \end{cases}$$

The horizontal displacement is therefore:

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2$$

$$\rightarrow d = v_{ix}t = \frac{2v_{ix}v_{iy}}{g} = \frac{2\left(5.73\frac{m}{s}\right)^2}{9.8\frac{m}{s^2}} = 6.7m$$

5. What is the net work done on the block from the point at which it's launched at the top of the incline to where it lands a horizontal distance d later?

Ignoring air resistance, the only force that acts on the projectile while in flight is due to the force of gravity. And, the force of gravity only acts in the vertical direction. Thus:

$$W_{horizontal} = F_g d\cos\phi = mgd\cos 90 = 0J$$

$$W_{vertical} = W_{zero\,to\,y_{\max}} + W_{y_{\max}\,to\,zero} = F_g y_{\max}\cos\alpha + F_g y_{\max}\cos\beta$$

$$W_{vertical} = F_g y_{\max}\cos 180 + F_g y_{\max}\cos 0 = -F_g y_{\max} + F_g y_{\max} = 0J$$

$$\therefore W_{net} = W_{horizontal} + W_{vertical} = 0J$$

Useful formulas:

Motion in the r = x, y or z-directions

$$r_f = r_0 + v_{0r}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{0r} + a_r t$$

$$v_{fr}^2 = v_{0r}^2 + 2a_r\Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles $C = 2\pi r$, $A = \pi r^2$

Triangles $A = \tfrac{1}{2}bh$

Spheres $A = 4\pi r^2$, $V = \tfrac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$\text{magnitude of a vector} = \sqrt{{v_x}^2 + {v_y}^2}$$

$$\text{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\,{}^{m}/_{s^2}$ $G = 6.67\times10^{-11}\,{}^{Nm^2}/_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}/_{mole}$ $k_B = 1.38\times10^{-23}\,{}^{J}/_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}/_{m^2K^4}$ $v_{sound} = 343\,{}^{m}/_{s}$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$$

Heat

$$T_C = \tfrac{5}{9}[T_F - 32]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}(1 + \alpha\Delta T)$$

$$A_{new} = A_{old}(1 + 2\alpha\Delta T)$$

$$V_{new} = V_{old}(1 + \beta\Delta T) : \beta = 3\alpha$$

$$PV = Nk_BT$$

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$$

$$a_r = r\omega^2$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho gd$$

$$F_B = \rho gV$$

$$A_1v_1 = A_2v_2$$

$$\rho_1A_1v_1 = \rho_2A_2v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho gh_2$$

Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2 \rho v A^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$$