Name____

Physics 110 Quiz #3, October 11, 2019

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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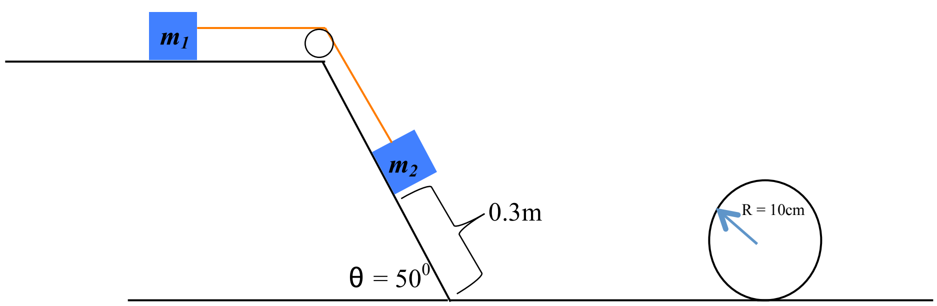
Consider the arrangement of masses shown below. Mass $m_1 = 0.5kg$ is initially at rest on a horizontal surface and is connected to a mass $m_2 = 2.5kg$, also initially at rest, on the ramp inclined at $\theta = 50^0$ with respect to the horizontal.

a. If the both masses are simultaneously released from rest and if all of the surfaces are frictionless, what is the speed of mass m_2 when it reaches the bottom of the ramp, a distance of $0.3m$ from where it was released?



Both masses will acquire the same final speed because the rope connects them. The increase in kinetic energy of both masses comes from the decrease in gravitational potential energy of the mass on the incline.

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s = \left(\Delta K_1 + \Delta U_{g1}\right) + \left(\Delta K_2 + \Delta U_{g2}\right) + \Delta U_s = 0$$

$$0 = \left[\left(\tfrac{1}{2}m_1 v_{1f}^2 - \tfrac{1}{2}m_1 v_{1i}^2\right) + \left(m_1 g y_{1f} - m_1 g y_{1i}\right)\right]$$

$$+\left[\left(\tfrac{1}{2}m_2 v_{2f}^2 - \tfrac{1}{2}m_2 v_{2i}^2\right) + \left(m_2 g y_{2f} - m_2 g y_{2i}\right)\right] + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right)$$

$$0 = \tfrac{1}{2}m_1 v_{1f}^2 + \tfrac{1}{2}m_2 v_{2f}^2 - m_2 g y_{2i}$$

$$0 = \tfrac{1}{2}\left(m_1 + m_2\right)v_f^2 - m_2 g d\sin\theta$$

$$\therefore v_f = \sqrt{\frac{2m_2 g d\sin\theta}{m_1 + m_2}} = \sqrt{\frac{2\times 2.5kg \times 9.8\tfrac{m}{s^2} \times 0.3m \times \sin 50}{0.5kg + 2.5kg}} = 1.94\tfrac{m}{s}$$

b. Suppose that the instant m_2 reaches the bottom of the ramp the cord connecting to breaks. Mass m_2 continues moving to the right and around the loop-the-loop porting of the track. When mass m_2 is halfway up the right-hand side of the loop, how much work was done on mass m_2 by the force of gravity?

1. $W_g = 0$.
2. $W_g = m_2 g$.
3. $W_g = (m_1 + m_2) gR$.
4. $W_g = -m_2 gR$.
5. $W_g = -\left(\frac{m_2}{m_1 + m_2}\right) gR$

c. What is the magnitude of the normal force on mass m_2 when it is halfway up the right-hand side of the loop-the-loop, if the loop-the-loop has a radius $R = 10cm$?

The mass slows down as it rises due to gravity. If we take the mass, the loop-the-loop and the world as the system then we can apply conservation of energy. This will allow us to determine the speed when the mass is halfway up the right-hand side of the loop-the-loop. We have

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s = 0$$

$$0 = \left(\tfrac{1}{2} m_2 v_{2f}^2 - \tfrac{1}{2} m_2 v_{2i}^2\right) + \left(m_2 g y_{2f} - m_2 g y_{2i}\right)$$

$$0 = \tfrac{1}{2} m_2 v_{2f}^2 - \tfrac{1}{2} m_2 v_{2i}^2 + m_2 g y_{2f}$$

$$v_{2f} = \sqrt{v_{2i}^2 - 2gR -} = \sqrt{\left(1.94 \tfrac{m}{s}\right)^2 - \left(2 \times 9.8 \tfrac{m}{s^2} \times 0.1m\right)} = 1.34 \tfrac{m}{s}$$

Now the normal force points towards the center of the circle. When the mass is halfway up the right-hand side of the loop-the-loop the normal force is given by

$$F_N = \frac{mv^2}{R} = \frac{2.5kg\left(1.34 \tfrac{m}{s}\right)^2}{0.1m} 44.9N$$

Physics 110 Formulas

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles   *Triangles*   *Spheres*

$C = 2\pi r \quad A = \frac{1}{2}bh \quad A = 4\pi r^2$

$A = \pi r^2 \quad V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x{}^2 + v^2{}_y}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8 \, m/s^2 \qquad G = 6.67\times10^{-11} \, Nm^2/kg^2$

$N_A = 6.02\times10^{23} \, atoms/mole \qquad k_B = 1.38\times10^{-23} \, J/K$

$\sigma = 5.67\times10^{-8} \, W/m^2K^4 \qquad v_{sound} = 343 \, m/s$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\cdot dt$

$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$

$\vec{F}_s = -k\vec{x}$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

Work/Energy

$K_T = \frac{1}{2}mv^2$

$K_R = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = Fd\,\mathrm{Cos}\,\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T): \ \beta = 3\alpha$

$PV = Nk_B T$

$\frac{3}{2}k_B T = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta: \ v = r\omega: \ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = f\lambda = (331 + 0.6T)\,\frac{m}{s}$

$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\,\frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho g d$

$F_B = \rho g V$

$A_1 v_1 = A_2 v_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho g h_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho v A^2$