Name____

Physics 110 Quiz #3, April 18, 2020

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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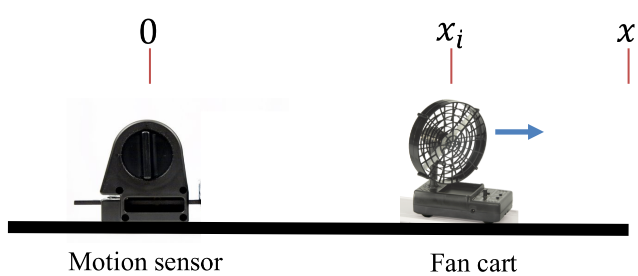
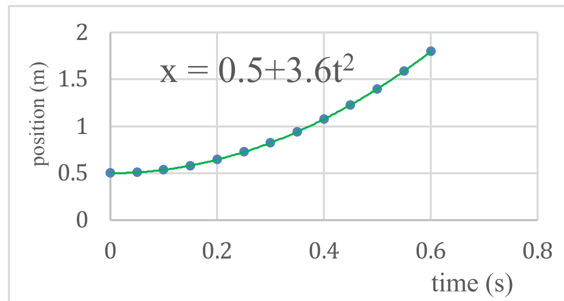
A fan cart is used to study motion. Suppose that a fan cart of mass $m = 0.5kg$ is released from rest and its position a function of time is measured with a motion sensor. The setup is shown on the left and the data of the position of the fan cart as a function to time collected, is plotted on the right.



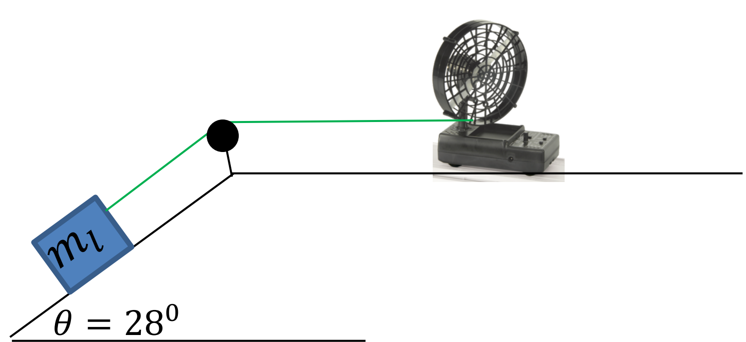


a. What is the magnitude of the constant force F_{cart} produced by the fan on the cart?

$$x = x_i + v_{ix}t + \frac{1}{2}a_x t^2 = 0.5 + 3.6t^2 \rightarrow \frac{1}{2}a_x = 3.6\frac{m}{s^2} \rightarrow a_x = 7.2\frac{m}{s^2}$$

$$F_{cart} = m_{cart}a_{cart} = 0.5kg \times 7.2\frac{m}{s^2} = 3.6N$$

Suppose that the fan cart from above was used in a second experiment shown below. The fan cart is tied to a load of mass $m_l = 0.25kg$ by a massless string passed over a massless pulley. There is friction between both masses and the track with coefficient of friction $\mu = 0.3$. The system is released from rest and the fan cart moves toward the right pulling the load up the incline. Assume the force the fan cart applies is the same as in part a.



b. Using Newton's laws of motion, what is the expression for the net horizontal force on the fan cart?

Assume a standard cartesian coordinate system

$$F_{x,cart} = F_{cart} - F_T - F_{fr} = F_{cart} - F_T - \mu F_{Ncart} = F_{cart} - F_T - \mu m_{cart} g = m_{cart} a$$

where from the vertical motion $F_{Ncart} = m_{cart} g$.

c. Using Newton's laws of motion, what is the expression for the net horizontal force on the load on the incline?

Assume a tilted cartesian coordinate system

$$F_{x,load} = F_T - F_{fr} - F_{wx} = F_T - \mu F_N - F_w \sin\theta = F_T - \mu m_l g \cos\theta - m_l g \sin\theta = m_l a$$

where from the vertical motion $F_{N,l} = m_l g \cos\theta$

d. From your expressions in parts b and c, what is the magnitude of the acceleration of the load up the incline?

Taking the expression from part b, add it to part a eliminating the tension force.

Solving for the acceleration we get after some algebra

$$a = \frac{F_{cart} - \mu g (m_l \cos\theta + m_{cart}) - m_l g \sin\theta}{m_{cart} + m_l}$$

$$a = \frac{3.6N - 0.3 \times 9.8\frac{m}{s^2}(0.25kg \cos 28 + 0.5kg) - 0.25 \times 9.8\frac{m}{s^2} \sin 28}{0.5kg + 0.25kg} = 0.44\frac{m}{s^2}$$

e. From your expressions in b and c, what is the magnitude of the tension in the rope connecting the two masses?

Using either the expression from part b or part c, we can solve for the tension force. I'm going to use part b and then part c to solve for F_T.

$$F_T = F_{cart} - \mu m_{cart} g - m_{cart} a = 3.6N - 0.5kg\left(0.3 \times 9.8\frac{m}{s^2} + 0.44\frac{m}{s^2}\right) = 1.91N$$

$$F_T = m_l a + \mu m_l g \cos\theta - m_l g \sin\theta = 0.25kg\left(0.44\frac{m}{s^2} + 9.8\frac{m}{s^2}(0.3 \cos 28 + \sin 28)\right)$$
$$= 1.91N$$

Physics 110 Formulas

Motion

$\Delta x = x_f - x_i$ $\quad v_{avg} = \frac{\Delta x}{\Delta t}$ $\quad a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles — $C = 2\pi r$, $A = \pi r^2$

Triangles — $A = \frac{1}{2}bh$

Spheres — $A = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\, m/s^2$ $\quad G = 6.67\times10^{-11}\, Nm^2/kg^2$

$N_A = 6.02\times10^{23}\, atoms/mole$ $\quad k_B = 1.38\times10^{-23}\, J/K$

$\sigma = 5.67\times10^{-8}\, W/m^2K^4$ $\quad v_{sound} = 343\, m/s$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T) : \beta = 3\alpha$

$PV = Nk_B T$

$\frac{3}{2}k_B T = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = f\lambda = (331 + 0.6T)\, \frac{m}{s}$

$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\, \frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1 v_1 = A_2 v_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho v A^2$