Name_______________________________________

Physics 110 Quiz #3, April 16, 2021

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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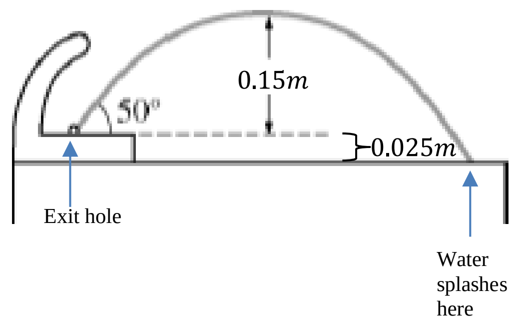
1. A water fountain shown below projects water out of an exit hole at an initial speed of v_i at an angle of 50^0measured with respect to the horizontal. If the water reaches a height of $0.15m$ above its launch point, what was the speed v_i of water as it leaves the exit hole?



$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$$

$$0^0 = (v_i \sin\theta)^2 - 2g\Delta y \rightarrow v_i = \frac{2g\Delta y}{\sin^2\theta}$$

$$v_i = \frac{2 \times 9.8\frac{m}{s^2} \times 0.15m}{\sin^{-1} 50} = 2.24\frac{m}{s}$$

2. How far horizontally does the water travel before it splashes into the bottom of the fountain?

$$x_f = x_i + v_{ix}t + \tfrac{1}{2}a_x t^2 \rightarrow x_f = v_{ix}t = (v_i\cos\theta)t \rightarrow t = \frac{x_f}{v_i\cos\theta}$$

$$y_f = y + v_{iy}t + \tfrac{1}{2}a_y t^2 = (v_i\sin\theta)\left(\frac{x_f}{v_i\cos\theta}\right) - \frac{g}{2}\left(\frac{x_f}{v_i\cos\theta}\right)^2 = x_f\tan\theta - \frac{g x_f^2}{2v_i^2\cos^2\theta}$$

$$0 = 0.025 + x_f\tan 50 - \left(\frac{9.8\frac{m}{s^2}}{2\left(2.24\frac{m}{s}\right)^2\cos^2 50}\right)x_f^2$$

$$x_f = \begin{cases} 0.52m \\ -0.02m \end{cases}$$

The water lands a horizontal distance of $0.52m$ to the right of where it was launched.

3. What is the impact speed of the water with the bottom of the fountain?

$$v_{fx} = v_{ix} + a_x t \rightarrow v_{fx} = v_{ix} = v_i \cos\theta = 2.24\frac{m}{s}\cos 50 = 1.44\frac{m}{s}$$

$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y \rightarrow v_{fy} = \sqrt{v_{iy}^2 - 2g\Delta y} = \sqrt{(v_i \sin\theta)^2 - 2gy_f}$$

$$v_{fy} = -\sqrt{\left(2.24\frac{m}{s}\sin 50\right)^2 - 2 \times 9.8\frac{m}{s^2} \times (-0.025m)} = -1.85\frac{m}{s}$$

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(1.44\frac{m}{s}\right)^2 + \left(-1.85\frac{m}{s}\right)^2} = 2.3\frac{m}{s}$$

4. At what angle, measured with respect to the horizontal, does the water make when it strikes the bottom of the fountain?

$\tan\phi = \frac{v_{fy}}{v_{fx}} = \frac{-1.85\frac{m}{s}}{1.44\frac{m}{s}} = -1.29 \rightarrow \phi = -52.1^0$ or 52.1^0 below the positive x-axis.

5. To get the water (of mass m) out of the exit hole, a pump (not shown) must accelerate the water from rest to its launch speed out of the exit hole. It takes a time t, from when you push the button to get water until the water leaves the exit hole, to accelerate this water to its launch speed. What magnitude of force would be needed to accelerate the water out of the exit hole?

a. $F = \frac{mv_f}{t}$.
b. $F = \frac{mv_f^2}{2t}$.
c. $F = mv_f t$.
d. $F = \frac{v_f}{t}$
e. None of the above give the correct magnitude of the force required.

Physics 110 Formulas

Motion

$\Delta x = x_f - x_i$ $\quad v_{avg} = \frac{\Delta x}{\Delta t}$ $\quad a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles	Triangles	Spheres
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

Quadratic equation: $ax^2 + bx + c = 0$,

whose solutions are given by: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\, m/s^2 \quad G = 6.67 \times 10^{-11}\, Nm^2/kg^2$

$N_A = 6.02 \times 10^{23}\, atoms/mole \quad k_B = 1.38 \times 10^{-23}\, J/K$

$\sigma = 5.67 \times 10^{-8}\, W/m^2K^4 \quad v_{sound} = 343\, m/s$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = Fd\cos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T) : \beta = 3\alpha$

$PV = Nk_BT$

$\frac{3}{2}k_BT = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = e\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = f\lambda = (331 + 0.6T)\, \frac{m}{s}$

$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1 \times 10^{-12}\, \frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}; \quad f_n = nf_1 = n\frac{v}{4L}$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1v_1 = A_2v_2$

$\rho_1A_1v_1 = \rho_2A_2v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{m}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho vA^2$