Name $\qquad$
Physics 110 Quiz \#3, April 15, 2022
Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

As spring is springing and the grass is greening, sports are taking to the field. One of the sports that many people of all ages look forward to is the opening of golf courses. Suppose that you go golfing on a windless day and when you hit the ball it leaves the club head at a speed of $67 \frac{\mathrm{~m}}{\mathrm{~s}}(\sim 150 \mathrm{mph})$ at an angle $40^{\circ}$ with respect to the horizontal. In the problems below assume a standard coordinate system with up and to the right (the directions the ball is hit) as the positive $y$ and x -directions respectively.

1. What is the time-of-flight of the golf ball through the air?

$$
y_{f}=y_{i}+v_{i y} t+\frac{1}{2} a_{y} t^{2} \rightarrow 0=\left(v_{i} \sin \theta\right) t-\frac{1}{2} g t^{2}=\left(v_{i} \sin \theta-\frac{1}{2} g t\right) t
$$

$t=0$ when the ball was struck.

$$
0=v_{i} \sin \theta-\frac{1}{2} g t \rightarrow t_{t o f}=\frac{2 v_{i} \sin \theta}{g}=\frac{2 \times 67 \frac{\mathrm{~m}}{s} \times \sin 40}{9.8 \frac{\mathrm{~m}}{s^{2}}}=8.8 \mathrm{~s}
$$

2. How far from where the golf ball was hit by the club does the golf ball land?

$$
x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}=v_{i x} t=\left(v_{i} \cos \theta\right) t_{t o f}=67 \frac{\mathrm{~m}}{\mathrm{~s}} \times 8.8 \mathrm{~s} \times \cos 40=451 \mathrm{~m}
$$

3. What was the maximum height above the ground reached by the golf ball?

$$
\begin{aligned}
& v_{f y}=v_{i y}+a_{y} t \rightarrow 0=v_{i} \sin \theta-g t \rightarrow t_{\text {rise }}=\frac{v_{i} \sin \theta}{g}=\frac{67 \frac{\mathrm{~m}}{s} \sin 40}{9.8 \frac{\mathrm{~m}}{s^{2}}}=4.4 \mathrm{~s} \\
& y_{f}=y_{i}+v_{i y} t+\frac{1}{2} a_{y} t^{2}=\left(v_{i} \sin \theta\right) t_{\text {rise }}-\frac{1}{2} g t_{\text {rise }}^{2} \\
& y_{\max }=67 \frac{\mathrm{~m}}{\mathrm{~s}} \times 4.4 \mathrm{~s} \times \sin 40-\frac{1}{2} \times 9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}(4.4 \mathrm{~s})^{2}=94.6 \mathrm{~m}
\end{aligned}
$$

4. What was the impact velocity of the golf ball with the ground?

$$
\begin{aligned}
& v_{f x}=v_{i x}+a_{x} t \rightarrow v_{f x}=v_{i x}=v_{i} \cos \theta=67 \frac{\mathrm{~m}}{\mathrm{~s}} \cos 40=51.3 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& v_{f y}=v_{i y}+a_{y} t=v_{i} \sin \theta-g t_{t o f}=67 \frac{\mathrm{~m}}{\mathrm{~s}} \sin 40-9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \times 8.8 s=-43.2 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& v_{f}=\sqrt{v_{f x}^{2}+v_{f y}^{2}}=\sqrt{\left(51.3 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(-43.2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}}=67 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& \tan \phi=\frac{v_{f y}}{v_{f x}} \rightarrow \phi=\tan ^{-1}\left(\frac{v_{f y}}{v_{f x}}\right)=\tan ^{-1}\left(\frac{-43.2 \frac{\mathrm{~m}}{\mathrm{~s}}}{51.3 \frac{\mathrm{~m}}{\mathrm{~s}}}\right)=-40^{0} \text { or } 40^{\circ} \text { below the positive x-axis. }
\end{aligned}
$$

5. Suppose that while you're playing the wind picks up such that the wind creates a constant acceleration of $2.7 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$ in the negative x -direction. Supposing that the golf ball leaves the club with the same initial velocity, how far away does the golf ball land?
$y_{f}=y_{i}+v_{i y} t+\frac{1}{2} a_{y} t^{2} \rightarrow 0=\left(v_{i} \sin \theta\right) t-\frac{1}{2} g t^{2}=\left(v_{i} \sin \theta-\frac{1}{2} g t\right) t$
$t=0$ when the ball was struck.
$0=v_{i} \sin \theta-\frac{1}{2} g t \rightarrow t_{t o f}=\frac{2 v_{i} \sin \theta}{g}=\frac{2 \times 67 \frac{\mathrm{~m}}{\mathrm{~s}} \times \sin 40}{9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=8.8 \mathrm{~s}$
$x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}=\left(v_{i} \cos \theta\right) t_{t o f}-\frac{1}{2} a t_{t o f}^{2}$
$x_{f}=67 \frac{\mathrm{~m}}{\mathrm{~s}} \times 8.8 \mathrm{~s} \times \cos 40-\frac{1}{2} \times 2.7 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}(8.8 \mathrm{~s})^{2}=347 \mathrm{~m}$

Vectors
$v=\sqrt{v_{x}^{2}+v_{y}^{2}}$
$\phi=\tan ^{-1}\left(\frac{v_{y}}{v_{x}}\right)$

## Motion Definitions

Displacement: $\Delta x=x_{f}-x_{i}$
Average velocity: $v_{\text {avg }}=\frac{\Delta x}{\Delta t}$
Average acceleration: $a_{a v g}=\frac{\Delta v}{\Delta t}$

## Equations of Motion

displacement: $\left\{\begin{array}{l}x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2} \\ y_{f}=y_{i}+v_{i y} t+\frac{1}{2} a_{y} t^{2}\end{array}\right.$
velocity: $\left\{\begin{array}{l}v_{f x}=v_{i x}+a_{x} t \\ v_{f y}=v_{i y}+a_{y} t\end{array}\right.$
time-independent: $\left\{\begin{array}{l}v_{f x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x \\ v_{f y}^{2}=v_{i y}^{2}+2 a_{y} \Delta y\end{array}\right.$

Rotational Motion Definitions
Angular displacement: $\Delta s=R \Delta \theta$
Angular velocity: $\omega=\frac{\Delta \theta}{\Delta t} \rightarrow v=R \omega$
Angular acceleration: $\alpha=\frac{\Delta \omega}{\Delta t} \rightarrow\left\{\begin{array}{c}a_{t}=r \alpha \\ a_{c}=r \omega^{2}\end{array}\right.$
Rotational Equations of Motion

$$
\begin{aligned}
& \theta_{f}=\theta_{i}+\omega_{i} t+\frac{1}{2} \alpha t^{2} \\
& \omega_{f}=\omega_{i}+\alpha t \\
& \omega_{f}^{2}=\omega_{i}^{2}+2 \alpha \Delta \theta
\end{aligned}
$$

Momentum \& Force
$\vec{p}=m \vec{v} \rightarrow p_{x}=m v_{x} ; p_{y}=m v_{y}$
$\Delta \vec{p}=\vec{F} \Delta t \rightarrow \vec{p}_{f}=\vec{p}_{i}+\vec{F} \Delta t$
$\vec{F}=\frac{d \vec{p}}{d t}=m \vec{a} \rightarrow F_{x}=m a_{x} ; F_{y}=m a_{y}$
$F_{f r}=\mu F_{N}$
$F_{w}=m g$
$F_{S}=-k x$
$F_{G}=G \frac{M_{1} M_{2}}{r^{2}}$
$F_{c}=m a_{c}=m \frac{v^{2}}{R}$
Work \& Energy
$\left\{\begin{array}{c}W_{T}=\int \vec{F} \cdot d \vec{r}=F d r \cos \theta=\Delta K_{T} \\ W_{R}=\int \vec{\tau} \cdot d \vec{\theta}=\tau d \theta=\Delta K_{R}\end{array}\right.$
$W_{n e t}=W_{T}+W_{R}=\Delta K_{T}+\Delta K_{R}=-\Delta U$
$K_{T}=\frac{1}{2} m v^{2}$
$K_{R}=\frac{1}{2} I \omega^{2}$
$U_{g}=m g y$
$U_{s}=\frac{1}{2} k x^{2}$
$\Delta E=\Delta E_{R}+\Delta E_{T}$
$\Delta E=\Delta K_{R}+\Delta K_{T}+\Delta U_{g}+\Delta U_{s}=\left\{\begin{array}{c}0 \\ W_{f r}\end{array}\right.$
Rotational Momentum \& Force

$$
\begin{aligned}
& \vec{\tau}=\vec{r} \times \vec{F} ; \tau=r_{\perp} F=r F_{\perp}=r F \sin \theta \\
& \tau=\frac{\Delta L}{\Delta t}=I \alpha \\
& L=I \omega \\
& \Delta \vec{L}=\vec{\tau} \Delta t \rightarrow \vec{L}_{f}=\vec{L}_{i}+\vec{\tau} \Delta t
\end{aligned}
$$

Fluids
$\rho=\frac{m}{V}$
$P=\frac{F}{A}$
$P_{y}=P_{\text {air }}+\rho g y$
$F_{B}=\rho g V$
$\rho_{1} A_{1} v_{1}=\rho_{2} A_{2} v_{2}$; compressible $A_{1} v_{1}=A_{2} v_{2}$; incompressible
$P_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1}=P_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho g y_{2}$

Simple Harmonic Motion
$\omega=2 \pi f=\frac{2 \pi}{T}$
$T_{s}=2 \pi \sqrt{\frac{m}{k}} ; \quad \omega=\sqrt{\frac{k}{m}}$
$T_{p}=2 \pi \sqrt{\frac{l}{g}} ; \quad \omega=\sqrt{\frac{g}{l}}$

Geometry/Algebra
Circles:

$$
A=\pi r^{2} \quad C=2 \pi r=\pi D
$$

Spheres: $\quad A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$
Triangles: $\quad A=\frac{1}{2} b h$
Quadratics: $\quad a x^{2}+b x+c=0 \rightarrow x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \quad v= \pm \omega x_{\max } \sqrt{1-\left(\frac{x}{x_{\max }}\right)^{2}}$

Sound
$v_{s}=f \lambda=(331+0.6 T) \frac{m}{s}$
$\beta=10 \log \frac{I}{I_{o}}$
$f_{n}=n f_{1}=n \frac{v}{2 L} ; n=1,2,3, \ldots$ open pipes
$f_{n}=n f_{1}=n \frac{v}{4 L} ; n=1,3,5, \ldots$ closed pipes
Waves
$v=f \lambda=\sqrt{\frac{F_{T}}{\mu}}$
$f_{n}=n f_{1}=n \frac{v}{2 L} ; n=1,2,3, \ldots$
$I=2 \pi^{2} f^{2} \rho v A^{2}$

Equations of Motion for SHM

$$
x(t)=\left\{\begin{array}{l}
x_{\max } \sin \left(\frac{2 \pi}{T} t\right) \\
x_{\max } \cos \left(\frac{2 \pi}{T} t\right)
\end{array}\right.
$$

$$
v(t)=\left\{\begin{array}{c}
v_{\max } \cos \left(\frac{2 \pi}{T} t\right) \\
-v_{\max } \sin \left(\frac{2 \pi}{T} t\right)
\end{array}\right.
$$

$a(t)=\left\{\begin{array}{l}-a_{\text {max }} \sin \left(\frac{2 \pi}{T} t\right) \\ -a_{\text {max }} \cos \left(\frac{2 \pi}{T} t\right)\end{array}\right.$
$v= \pm v_{\max } \sqrt{1-\left(\frac{x}{x_{\max }}\right)^{2}}$
nano $=1 \times 10^{-9}$
micro $=1 \times 10^{-6}$
milli $=1 \times 10^{-3}$
cent $i=1 \times 10^{-2}$
kilo $=1 \times 10^{3}$
mega $=1 \times 10^{6}$


