Name____

Physics 110 Quiz #4, October 18, 2019

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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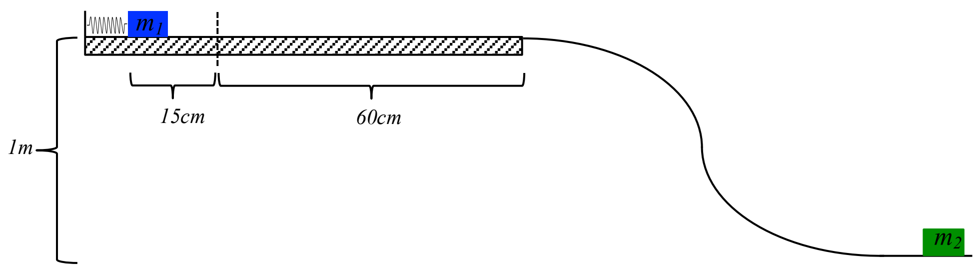
A spring, with unknown spring constant, is compressed by an amount $\Delta x = 15cm$ from its equilibrium position and a mass $m_1 = 0.5kg$ is placed at rest on the end of the spring. When the mass is released from rest, the spring extends by $\Delta x = 15cm$ at which point the mass loses contact with the spring. There is friction between the mass and the horizontal surface with a coefficient of friction of $\mu = 0.7$.

a. What spring, or stiffness, constant for would be required so that the mass leaves the end of the horizontal surface at a speed of $2.7\frac{m}{s}$? Assume that from the equilibrium position, shown by the vertical dashed line, when the mass loses contact with the spring, to the end of the horizontal section of track the distance is $60cm$, as is shown in the diagram below.



Let A be the initial position of the compressed spring, B be the point the mass loses contact with the spring and C be the end of the horizontal track. Conservation of energy between points A and B gives

$$\Delta E_{AB} = \Delta K + \Delta U_g + \Delta U_s = \Delta K + \Delta U_s = W_{fr}$$

$$-F_{fr}\Delta x_{AB} = \left(\tfrac{1}{2}m_1v_B^2 - \tfrac{1}{2}m_1v_A^2\right) + \left(\tfrac{1}{2}kx_B^2 - \tfrac{1}{2}kx_A^2\right) = \tfrac{1}{2}m_1v_B^2 - \tfrac{1}{2}kx_A^2$$

Conservation of energy between points B and C gives

$$\Delta E_{BC} = \Delta K + \Delta U_g + \Delta U_s = \Delta K = W_{fr}$$

$$-F_{fr}\Delta x_{BC} = \tfrac{1}{2}m_1v_C^2 - \tfrac{1}{2}m_1v_B^2 \rightarrow \tfrac{1}{2}m_1v_B^2 = \tfrac{1}{2}m_1v_C^2 + F_{fr}\Delta x_{BC}$$

Combining the two expressions for the kinetic energy at point B, we have

$$-F_{fr}\Delta x_{AB} = \tfrac{1}{2}m_1v_B^2 - \tfrac{1}{2}kx_A^2 = \tfrac{1}{2}m_1v_C^2 + F_{fr}\Delta x_{BC} - \tfrac{1}{2}kx_A^2$$

$$k = \frac{m_1v_C^2 + 2F_{fr}\left(\Delta x_{AB} + \Delta x_{BC}\right)}{x_A^2} = \frac{0.5kg\left(2.7\frac{m}{s}\right)^2 + 2\times 0.7\times 0.5kg\times 9.8\frac{m}{s^2}\times\left(0.75m\right)}{\left(0.15m\right)^2}$$

$$k = 391\tfrac{N}{m}$$

b. With what speed did mass m_1 lose contact with the spring?

From part a, we have $\frac{1}{2}m_1v_B^2 = \frac{1}{2}m_1v_C^2 + F_{fr}\Delta x_{BC}$. Thus

$$\frac{1}{2}m_1v_B^2 = \frac{1}{2}m_1v_C^2 + F_{fr}\Delta x_{BC} = \frac{1}{2}m_1v_C^2 + \mu m_1 g\Delta x_{BC}$$

$$v_B = \sqrt{v_C^2 + 2\mu g\Delta x_{BC}} = \sqrt{\left(2.7\tfrac{m}{s}\right)^2 + 2\times 0.7\times 9.8\tfrac{m}{s^2}\times 0.6m} = 3.9\tfrac{m}{s}$$

c. Suppose that after sliding across the horizontal surface, the mass slides down a frictionless ramp. At the bottom of the ramp, there is a stationary mass $m_2 = 2.0kg$. Mass m_1 makes an inelastic collision with mass m_2 and the two masses stick together after the collision. How fast are the two masses moving to the right after the collision?

Applying conservation of energy between the top and bottom of the track we have

$$\Delta E_{AB} = \Delta K + \Delta U_g + \Delta U_s = \Delta K + \Delta U_g = 0$$

$$0 = \left(\tfrac{1}{2}m_1v_{bottom}^2 - \tfrac{1}{2}m_1v_{top}^2\right) + \left(m_1 g y_{bottom} - m_1 g y_{top}\right)$$

$$v_{bottom} = \sqrt{v_{top}^2 + 2gy_{top}} = \sqrt{\left(2.7\tfrac{m}{s}\right)^2 + 2\times 9.8\tfrac{m}{s^2}\times 1m} = 5.2\tfrac{m}{s}$$

Applying conservation of momentum in the collision we have

$$p_{i,system} = p_{f,system} \rightarrow m_1v_{bottom} = \left(m_1 + m_2\right)V$$

$$V = \left(\frac{m_1}{m_1 + m_2}\right)v_{bottom} = \left(\frac{0.5kg}{2.5kg}\right)5.2\tfrac{m}{s} = 1.0\tfrac{m}{s}$$

d. How much energy is lost to the collision of the two masses?

$$\Delta E_{lost} = K_f - K_i = \tfrac{1}{2}\left(2.5kg\right)\left(1\tfrac{m}{s}\right)^2 - \tfrac{1}{2}\left(0.5kg\right)\left(5.2\tfrac{m}{s}\right)^2 = -5.51J$$

or as a fraction of the incident energy,

$$f = \left|\left(\frac{K_f - K_i}{K_i}\right)\right| = \left|\left(\frac{\frac{1}{2}\left(m_1 + m_2\right)V^2 - \frac{1}{2}m_1v_{bottom}^2}{\frac{1}{2}m_1v_{bottom}^2}\right)\right| = \left|\left(\frac{\frac{1}{2}\left(2.5kg\right)\left(1\frac{m}{s}\right)^2 - \frac{1}{2}\left(0.5kg\right)\left(5.2\frac{m}{s}\right)^2}{\frac{1}{2}\left(0.5kg\right)\left(5.2\frac{m}{s}\right)^2}\right)\right|$$

$$f = 0.82$$

Physics 110 Formulas

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Uniform Circular Motion

$$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x^{\,2} + v_y^{\,2}}$

direction of a vector: $\phi = \tan^{-1}\left(\dfrac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\,{}^{m}/_{s^2}$ $\quad G = 6.67\times10^{-11}\,{}^{Nm^2}/_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}/_{mole}$ $\quad k_B = 1.38\times10^{-23}\,{}^{J}/_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}/_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}/_{s}$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\cdot dt$$

$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

$$\vec{F}_s = -k\vec{x}$$

$$|\vec{F}_{fr}| = \mu|\vec{F}_N|$$

Work/Energy

$$K_T = \tfrac{1}{2}mv^2$$

$$K_R = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = F\Delta x \operatorname{Cos}\theta = \Delta K_T$$

$$W_R = \tau\theta = \Delta K_R$$

$$W_{net} = W_R + W_T = \Delta K_R + \Delta K_T$$

$$\Delta K_R + \Delta K_T + \Delta U_g + \Delta U_S = \Delta E_{system} = 0$$

$$\Delta K_R + \Delta K_T + \Delta U_g + \Delta U_S = \Delta E_{system} = W_{fr} = -F_{fr}\Delta x$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta : \; v = r\omega : \; a_t = r\alpha$$

$$a_r = r\omega^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L}; \; f_n = nf_1 = n\frac{v}{4L}$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho gd$$

$$F_B = \rho gV$$

$$A_1v_1 = A_2v_2$$

$$\rho_1A_1v_1 = \rho_2A_2v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho gh_2$$

Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2 \rho v A^2$$