Name____

Physics 110 Quiz #4, October 16, 2020

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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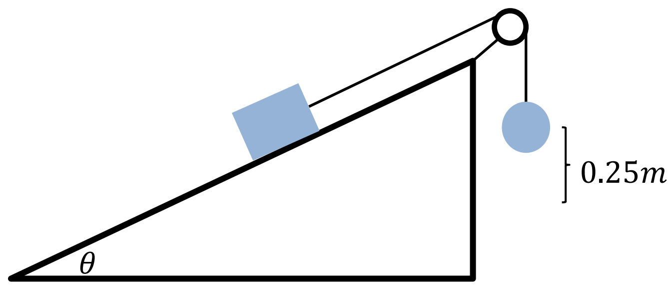
A block of mass $m_1 = 2kg$ is tied by a light rope to a sphere of mass $m_2 = 4kg$. The ramp is inclined at angle $\theta = 42^0$measured with respect to the horizontal.

1. If the $4kg$ sphere is released from rest and is allowed to fall through a distance $d = 0.25m$, what is the speed of the sphere if the ramp is considered frictionless?



$$\Delta E = \Delta K_1 + \Delta U_{g1} + \Delta K_2 + \Delta U_{g2} = 0$$

$$\frac{1}{2}m_1 v_{1f}^2 + m_1 g y_{1f} + \frac{1}{2}m_2 v_{2f}^2 - m_2 g y_{2f} = 0$$

$$\frac{1}{2}(m_1 + m_2)v_f^2 = m_2 g d - m_1 g d \sin\theta$$

$$v_f = \sqrt{\frac{2(m_2 - m_1 \sin\theta)gd}{m_1 + m_2}}$$

$$v_f = \sqrt{\frac{2(4kg - 2kg \sin 42) \times 9.8\frac{m}{s^2} \times 0.25m}{2kg + 4kg}} = 1.47\frac{m}{s}$$

2. Suppose instead that the $4kg$ sphere were again released from rest and is allowed to fall through a distance $d = 0.25m$ but this time the ramp is not frictionless. If the speed of the sphere is $v = 1.2\frac{m}{s}$, what is the coefficient of friction between the block and the ramp?

$$\Delta E = \Delta K_1 + \Delta U_{g1} + \Delta K_2 + \Delta U_{g2} = W_{fr}$$

$$\frac{1}{2}m_1 v_{1f}^2 + m_1 g y_{1f} + \frac{1}{2}m_2 v_{2f}^2 - m_2 g y_{2f} = -\mu(m_1 g \cos\theta)d$$

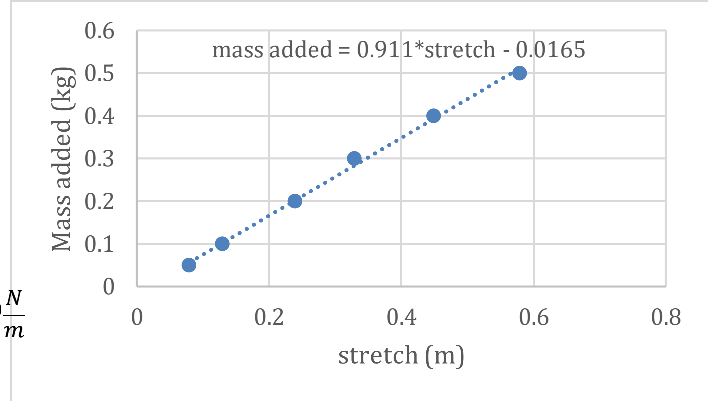
$$\mu = -\frac{\frac{1}{2}(m_1 + m_2)v_f^2 - m_2 g d + m_1 g d \sin\theta}{m_1 g d \cos\theta}$$

$$\mu = -\frac{\frac{1}{2}(2kg + 4kg)\left(1.2\frac{m}{s}\right)^2 + (2kg \sin 42 - 4kg) \times 9.8\frac{m}{s^2} \times 0.25m}{2kg \times 9.8\frac{m}{s^2} \times 0.25m \cos 42}$$

$$\mu = 0.6$$

3. Which of the following gives the change in gravitational potential energy of the $m = 1kg$ block compared to the $M = 4kg$ sphere?
   a. $\Delta U_{g,m} > \Delta U_{g,M}$.
   b. $\Delta U_{g,m} = \Delta U_{g,M}$.
   c. $\Delta U_{g,m} < \Delta U_{g,M}$.
   d. The change in gravitational energy cannot be determined from the information given.

A spring of stiffness k is used in a separate experiment. To determine the stiffness of the spring, the spring is initially suspended vertically, and various masses are hung and the corresponding stretch of the spring from equilibrium is measured. A plot of the mass added to the spring versus the stretch of the spring is shown below.



4. What is the stiffness k of the spring?

$$F_s - F_w = ma_y = 0$$
$$ky = mg \rightarrow m = \frac{k}{g}y$$
$$\frac{k}{g} = 0.91\frac{kg}{m} \rightarrow k = 0.91\frac{kg}{m} \times 9.8\frac{m}{s^2} = 8.9\frac{N}{m}$$

5. Suppose that the spring is laid horizontal and the $2kg$ mass is attached to the spring at its equilibrium length. The surface is frictionless and the $2kg$ mass is pulled out to a distance of $0.75m$ from equilibrium and released from rest. What is the speed of the mass when the spring returns to its equilibrium length?

$$\Delta E = \Delta K + \Delta U_s \rightarrow 0 = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}kx_i^2 \rightarrow v_f = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{8.9\frac{N}{m}}{2kg}} \times 0.75m = 1.6\frac{m}{s}$$

Physics 110 Formulas

Motion

$\Delta x = x_f - x_i$ $\quad v_{avg} = \frac{\Delta x}{\Delta t}$ $\quad a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles $\quad$ *Triangles* $\quad$ *Spheres*

$C = 2\pi r \quad A = \frac{1}{2}bh \quad A = 4\pi r^2$

$A = \pi r^2 \quad V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x^{\ 2} + v_{\ y}^2}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\ ^{m}/_{s^2} \qquad G = 6.67 \times 10^{-11}\ ^{Nm^2}/_{kg^2}$

$N_A = 6.02 \times 10^{23}\ ^{atoms}/_{mole} \qquad k_B = 1.38 \times 10^{-23}\ ^{J}/_{K}$

$\sigma = 5.67 \times 10^{-8}\ ^{W}/_{m^2K^4} \qquad v_{sound} = 343\ ^{m}/_{s}$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = Fd\cos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T): \ \beta = 3\alpha$

$PV = Nk_BT$

$\frac{3}{2}k_BT = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = e\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = f\lambda = (331 + 0.6T)\ \frac{m}{s}$

$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1 \times 10^{-12}\ \frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}; \quad f_n = nf_1 = n\frac{v}{4L}$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1v_1 = A_2v_2$

$\rho_1A_1v_1 = \rho_2A_2v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{m}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho v A^2$