Name____

Physics 110 Quiz #4, October 16, 2013

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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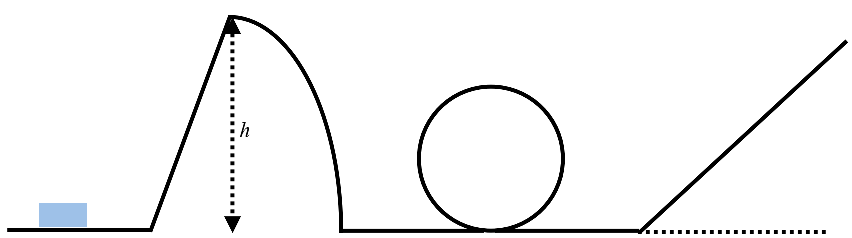
Suppose that you are a roller coaster designer tasked with a new thrill ride. The cars and their riders (of mass $m_{total} = 1000kg$) start on the ground an immediately a mechanism pulls them up a large hill of height h. The cart and riders crest the hill and starting essentially from rest at the top of the hill, travel down to the hill to the first loop. The top of the loop is $15m$ above the ground and the cart and riders travel around the loop and exit the loop moving to the right. The cart and riders then travel towards a ramp that is inclined at an angle of 45^0 with respect to the ground. The entire track is frictionless with the exception of the inclined portion with the ramp which as a coefficient of friction μ.

1. What is the minimum height h that is needed so that the cart and riders (the blue box) will make it around the loop?



To calculate the minimum height, we'll use the fact that energy is conserved and that at the top of loop, the minimum speed will lead to the minimum height. We find the minimum speed by examining the forces on the cart and riders assuming that vertically up is the positive y-direction. When the normal force vanishes we have the condition for the minimum speed.

$$\sum F_y : -F_N - F_W = ma_c = -m\frac{v^2}{r} \to F_W = mg = m\frac{v_{\min}^2}{r} \to v_{\min}^2 = rg$$

Applying conservation of energy we have for the minimum height

$$\Delta E = \Delta KE + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_f^2\right) + \left(mgy_f - mgy_i\right) = \tfrac{1}{2}mv_{\min}^2 + \left(mg(2r) - mgh\right) = 0$$

$$\therefore h = \frac{2mgr + \tfrac{1}{2}mrg}{mg} = 2.5r = 2.5 \times 7.5m = 18.8m$$

2. Suppose that to ensure rider safety you make the height of the hill 20% greater than the value you found in part 1. What will the speed at the bottom of the loop?

Assuming that the height of the starting hill is 20% greater than the value in part 1, and taking the ground as the zero of our potential energy, we have by conservation of energy

$$\Delta E = \Delta KE + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_f^2\right) + \left(mgy_f - mgy_i\right) = \tfrac{1}{2}mv_{bottom}^2 - mg(1.2h) = 0$$

$$\therefore v_{bottom} = \sqrt{2(1.2gh)} = \sqrt{2.4 \times 9.8\tfrac{m}{s^2} \times 18.8m} = 21.0\tfrac{m}{s}$$

3. When the cart and riders enter the inclined portion of the ride they come to rest at a distance x measured along the incline. Considering just the cart and riders, the net work done by all external forces on the cart and rider as the cart and riders travel along the incline is given by

a. $W_{net} = 0J$.
b. $W_{net} = -1.2mgh$.
c. $W_{net} = mgx\sin\theta$.
d. $W_{net} = -\mu mgx\cos\theta$.

The net work done on the cart and riders by either calculating the change in kinetic energy or by determining the net force that acts on the cart and riders over the distance given.

From the change in kinetic energy we have:

$$W_{net} = \Delta KE = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = -\tfrac{1}{2}mv_{bottom}^2 = -1.2mgh$$

From the net force we have:

$$W_{net} = \vec{F}_{net} \cdot \Delta\vec{x} = F_{net}\Delta x\cos\phi = \left(mg\sin\theta + \mu mg\cos\theta\right)x\cos(180) = -\left(mg\sin\theta + \mu mg\cos\theta\right)x$$

which one could work out to show gives choice B.

Useful formulas:

Motion in the r = x, y or z-directions

$$r_f = r_0 + v_{0r}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{0r} + a_r t$$

$$v_{fr}^{\ 2} = v_{0r}^{\ 2} + 2a_r \Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$magnitude\ of\ a\,vector = \sqrt{v_x^{\ 2} + v_y^{\ 2}}$$

$$direction\ of\ a\,vector \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\,{}^{m}/_{s^2}$ $\quad G = 6.67\times10^{-11}\,{}^{Nm^2}/_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}/_{mole}$ $\quad k_B = 1.38\times10^{-23}\,{}^{J}/_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}/_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}/_{s}$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$$

Heat

$$T_C = \tfrac{5}{9}\left[T_F - 32\right]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}(1 + \alpha\Delta T)$$

$$A_{new} = A_{old}(1 + 2\alpha\Delta T)$$

$$V_{new} = V_{old}(1 + \beta\Delta T) : \beta = 3\alpha$$

$$PV = Nk_B T$$

$$\tfrac{3}{2}k_B T = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$$

$$a_r = r\omega^2$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho gd$$

$$F_B = \rho gV$$

$$A_1 v_1 = A_2 v_2$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho gh_2$$

Simple Harmonic Motion/Waves

$$\omega = \sqrt{\frac{k}{m}} = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = x_{\max}\sin(\omega t) \ \ or \ \ x_{\max}\cos(\omega t)$$

$$v(t) = v_{\max}\cos(\omega t) \ \ or \ \ -v_{\max}\sin(\omega t)$$

$$a(t) = -a_{\max}\sin(\omega t) \ \ or \ \ -a_{\max}\cos(\omega t)$$

$$v_{\max} = \omega x_{\max}; \quad a_{\max} = \omega^2 x_{\max}$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2 \rho v A^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$$