Name____

Physics 110 Quiz #5, October 21, 2016

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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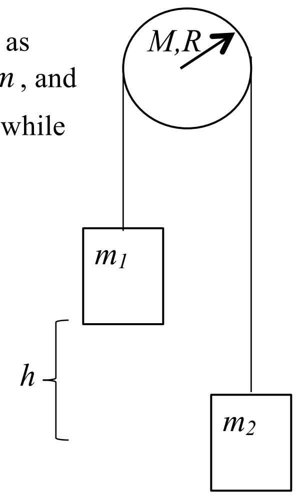
Two blocks are connected by a light rope that passes around a massive pulley as shown on the right. The pulley has a mass of $M = 6kg$, radius of $R = 0.45m$, and moment of inertia $I_p = \frac{1}{2}MR^2$. The block on the left has a mass $m_1 = 75kg$ while the block on the right has mass $m_2 = 65kg$.



1. What is the expression angular acceleration of the system about the axis of the pulley if the left block is released from rest? Choose counter-clockwise as the positive direction for the torques.

$$\sum \tau : F_{TL}R - F_{TR}R = I\alpha$$

$$\alpha = \frac{\left(F_{TL} - F_{TR}\right)R}{I} = \frac{2\left(F_{TL} - F_{TR}\right)}{MR}$$

2. From free-body diagrams of the forces that act on the blocks m_1 and m_2 and using your result for the angular acceleration, what is the acceleration of the block with mass m_1?

The forces on each mass:

m_1 :

$$\sum F_{y:}\ F_{TL} - m_1 g = -m_1 a \rightarrow F_{TL} = m_1 g - m_1 a$$

m_2 :

$$\sum F_{y:}\ F_{TR} - m_2 g = m_2 a \rightarrow F_{TR} = m_2 g + m_2 a$$

$$\alpha = \frac{a}{R} = \frac{2\left(F_{TL} - F_{TR}\right)}{MR} = \frac{2\left(m_1 g - m_1 a - m_2 g - m_2 a\right)}{MR}$$

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2 + M/2}\right)g = \left(\frac{75kg - 65kg}{75kg + 65kg + 6kg/2}\right)9.8\tfrac{m}{s^2} = 0.69\tfrac{m}{s^2}$$

3. When released from rest, mass m_1 falls through a distance $h = 0.2m$. How fast is the *pulley spinning*?

$$v_f^2 = v_i^2 + 2a\Delta y = 2a\Delta y$$
$$v_f = \sqrt{2a\Delta y} = \sqrt{2 \times 0.69 \tfrac{m}{s^2} \times 0.2m} = 0.53 \tfrac{m}{s}$$

$$v = R\omega \rightarrow \omega = \frac{v}{R} = \frac{0.53 \tfrac{m}{s}}{0.45m} = 1.17 \tfrac{rad}{s}$$

4. What is the magnitude of the angular momentum of the pulley about its axis of rotation?

$$L = I\omega = \left(\tfrac{1}{2}MR^2\right)\omega = 0.5 \times 6kg \times (0.45m)^2 \times 1.17 \tfrac{rad}{s} = 0.71 \tfrac{kgm^2}{s}$$

5. Using energy ideas, when the block is released from rest, determine the speed of the mass m_2 after it rises through a height of $h = 0.2m$. Assume that there is no friction in the axle that the pulley spins around.

$$\Delta E_{system} = 0 = \Delta K_{T1} + \Delta K_{T2} + \Delta K_R + \Delta U_{g1} + \Delta U_{g2}$$
$$0 = \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2 + \tfrac{1}{2}I\omega^2 - m_1gh + m_2gh$$
$$0 = \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 - m_1gh + m_2gh$$
$$v = \sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2 + M/2}} = \sqrt{\frac{2 \times 9.8 \tfrac{m}{s^2} \times 0.2m(75kg - 65kg)}{75kg + 65kg + 6kg/2}} = 0.52 \tfrac{m}{s}$$

Useful formulas:

Motion in the r = x, y or z-directions

$$r_f = r_0 + v_{0r}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{0r} + a_r t$$

$$v_{fr}^{\ 2} = v_{0r}^{\ 2} + 2a_r \Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2\pi r$	$A = \frac{1}{2}bh$	$A = 4\pi r^2$
$A = \pi r^2$		$V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$\textit{magnitude of a vector} = \sqrt{v_x^{\ 2} + v_y^{\ 2}}$$

$$\textit{direction of a vector} \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\,{}^{m}\!/_{s^2}$ $\quad G = 6.67\times 10^{-11}\,{}^{Nm^2}\!/_{kg^2}$

$N_A = 6.02\times 10^{23}\,{}^{atoms}\!/_{mole}$ $\quad k_B = 1.38\times 10^{-23}\,{}^{J}\!/_{K}$

$\sigma = 5.67\times 10^{-8}\,{}^{W}\!/_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}\!/_{s}$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$$

Heat

$$T_C = \tfrac{5}{9}[T_F - 32]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}(1 + \alpha\Delta T)$$

$$A_{new} = A_{old}(1 + 2\alpha\Delta T)$$

$$V_{new} = V_{old}(1 + \beta\Delta T):\ \beta = 3\alpha$$

$$PV = Nk_BT$$

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta:\ v = r\omega:\ a_t = r\alpha$$

$$a_r = r\omega^2$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho gd$$

$$F_B = \rho gV$$

$$A_1v_1 = A_2v_2$$

$$\rho_1A_1v_1 = \rho_2A_2v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho gh_2$$

Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2 \rho v A^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\,\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0};\quad I_o = 1\times 10^{-12}\,\tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L};\ f_n = nf_1 = n\frac{v}{4L}$$