Name____

Physics 110 Quiz #5, October 25, 2019

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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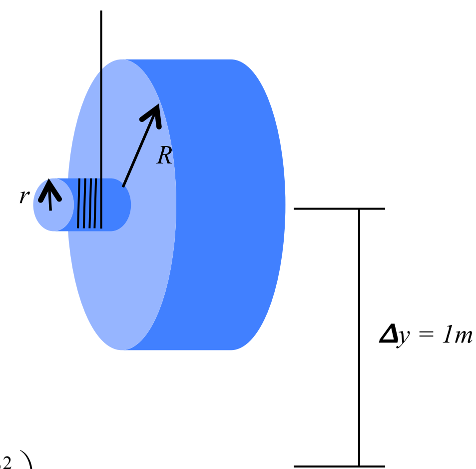
A $M = 0.5kg$ yo-yo has an outer radius (R) that is six times larger than the radius of the axle (r) through its center around which a light string is wrapped. That is $R = 6r$. The yo-yo is released from rest and the center of the axle through the center of the yo-yo falls through a distance of $1.0m$.



a. What is the translational acceleration of the system?
Hint: $I = \frac{1}{2}MR^2$ for the yo-yo.

$$\sum \tau: \quad rF_T = I\alpha \rightarrow F_T = \frac{I\alpha}{r} = \frac{\frac{1}{2}MR^2\left(\frac{a}{r}\right)}{r} = \frac{1}{2}M\left(\frac{R^2}{r^2}\right)a$$

$$\sum F_y: \quad F_T - Mg = -Ma \rightarrow F_T = Mg - Ma$$

$$F_T = \frac{1}{2}M\left(\frac{R^2}{r^2}\right)a = Mg - Ma \rightarrow a = \left(\frac{1}{\frac{R^2}{2r^2}+1}\right)g$$

$$\therefore a = \left(\frac{1}{\frac{R^2}{2r^2}+1}\right)g = \left(\frac{1}{\frac{36r^2}{2r^2}+1}\right)\times 9.8\tfrac{m}{s^2} = 0.52\tfrac{m}{s^2}$$

b. What is the magnitude of the tension force in the string?

$$F_T = Mg - Ma = 0.5kg \times \left(9.8\tfrac{m}{s^2} - 0.52\tfrac{m}{s^2}\right) = 4.64N$$

c. What is the angular speed of the yo-yo, in revolutions per second, after the yo-yo has fallen through a distance of $1.0m$ from rest? Assume that $r = 2cm$ and do this problem using one of your equations of motion.

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$\alpha = \frac{a}{r} = \frac{0.52\,\frac{m}{s^2}}{0.02m} = 26\,\frac{rad}{s^2}$$

$$\Delta s = r\Delta\theta \rightarrow \Delta\theta = \frac{\Delta s}{r} = \frac{1m}{0.02m} = 50rad$$

$$\omega_f = \sqrt{\omega_i^2 + 2\alpha\Delta\theta} = \sqrt{2\alpha\Delta\theta} = \sqrt{2 \times 26\,\frac{rad}{s^2} \times 50rad} = 51\,\frac{rad}{s}$$

$$\omega_f = 51\,\frac{rad}{s} \times \frac{1rev}{2\pi rad} = 8.1\,\frac{rev}{s}$$

d. Taking the system to be the yo-yo and the rest of the world, determine the rotational speed of the yo-yo after the yo-yo has fallen a distance of $1.0m$ by using energy ideas.

$$\Delta E = 0 = \Delta K_T + \Delta K_R + \Delta_g + \Delta U_s = \left(\tfrac{1}{2}Mv_f^2 - \tfrac{1}{2}Mv_i^2\right) + \left(\tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2\right) + \left(Mgy_f - Mgy_i\right)$$

$$0 = \tfrac{1}{2}Mv_f^2 + \tfrac{1}{2}I\omega_f^2 - Mgy_i$$

$$Mgy_i = \tfrac{1}{2}M\left(r\omega_f\right)^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega_f^2 = \tfrac{1}{2}M\left(r^2 + R^2\right)\omega_f^2$$

$$\omega_f = \sqrt{\frac{2gy_i}{r^2 + R^2}} = \sqrt{\frac{2 \times 9.8\,\frac{m}{s^2} \times 1m}{\left(0.02m\right)^2 + \tfrac{1}{2}\left(6 \times 0.02m\right)^2}} = 50.8\,\frac{rad}{s} \times \frac{1rev}{2\pi rad} = 8.1\,\frac{rad}{s}$$

Physics 110 Formulas

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles   *Triangles*   *Spheres*

$C = 2\pi r$ $A = \frac{1}{2}bh$ $A = 4\pi r^2$

$A = \pi r^2$ $V = \frac{4}{3}\pi r^3$

$Quadratic\ equation: ax^2 + bx + c = 0,$

$whose\ solutions\ are\ given\ by: \ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x{}^2 + v^2{}_y}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8 \, m/s^2 \qquad G = 6.67\times10^{-11} \, Nm^2/kg^2$

$N_A = 6.02\times10^{23} \, atoms/mole \qquad k_B = 1.38\times10^{-23} \, J/K$

$\sigma = 5.67\times10^{-8} \, W/m^2K^4 \qquad v_{sound} = 343 \, m/s$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\cdot dt$

$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$

$\vec{F}_s = -k\vec{x}$

$|\vec{F}_{fr}| = \mu|\vec{F}_N|$

Work/Energy

$K_T = \frac{1}{2}mv^2$

$K_R = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = F\Delta x \mathrm{Cos}\theta = \Delta K_T$

$W_R = \tau\theta = \Delta K_R$

$W_{net} = W_R + W_T = \Delta K_R + \Delta K_T$

$\Delta K_R + \Delta K_T + \Delta U_g + \Delta U_S = \Delta E_{system} = 0$

$\Delta K_R + \Delta K_T + \Delta U_g + \Delta U_S = \Delta E_{system} = W_{fr} = -F_{fr}\Delta x$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta: \ v = r\omega: \ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = f\lambda = (331 + 0.6T)\frac{m}{s}$

$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho g d$

$F_B = \rho g V$

$A_1 v_1 = A_2 v_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho g h_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho v A^2$