Name____

Physics 110 Quiz #5, October 23, 2020

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

____

A ball of mass $m_{ball} = 200g$ is fired horizontally into a block of mass $m_{block} = 1.7kg$ initially at rest on a horizontal surface. After the collision the ball and block slide across the horizontal surface where there is friction with coefficient of friction $\mu = 0.2$. The ball and block then collide with a spring (with stiffness $k = 10\frac{N}{m}$) initially at its equilibrium length.

1. If the spring compresses by an amount $x_f = 0.25m$ from equilibrium and comes to rest, what was the speed of the ball and block system just before it collided with the spring?

$$\Delta E = -F_{fr}\Delta x = \Delta K + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}Mv_f^2 - \tfrac{1}{2}Mv_i^2\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right)$$

$$-\mu F_N\Delta x = -\mu Mgx_f - \tfrac{1}{2}kx_f^2 = -\tfrac{1}{2}Mv_i^2 \rightarrow v_i = \sqrt{2\mu g x_f + \frac{k}{M}x_f^2}$$

$$v_i = \sqrt{\left(2 \times 0.2 \times 9.8\tfrac{m}{s^2} \times 0.25m\right) + \frac{10\frac{N}{m}}{(0.2kg + 1.7kg)}(0.25m)^2} = 1.14\tfrac{m}{s}$$

2. What was the speed of the block and ball immediately after the collision if the block and ball slide across the surface a distance of $\Delta x = 0.5m$ before striking the spring?

$$\Delta E = -F_{fr}\Delta x = -\mu F_N = -\mu Mgx_f = \Delta K + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}Mv_f^2 - \tfrac{1}{2}Mv_i^2\right)$$

$$v_i = \sqrt{2\mu g x_f + v_f^2} = \sqrt{\left(2 \times 0.2 \times 9.8\tfrac{m}{s^2} \times 0.5m\right) + \left(1.14\tfrac{m}{s}\right)^2} = 1.8\tfrac{m}{s}$$

3. What was the speed of the ball before the collision with the block?

$$\Delta p = 0 \rightarrow p_f - p_i = 0 \rightarrow p_i = p_f \rightarrow mv_i = MV \rightarrow v_i = \frac{M}{m_{ball}}V$$

$$v_i = \frac{(0.2kg + 1.7kg)}{0.2kg} \times 1.8\tfrac{m}{s} = 17.2\tfrac{m}{s}$$

4. Suppose that a merry-go-round (with radius $R = 1.5m$) starts from rest rotationally and is accelerated by a constant force applied parallel to the outer edge of the merry-go-round. The force is applied for a time of $t = 6s$ in which time the merry-go-round achieves a constant rotational velocity of $30rpm$. Through what angle $\Delta\theta$ was the merry-go-round rotated by this force?

$$\omega_f = 30\frac{rev}{min} \times \frac{1min}{60s} \times \frac{2\pi rad}{1rev} = \pi\frac{rad}{s}$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \rightarrow \Delta\theta = \frac{\omega_f^2}{2\alpha} = \frac{\left(\pi\frac{rad}{s}\right)^2}{0.52\frac{rad}{s^2}} = 9.5rad$$

Where, the angular acceleration is: $\omega_f = \omega_i + \propto t \rightarrow \alpha = \frac{\omega_f}{t} = \frac{\pi\frac{rad}{s}}{6s} = 0.52\frac{rad}{s^2}$.

You could also have done this from the angular trajectory:
$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(0.52\frac{rad}{s^2}\right)(6s)^2 = 9.4rad$

5. Suppose that two children, Alex and Samantha, sit on the merry-go-round. Alex (with mass m_A) sits on the outer edge of a merry-go-round and Samantha (with mass $m_S = \frac{1}{2}m_A$) sits midway between the center of the merry-go-round and the outer edge. The merry-go-round make one complete revolution every $t = 2s$. Compared to the tangential velocity of Samantha, which of the following gives the correct relation for the tangential velocity of Alex?
   a. The tangential velocity of Alex is one fourth of that of Samantha
   b. The tangential velocity of Alex is twice that of Samantha.
   c. The tangential velocity of Alex is equal to that of Samantha.
   d. The tangential velocity of Alex is half than that of Samantha.
   d. The tangential velocity of Alex is one quarter that of Samantha.

Physics 110 Formulas

Motion

$\Delta x = x_f - x_i \qquad v_{avg} = \frac{\Delta x}{\Delta t} \qquad a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \text{D}x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \text{D}y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2pr}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles	*Triangles*	*Spheres*
$C = 2pr$	$A = \frac{1}{2}bh$	$A = 4pr^2$
$A = pr^2$		$V = \frac{4}{3}pr^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x^{\ 2} + v^2_{\ y}}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8 \, {}^{m}\!/\!{}_{s^2} \qquad G = 6.67 ´ \ 10^{-11} \, {}^{Nm^2}\!/\!{}_{kg^2}$

$N_A = 6.02 ´ \ 10^{23} \, {}^{atoms}\!/\!{}_{mole} \qquad k_B = 1.38 ´ \ 10^{-23} \, {}^{J}\!/\!{}_{K}$

$s = 5.67 ´ \ 10^{-8} \, {}^{W}\!/\!{}_{m^2K^4} \qquad v_{sound} = 343 \, {}^{m}\!/\!{}_{s}$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\text{D}t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = mF_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}Iw^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCosq = \text{D}E_T$

$W_R = tq = \text{D}E_R$

$W_{net} = W_R + W_T = \text{D}E_R + \text{D}E_T$

$\text{D}E_R + \text{D}E_T + \text{D}U_g + \text{D}U_S = 0$

$\text{D}E_R + \text{D}E_T + \text{D}U_g + \text{D}U_S = -\text{D}E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + a\text{D}T)$

$A_{new} = A_{old}(1 + 2a\text{D}T)$

$V_{new} = V_{old}(1 + b\text{D}T) : \ b = 3a$

$PV = Nk_BT$

$\frac{3}{2}k_BT = \frac{1}{2}mv^2$

$\text{D}Q = mc\text{D}T$

$P_C = \frac{\text{D}Q}{\text{D}t} = \frac{kA}{L}\text{D}T$

$P_R = \frac{\text{D}Q}{\text{D}T} = esA\text{D}T^4$

$\text{D}U = \text{D}Q - \text{D}W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta : \ v = r\omega : \ a_t = r\alpha$

$a_r = r\omega^2$

Sound

$v = fl = (331 + 0.6T)\frac{m}{s}$

$b = 10\log\frac{I}{I_0}; \quad I_o = 1 ´ \ 10^{-12} \frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}; \ f_n = nf_1 = n\frac{v}{4L}$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1v_1 = A_2v_2$

$\rho_1A_1v_1 = \rho_2A_2v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$w = 2pf = \frac{2p}{T}$

$T_S = 2p\sqrt{\frac{m}{k}}$

$T_P = 2p\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2pt}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2pt}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2pt}{T}\right)$

$v = fl = \sqrt{\frac{F_T}{m}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2p^2f^2rvA^2$