Name_______________________________________

Physics 110 Quiz #5, May 8, 2020

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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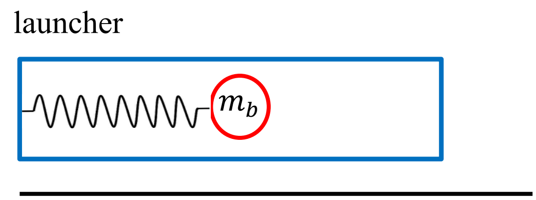
A small steel ball of mass $m_b = 50g$ is shot from a horizontal launcher in to a block of wood of mass $M = 5kg$, initially at rest.

a. To launch the ball from the launcher, the ball is compressed against a horizontal spring of stiffness k by an amount $|x| = 10cm$ (measured from the equilibrium position of the spring.) If the ball is launched from rest and loses contact with the spring when the spring returns to its equilibrium length with a speed of $v_b = 300\frac{m}{s}$, what is the stiffness k of the spring?



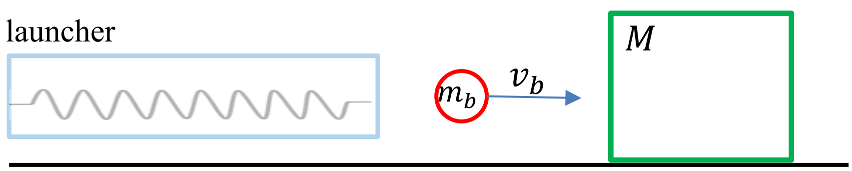
Since the ball is fired horizontally $\Delta U_g = 0$. The initial stored elastic potential energy is transformed to kinetic energy.

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s = 0$$

$$\Delta E = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right) = 0$$

$$k = \frac{mv_f^2}{x_i^2} = \frac{0.05kg\left(300\frac{m}{s}\right)^2}{(0.1m)^2} = 4.5 \times 10^5 \frac{N}{m}$$

b. After launch, the ball strikes the stationary block and imbeds itself in the block. What is the speed of the ball & block after the bullet comes to rest in the block?



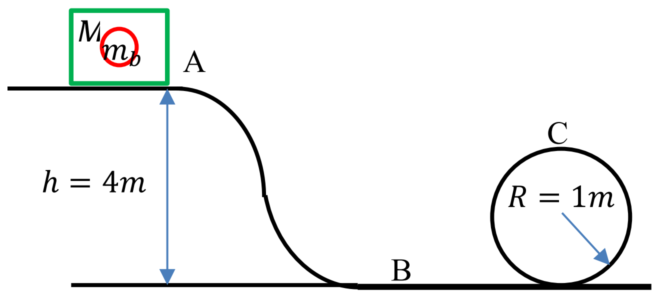
We have a collision and we apply conservation of momentum.

$$\Delta p_x = p_{fx} - p_{ix} = 0$$

$$p_{ix} = p_{fx} \rightarrow m_b v_b = (m_b + M)V$$

$$V = \frac{m_b}{m_b + M} v_b = \frac{0.05kg}{0.05kg + 5kg} \times 300\frac{m}{s} = 2.97\frac{m}{s}$$

c. Suppose that the ball and block are actually located on the top of a frictionless track as shown on the right. What are the speeds of the ball and block at the bottom of the hill (labeled as point B) and the top of the loop-the-loop (labeled as point C)?



$\Delta E_{AB} = \Delta K_{AB} + \Delta U_{gAB} + \Delta U_s = 0$

$\Delta E_{AB} = \left(\frac{1}{2}m_{total}v_B^2 - \frac{1}{2}m_{total}v_A^2\right) +$
$(m_{total}gy_B - m_{total}gy_A) = 0$

$$v_B = \sqrt{v_A^2 + 2gy_a} = \sqrt{\left(2.97\tfrac{m}{s}\right)^2 + 2 \times 9.8\tfrac{m}{s^2} \times 4m} = 9.3\tfrac{m}{s}$$

$\Delta E_{BC} = \Delta K_{BC} + \Delta U_{gBC} + \Delta U_s = 0$

$\Delta E_{BC} = \left(\frac{1}{2}m_{total}v_C^2 - \frac{1}{2}m_{total}v_B^2\right) + (m_{total}gy_C - m_{total}gy_B) = 0$

$$v_C = \sqrt{v_B^2 - 2gy_C} = \sqrt{\left(9.3\tfrac{m}{s}\right)^2 - 2 \times 9.8\tfrac{m}{s^2} \times 2m} = 6.9\tfrac{m}{s}$$

d. What is the difference between the magnitudes of the reaction forces of the track at point B and point C?

At point B:
$F_{NB} - F_W = 0 \rightarrow F_{NB} = m_{total}g = 5.05kg \times 9.8\frac{m}{s^2} = 49.5N$

At point C:

$$-F_{NC} - F_W = -F_N - m_{total}g = -m_{total}\frac{v_C^2}{R}$$

$$F_{NC} = m_{total}\frac{v_C^2}{R} - m_{total}g = (0.05kg + 5kg)\left[\frac{\left(6.9\frac{m}{s}\right)^2}{1m} - 9.8\frac{m}{s^2}\right] = 190.1N$$

The difference is $\Delta F = F_{NB} - F_{NC} = -140.6N$

e. Suppose that the entire track was not frictionless and further that you measure the speeds at the top of the hill (labeled point A) and at the top of the loop-the-loop (labeled point C). Can you determine the work done by friction between points A and C? You will earn zero credit for simply saying yes or no. You need to explain in complete sentences why you think you can or why you think you cannot determine the work done by friction between points A and C. There is no actual calculation required for this question.

Yes, you can determine the work done by friction since you know the initial and final speeds of the system and the heights through which the masses fell. Thus $\Delta E = W_{fr} = \Delta K + \Delta U_g$. The one thing you could not determine is the frictional force since we don't know the actual path taken by the masses.

Physics 110 Formulas

Motion

$\Delta x = x_f - x_i$ $\quad v_{avg} = \frac{\Delta x}{\Delta t}$ $\quad a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}; \quad a_r = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles *Triangles* *Spheres*

$C = 2\pi r$ $\quad A = \frac{1}{2}bh$ $\quad A = 4\pi r^2$

$A = \pi r^2$ $\quad V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x{}^2 + v^2{}_y}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8\,{}^{m}/_{s^2}$ $\quad G = 6.67\times10^{-11}\,{}^{Nm^2}/_{kg^2}$

$N_A = 6.02\times10^{23}\,{}^{atoms}/_{mole}$ $\quad k_B = 1.38\times10^{-23}\,{}^{J}/_{K}$

$\sigma = 5.67\times10^{-8}\,{}^{W}/_{m^2K^4}$ $\quad v_{sound} = 343\,{}^{m}/_{s}$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T): \ \beta = 3\alpha$

$PV = Nk_BT$

$\frac{3}{2}k_BT = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta: \ v = r\omega: \ a_t = r\alpha$

$a_r = r\omega^2$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1v_1 = A_2v_2$

$\rho_1A_1v_1 = \rho_2A_2v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin\left(\frac{2\pi t}{T}\right)$

$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\frac{2\pi t}{T}\right)$

$a(t) = -A\frac{k}{m}\sin\left(\frac{2\pi t}{T}\right)$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho v A^2$

Sound

$v = f\lambda = (331 + 0.6T)\,\frac{m}{s}$

$\beta = 10\log\frac{I}{I_0}; \quad I_o = 1\times10^{-12}\,\frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}; \quad f_n = nf_1 = n\frac{v}{4L}$