Name $\qquad$
Physics 110 Quiz \#5, May 7, 2021
Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

Consider two blocks of mass $m=0.5 \mathrm{~kg}$ is connected by a very light cord that passes over a frictionless and massless pulley. The block on the left is connected to a spring of stiffness $k=$ $10 \frac{\mathrm{~N}}{\mathrm{~m}}$ initially at its equilibrium length, as shown below and the system of masses is released from rest.


1. Using energy ideas, what is the maximum extension of the spring from equilibrium? Assume that the ramp surfaces are frictionless.
$\Delta E=\Delta K_{1}+\Delta U_{g 1}+\Delta K_{2}+\Delta U_{g 2}+\Delta U_{s} \rightarrow 0=\Delta U_{g 1}+\Delta+\Delta U_{s}$
$0=\left(m g y_{1 f}-m g y_{1 i}\right)+\left(m g y_{2 f}-m g y_{i}\right)+\left(\frac{1}{2} k x_{f}^{2}-\frac{1}{2} k x_{i}^{2}\right)$
$0=(m g d \sin \theta-0)+(0-m g d \sin \phi)+\left(\frac{1}{2} k d^{2}-0\right) \rightarrow d=\frac{2 m g}{k}(\sin \phi-\sin \theta)$
$d=\frac{2 \times 0.5 \mathrm{~kg} \times 9.8 \frac{\mathrm{~m}}{s^{2}}}{10 \frac{N}{m}}(\sin 75-\sin 15)=0.69 \mathrm{~m}$
2. Suppose that the spring has been stretched by an amount $x_{f}=0.25 \mathrm{~m}$ from equilibrium.

What are the speeds of the masses at this point? Use Energy.
$\Delta E=\Delta K_{1}+\Delta U_{g 1}+\Delta K_{2}+\Delta U_{g 2}+\Delta U_{s}$
$0=\left(\frac{1}{2} m v_{f}^{2}-0\right)+(m g d \sin \theta-0)+\left(\frac{1}{2} m v_{f}^{2}-0\right)+(0-m g d \sin \phi)+\left(\frac{1}{2} k d^{2}-0\right)$
$v_{f}=\sqrt{g d(\sin \phi-\sin \theta)+\frac{k d^{2}}{4 m}}$
$v_{f}=\sqrt{9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \times 0.25 m(\sin 75-\sin 15)+\frac{10 \frac{\mathrm{~N}}{\mathrm{~m}}(0.25 \mathrm{~m})^{2}}{4 \times 0.5 \mathrm{~kg}}}=1.2 \frac{\mathrm{~m}}{\mathrm{~s}}$
3. Suppose friction existed between each block and the surfaces of the ramps with coefficient of friction $\mu=0.2$ for both blocks/ramps. If the blocks are released from rest what is the maximum stretch of the spring now?
$\Delta E=\Delta K_{1}+\Delta U_{g 1}+\Delta K_{2}+\Delta U_{g 2}+\Delta U_{s}$
$W_{f r 1}+W_{f r 2}=-\mu F_{N 1} d-\mu F_{N 2} d=m g d \sin \theta-m g d \sin \phi+\frac{1}{2} k d^{2}$
$\rightarrow-\mu m g d(\cos \theta+\cos \phi)=m g d \sin \theta-m g d \sin \phi+\frac{1}{2} k d^{2}$
$d=\frac{2 m g}{k}[(\sin \phi-\sin \theta)-\mu(\cos \theta+\cos \phi)]$
$d=\frac{2 \times 0.5 \mathrm{~kg} \times 9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}{10 \frac{\mathrm{~N}}{\mathrm{~m}}}[(\sin 75-\sin 15)-0.2(\cos 75+\cos 15)]=0.45 \mathrm{~m}$
4. Suppose that we let ramps again be frictionless further that the spring has just reached its maximum extension. At the point the spring reaches its maximum extension, the rope connecting the two masses is cut. What will be the period of oscillation of the mass on the left ramp?
$T=2 \pi \sqrt{\frac{m}{k}}=2 \pi \sqrt{\frac{0.5 k g}{10 \frac{N}{m}}}=1.4 \mathrm{~s}$
5. Using energy ideas, what will be the speed of the block on the right ramp, if the block slides $\Delta x=0.75 \mathrm{~m}$ along the ramp?
$\Delta E=\Delta K+\Delta U_{g}+\Delta U_{s} \rightarrow 0=\left(\frac{1}{2} m v_{f}^{2}-0\right)+(0-m g d \sin \phi)$
$v_{f}=\sqrt{2 g d \sin \phi}=\sqrt{2 \times 9.8 \frac{m}{s^{2}} \times 0.75 m \times \sin 75}=3.8 \frac{\mathrm{~m}}{\mathrm{~s}}$

Vectors
$v=\sqrt{v_{x}^{2}+v_{y}^{2}}$
$\phi=\tan ^{-1}\left(\frac{v_{y}}{v_{x}}\right)$

## Motion Definitions

Displacement: $\Delta x=x_{f}-x_{i}$
Average velocity: $v_{\text {avg }}=\frac{\Delta x}{\Delta t}$
Average acceleration: $a_{a v g}=\frac{\Delta v}{\Delta t}$

## Equations of Motion

displacement: $\left\{\begin{array}{l}x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2} \\ y_{f}=y_{i}+v_{i y} t+\frac{1}{2} a_{y} t^{2}\end{array}\right.$
velocity: $\left\{\begin{array}{l}v_{f x}=v_{i x}+a_{x} t \\ v_{f y}=v_{i y}+a_{y} t\end{array}\right.$
time-independent: $\left\{\begin{array}{l}v_{f x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x \\ v_{f y}^{2}=v_{i y}^{2}+2 a_{y} \Delta y\end{array}\right.$

Rotational Motion Definitions
Angular displacement: $\Delta s=R \Delta \theta$
Angular velocity: $\omega=\frac{\Delta \theta}{\Delta t} \rightarrow v=R \omega$
Angular acceleration: $\alpha=\frac{\Delta \omega}{\Delta t} \rightarrow\left\{\begin{array}{c}a_{t}=r \alpha \\ a_{c}=r \omega^{2}\end{array}\right.$
Rotational Equations of Motion

$$
\begin{aligned}
& \theta_{f}=\theta_{i}+\omega_{i} t+\frac{1}{2} \alpha t^{2} \\
& \omega_{f}=\omega_{i}+\alpha t \\
& \omega_{f}^{2}=\omega_{i}^{2}+2 \alpha \Delta \theta
\end{aligned}
$$

Momentum \& Force
$\vec{p}=m \vec{v} \rightarrow p_{x}=m v_{x} ; p_{y}=m v_{y}$
$\Delta \vec{p}=\vec{F} \Delta t \rightarrow \vec{p}_{f}=\vec{p}_{i}+\vec{F} \Delta t$
$\vec{F}=\frac{d \vec{p}}{d t}=m \vec{a} \rightarrow F_{x}=m a_{x} ; F_{y}=m a_{y}$
$F_{f r}=\mu F_{N}$
$F_{w}=m g$
$F_{S}=-k x$
$F_{G}=G \frac{M_{1} M_{2}}{r^{2}}$
$F_{c}=m a_{c}=m \frac{v^{2}}{R}$
Work \& Energy
$\left\{\begin{array}{c}W_{T}=\int \vec{F} \cdot d \vec{r}=F d r \cos \theta=\Delta K_{T} \\ W_{R}=\int \vec{\tau} \cdot d \vec{\theta}=\tau d \theta=\Delta K_{R}\end{array}\right.$
$W_{n e t}=W_{T}+W_{R}=\Delta K_{T}+\Delta K_{R}=-\Delta U$
$K_{T}=\frac{1}{2} m v^{2}$
$K_{R}=\frac{1}{2} I \omega^{2}$
$U_{g}=m g y$
$U_{s}=\frac{1}{2} k x^{2}$
$\Delta E=\Delta E_{R}+\Delta E_{T}$
$\Delta E=\Delta K_{R}+\Delta K_{T}+\Delta U_{g}+\Delta U_{s}=\left\{\begin{array}{c}0 \\ W_{f r}\end{array}\right.$
Rotational Momentum \& Force

$$
\begin{aligned}
& \vec{\tau}=\vec{r} \times \vec{F} ; \tau=r_{\perp} F=r F_{\perp}=r F \sin \theta \\
& \tau=\frac{\Delta L}{\Delta t}=I \alpha \\
& L=I \omega \\
& \Delta \vec{L}=\vec{\tau} \Delta t \rightarrow \vec{L}_{f}=\vec{L}_{i}+\vec{\tau} \Delta t
\end{aligned}
$$

Fluids
$\rho=\frac{m}{V}$
$P=\frac{F}{A}$
$P_{y}=P_{\text {air }}+\rho g y$
$F_{B}=\rho g V$
$\rho_{1} A_{1} v_{1}=\rho_{2} A_{2} v_{2}$; compressible $A_{1} v_{1}=A_{2} v_{2}$; incompressible
$P_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1}=P_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho g y_{2}$

Simple Harmonic Motion
$\omega=2 \pi f=\frac{2 \pi}{T}$
$T_{s}=2 \pi \sqrt{\frac{m}{k}} ; \quad \omega=\sqrt{\frac{k}{m}}$
$T_{p}=2 \pi \sqrt{\frac{l}{g}} ; \quad \omega=\sqrt{\frac{g}{l}}$

Geometry/Algebra
Circles: $\quad A=\pi r^{2} \quad C=2 \pi r=\pi D$
Spheres: $\quad A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$
Triangles: $\quad A=\frac{1}{2} b h$
Quadratics: $\quad a x^{2}+b x+c=0 \rightarrow x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \quad v= \pm \omega x_{\max } \sqrt{1-\left(\frac{x}{x_{\max }}\right)^{2}}$

Sound
$v_{s}=f \lambda=(331+0.6 T) \frac{m}{s}$
$\beta=10 \log \frac{I}{I_{o}}$
$f_{n}=n f_{1}=n \frac{v}{2 L} ; n=1,2,3, \ldots$ open pipes
$f_{n}=n f_{1}=n \frac{v}{4 L} ; n=1,3,5, \ldots$ closed pipes
Waves
$v=f \lambda=\sqrt{\frac{F_{T}}{\mu}}$
$f_{n}=n f_{1}=n \frac{v}{2 L} ; n=1,2,3, \ldots$
$I=2 \pi^{2} f^{2} \rho v A^{2}$

Equations of Motion for SHM
$x(t)=\left\{\begin{array}{l}x_{\text {max }} \sin \left(\frac{2 \pi}{T} t\right) \\ x_{\text {max }} \cos \left(\frac{2 \pi}{T} t\right)\end{array}\right.$
$v(t)=\left\{\begin{array}{c}v_{\text {max }} \cos \left(\frac{2 \pi}{T} t\right) \\ -v_{\text {max }} \sin \left(\frac{2 \pi}{T} t\right)\end{array}\right.$
$a(t)=\left\{\begin{array}{l}-a_{\text {max }} \sin \left(\frac{2 \pi}{T} t\right) \\ -a_{\text {max }} \cos \left(\frac{2 \pi}{T} t\right)\end{array}\right.$
$v= \pm v_{\max } \sqrt{1-\left(\frac{x}{x_{\max }}\right)^{2}}$

Periodic Table of the Elements


