Name $\qquad$
Physics 110 Quiz \#5, May 6, 2022
Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

1. A block of mass $m=2 \mathrm{~kg}$ is placed against an unstretched spring of stiffness $k_{g}=52000 \frac{N}{m}$. The mass and spring are pushed along the surface of the frictionless ramp a distance $d_{\text {ramp }}=0.25 \mathrm{~m}$. The mass is released from rest and the spring uncompresses. When the spring returns to its original length the mass is launched from the end of the spring at an angle of, $\theta=30^{\circ}$ measured
 with respect to the horizontal as shown below. Using energy ideas, what is the launch speed of the mass?

$$
\begin{aligned}
& \Delta E=0=\Delta K+\Delta U_{g}+\Delta U_{s} \\
& 0=\left(\frac{1}{2} m v_{f}^{2}-0\right)+\left(m g y_{f}-0\right)+\left(0-\frac{1}{2} k x_{i}^{2}\right) \\
& v_{f}=\sqrt{\frac{k}{m} x_{i}^{2}-m g y_{f}}=\sqrt{\frac{k}{m} x_{i}^{2}-2 g d_{r a m p} \sin \theta} \\
& v_{f}=\sqrt{\frac{52000 \frac{N}{m}}{2 k g}(0.25 m)^{2}-2 \times 9.8 \frac{\mathrm{~m}}{s^{2}} \times 0.25 m \sin 30}=40.3 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

2. A horizontal platform is located at a height $h$ above the ground such that the mass $m$ lands on the horizontal platform when the mass $m$ reaches the highest point in its motion. Using energy ideas, what is the height $h$ of the platform above the ground? Assume that the mass $m$ was launched from ground level.


$$
\begin{aligned}
& \Delta E=0=\Delta K+\Delta U_{g}+\Delta U_{s} \\
& 0=\left(\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2}\right)+\left(m g y_{f}-0\right) \rightarrow y_{f}=h=\frac{v_{i}^{2}-v_{f x}^{2}}{2 g} \\
& h=\frac{\left(40.3 \frac{m}{s}\right)^{2}-\left(40.3 \frac{m}{s} \cos 30\right)^{2}}{2 \times 9.8 \frac{m}{s^{2}}}=20.7 \mathrm{~m}
\end{aligned}
$$

3. After the mass lands on the horizontal platform, it slides across the horizontal frictionless surface until it collides with a spring of stiffness $k_{p}=1600 \frac{\mathrm{~N}}{\mathrm{~m}}$. How far will the spring have compressed when the mass comes to rest?

$$
\begin{aligned}
& \Delta E=0=\Delta K+\Delta U_{g}+\Delta U_{s} \\
& 0=\left(\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2}\right)+\left(\frac{1}{2} k x_{f}^{2}-\frac{1}{2} k x_{i}^{2}\right)=-\frac{1}{2} m v_{i}^{2}+\frac{1}{2} k x_{f}^{2} \rightarrow x_{f}=\sqrt{\frac{m}{k}} v_{i x} \\
& x_{f}=\sqrt{\frac{2 k g}{1600 \frac{N}{m}}} \times 40.3 \frac{m}{s} \cos 30=1.23 m
\end{aligned}
$$

4. Suppose that the horizontal platform was not frictionless, but that friction exists with coefficient of friction $\mu=0.8$. If the mass slides a distance $d=2 m$ before striking the spring, with what speed with the mass strike the spring?

$$
\begin{aligned}
& \Delta E=W_{f r}=-\mu m g d=\Delta K \\
& -\mu m g d=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2} \rightarrow v_{f}=\sqrt{v_{i x}^{2}-2 \mu g d} \\
& v_{f}=\sqrt{\left(40.3 \frac{\mathrm{~m}}{\mathrm{~s}} \cos 30\right)^{2}-\left(2 \times 0.8 \times 9.8 \frac{\mathrm{~m}}{s^{2}} \times 2 \mathrm{~m}\right)}=34.4 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

5. If friction exists under the spring as well, by how much will the spring compress in this case as the mass comes to rest?

$$
\begin{aligned}
& \Delta E=W_{f r}=\Delta K+\Delta U_{g}+\Delta U_{s} \\
& -\mu m g x_{f}=\left(\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2}\right)+\left(\frac{1}{2} k x_{f}^{2}-\frac{1}{2} k x_{i}^{2}\right)=-\frac{1}{2} m v_{i}^{2}+\frac{1}{2} k x_{f}^{2} \\
& \frac{1}{2} k x_{f}^{2}+\mu m g x_{f}-\frac{1}{2} m v_{i}^{2}=0 \rightarrow 800 x_{f}^{2}+15.7 x_{f}-1183.4=0 \\
& x_{f}=\left\{\begin{array}{c}
-1.23 m \\
1.20 m
\end{array} \rightarrow x_{f}=1.20 m\right.
\end{aligned}
$$

Vectors
$v=\sqrt{v_{x}^{2}+v_{y}^{2}}$
$\phi=\tan ^{-1}\left(\frac{v_{y}}{v_{x}}\right)$

## Motion Definitions

Displacement: $\Delta x=x_{f}-x_{i}$
Average velocity: $v_{\text {avg }}=\frac{\Delta x}{\Delta t}$
Average acceleration: $a_{a v g}=\frac{\Delta v}{\Delta t}$

## Equations of Motion

displacement: $\left\{\begin{array}{l}x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2} \\ y_{f}=y_{i}+v_{i y} t+\frac{1}{2} a_{y} t^{2}\end{array}\right.$
velocity: $\left\{\begin{array}{l}v_{f x}=v_{i x}+a_{x} t \\ v_{f y}=v_{i y}+a_{y} t\end{array}\right.$
time-independent: $\left\{\begin{array}{l}v_{f x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x \\ v_{f y}^{2}=v_{i y}^{2}+2 a_{y} \Delta y\end{array}\right.$
Rotational Motion Definitions
Angular displacement: $\Delta s=r \Delta \theta$
Angular velocity: $\omega=\frac{\Delta \theta}{\Delta t} \rightarrow v=r \omega$
Angular acceleration: $\alpha=\frac{\Delta \omega}{\Delta t} \rightarrow\left\{\begin{array}{c}a_{t}=r \alpha \\ a_{c}=r \omega^{2}\end{array}\right.$
Rotational Equations of Motion

$$
\begin{aligned}
& \theta_{f}=\theta_{i}+\omega_{i} t+\frac{1}{2} \alpha t^{2} \\
& \omega_{f}=\omega_{i}+\alpha t \\
& \omega_{f}^{2}=\omega_{i}^{2}+2 \alpha \Delta \theta
\end{aligned}
$$

Momentum \& Force
$\vec{p}=m \vec{v} \rightarrow p_{x}=m v_{x} ; p_{y}=m v_{y}$
$\Delta \vec{p}=\vec{F} \Delta t \rightarrow \vec{p}_{f}=\vec{p}_{i}+\vec{F} \Delta t$
$\vec{F}=\frac{d \vec{p}}{d t}=m \vec{a} \rightarrow F_{x}=m a_{x} ; F_{y}=m a_{y}$
$F_{f r}=\mu F_{N}$
$F_{w}=m g$
$F_{S}=-k x$
$F_{G}=G \frac{M_{1} M_{2}}{r^{2}}$
$F_{c}=m a_{c}=m \frac{v^{2}}{R}$
Work \& Energy
$\left\{\begin{array}{c}W_{T}=\int \vec{F} \cdot d \vec{r}=F d r \cos \theta=\Delta K_{T} \\ W_{R}=\int \vec{\tau} \cdot d \vec{\theta}=\tau d \theta=\Delta K_{R}\end{array}\right.$
$W_{n e t}=W_{T}+W_{R}=\Delta K_{T}+\Delta K_{R}=-\Delta U$
$K_{T}=\frac{1}{2} m v^{2}$
$K_{R}=\frac{1}{2} I \omega^{2}$
$U_{g}=m g y$
$U_{s}=\frac{1}{2} k x^{2}$
$\Delta E=\Delta E_{R}+\Delta E_{T}$
$\Delta E=\Delta K_{R}+\Delta K_{T}+\Delta U_{g}+\Delta U_{s}=\left\{\begin{array}{c}0 \\ W_{f r}\end{array}\right.$
Rotational Momentum \& Force

$$
\begin{aligned}
& \vec{\tau}=\vec{r} \times \vec{F} ; \tau=r_{\perp} F=r F_{\perp}=r F \sin \theta \\
& \tau=\frac{\Delta L}{\Delta t}=I \alpha \\
& L=I \omega \\
& \Delta \vec{L}=\vec{\tau} \Delta t \rightarrow \vec{L}_{f}=\vec{L}_{i}+\vec{\tau} \Delta t
\end{aligned}
$$

Fluids
$\rho=\frac{m}{V}$
$P=\frac{F}{A}$
$P_{y}=P_{\text {air }}+\rho g y$
$F_{B}=\rho g V$
$\rho_{1} A_{1} v_{1}=\rho_{2} A_{2} v_{2}$; compressible $A_{1} v_{1}=A_{2} v_{2}$; incompressible
$P_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1}=P_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho g y_{2}$

Simple Harmonic Motion
$\omega=2 \pi f=\frac{2 \pi}{T}$
$T_{s}=2 \pi \sqrt{\frac{m}{k}} ; \quad \omega=\sqrt{\frac{k}{m}}$
$T_{p}=2 \pi \sqrt{\frac{l}{g}} ; \quad \omega=\sqrt{\frac{g}{l}}$

Geometry/Algebra
Circles:

$$
A=\pi r^{2} \quad C=2 \pi r=\pi D
$$

Spheres: $\quad A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$
Triangles: $\quad A=\frac{1}{2} b h$
Quadratics: $\quad a x^{2}+b x+c=0 \rightarrow x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \quad v= \pm \omega x_{\max } \sqrt{1-\left(\frac{x}{x_{\max }}\right)^{2}}$

Sound
$v_{s}=f \lambda=(331+0.6 T) \frac{m}{s}$
$\beta=10 \log \frac{I}{I_{o}}$
$f_{n}=n f_{1}=n \frac{v}{2 L} ; n=1,2,3, \ldots$ open pipes
$f_{n}=n f_{1}=n \frac{v}{4 L} ; n=1,3,5, \ldots$ closed pipes
Waves
$v=f \lambda=\sqrt{\frac{F_{T}}{\mu}}$
$f_{n}=n f_{1}=n \frac{v}{2 L} ; n=1,2,3, \ldots$
$I=2 \pi^{2} f^{2} \rho v A^{2}$

Equations of Motion for SHM
$x(t)=\left\{\begin{array}{l}x_{\text {max }} \sin \left(\frac{2 \pi}{T} t\right) \\ x_{\text {max }} \cos \left(\frac{2 \pi}{T} t\right)\end{array}\right.$
$v(t)=\left\{\begin{array}{c}v_{\text {max }} \cos \left(\frac{2 \pi}{T} t\right) \\ -v_{\text {max }} \sin \left(\frac{2 \pi}{T} t\right)\end{array}\right.$
$a(t)=\left\{\begin{array}{l}-a_{\text {max }} \sin \left(\frac{2 \pi}{T} t\right) \\ -a_{\text {max }} \cos \left(\frac{2 \pi}{T} t\right)\end{array}\right.$
$v= \pm v_{\max } \sqrt{1-\left(\frac{x}{x_{\max }}\right)^{2}}$
nano $=1 \times 10^{-9}$
micro $=1 \times 10^{-6}$
milli $=1 \times 10^{-3}$
centi $=1 \times 10^{-2}$
kilo $=1 \times 10^{3}$
$m e g a=1 \times 10^{6}$


