Name____

Physics 110 Quiz #5, May 5, 2023

Please show all work, thoughts and/or reasoning to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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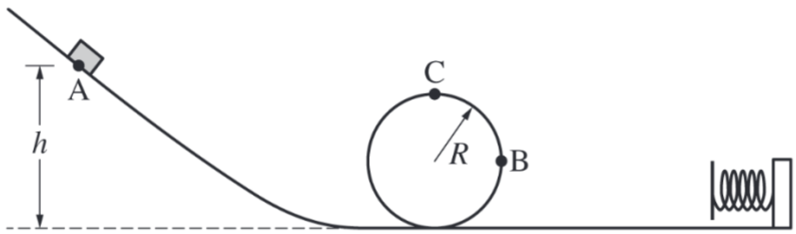
A block of mass m is released from rest at point A and travels without friction down the hill and around the loop onto a horizontal surface where it contacts a spring with stiffness $k = \frac{mg}{2R}$. Point C is the highest point on the loop and point B is the rightmost point on the loop as shown below.



a. Using energy ideas, what is the speed of the block at point B?

$$\Delta E = 0 = \Delta K + \Delta U_g + \Delta U_s = \left(\tfrac{1}{2}mv_B^2 - \tfrac{1}{2}mv_A^2\right) + (mgy_B - mgy_A)$$

$$0 = \tfrac{1}{2}mv_B^2 + (mgy_B - mgy_A) = \tfrac{1}{2}mv_B^2 + mg(R - h)$$

$$v_B = \sqrt{2g(h - R)}$$

b. What is the magnitude of the net force on the mass at point B?

In the x-direction: $F_N = -m\frac{v_B^2}{R} = -\frac{2mg}{R}(h - R)$

In the y-direction: $F_W = -mg$

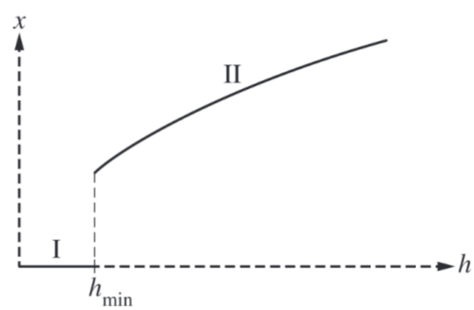
The net force: $F_{Net} = \sqrt{F_N^2 + F_W^2} = mg\sqrt{4\left(\frac{h-R}{R}\right)^2 + 1}$

c. Releasing the mass from rest at point A, what is the maximum compression of the spring? Assume that the spring is at a height $y = 0$ above the ground.

$$\Delta E = 0 = \Delta K + \Delta U_g + \Delta U_s = (mgy_B - mgy_A) + \left(\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2\right)$$

$$0 = -mgy_A + \frac{1}{2}kx_{max}^2 = x_{max} = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2mgh}{{mg}/{2R}}} = \sqrt{4hR}$$

A graph of the compression of the spring x as a function of the starting height h of the mass is shown below, where h_{min} is the minimum height needed for the block to be released at point A such that it makes it around the loop at point C.



d. Explain why section I of the plot is horizontal.

h_{min} is the minimum height needed for the mass to make it around the loop and compress the spring. At any height h smaller than h_{min}, the block never makes it around the loop and therefore cannot compress the spring. Thus, section I is horizontal and equal to zero.

e. Explain the mathematical shape of section II.

From part c, we have $x_{max} = \sqrt{4hR}$ which says that $x \propto \sqrt{h}$ and thus the mathematical shape of section II is the square root of h.