Name_______________________________________

Physics 110 Quiz #6, November 4, 2016

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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In the calendar year 2015 the Atlantic hurricane season saw 12 storms with 4 of them hurricanes. A hurricane is a low-pressure system that, when it moves over land from the open water, slows down and this dumps huge amounts of water on land and the storms are usually accompanied by strong winds. (Data taken from: http://www.nhc.noaa.gov/data/tcr/summary_atlc_2015.pdf)

1. Suppose that a hurricane is approaching your beachfront house with winds during the hurricane reaching $100mph$ $(\sim 45\frac{m}{s})$. A hurricane with wind speeds in the 86 – 110 mph range is called a category 2 hurricane (and if you're ever on Jeopardy this also corresponds to a 12 on the Beaufort wind scale). Let's assume you have closed up your house for the impending hurricane. If the speed inside is close to zero and if the density of air is $\rho_{air} = 1.3\frac{kg}{m^3}$ what is the difference in pressure between the inside and outside of your house? (Hint: Assume that your roof flat and is very thin.)

   $$P_{in} + \tfrac{1}{2}\rho v_{in}^2 + \rho g h_{in} = P_{out} + \tfrac{1}{2}\rho v_{out}^2 + \rho g h_{out}$$
   $$\Delta P = P_{in} - P_{out} = \tfrac{1}{2}\rho v_{out}^2 = \tfrac{1}{2}\times 1.3\tfrac{kg}{m^3}\times\left(45\tfrac{m}{s}\right)^2 = 1316.3\tfrac{N}{m^2}$$
   where we have assumed that the roof is negligibly thin and we have used the fact that $v_{in} \sim 0\frac{m}{s}$.

2. What total force (magnitude and direction) would act on the roof if it were *10 m* by *10 m*? *Express your answer in pounds where 1N ~ ¼ pound.* In what direction would your roof tend to move? Would the roof implode and fall to the floor of your house, or would it blow off and fly away? Explain your answer fully.

   $F = \Delta P \times A = 1316.3\frac{N}{m^2}\times(10m\times 10m) = 1.32\times 10^5\,N\times\dfrac{\frac{1}{4}lb}{1N} = 32{,}900lb$ and points vertically upwards from the inside of your house to the outside. To see this we turn to Bernoulli's equation. Since the energy is constant, the pressure is the greatest where the speed is the least. Thus the pressure in the house is greater than outside the house since the speed inside is lower than the speed outside. Thus the difference in pressure produces a lifting force and your roof blows off of your house.

3. Royal Caribbean's *Harmony of the Seas* (shown below) is the largest cruise ship on the water today. She has an empty weight of $125{,}000\,tons\,(\sim 1\times 10^{9}N)$ and sits in the water at rest on a dock. The ship itself is $366m\,(\sim 1188\,ft)$ long, $47m\,(\sim 156\,ft)$ wide, and $73m\,(\sim 236\,ft)$ high and part of the ship is under water. Calling this distance h, how far below the waterline is the bottom of the ship? Ignore the effects of air pressure.

https://en.wikipedia.org/wiki/MS_Harmony_of_the_Seas

The forces that act on the ship are given by: $\sum F_y :\; F_B - m_{ship}g = m_{ship}a_y = 0$. Thus the buoyant force is:

$$F_B = \rho_{saltwater}V_{saltwater}g = m_{ship}g \rightarrow \rho_{saltwater}V_{saltwater} = m_{ship}$$

$$\rho_{saltwater}A_{ship}h = m_{ship}$$

$$\therefore h = \frac{m_{ship}}{\rho_{saltwater}A_{ship}} = \frac{1\times 10^{8}kg}{1025\frac{kg}{m^3}\left(366m\times 47m\right)} = 5.6m \sim 18\,feet$$

4. The ship is capable of accommodating 6780 passengers and 2300 crew as well as all of their luggage and of course cargo like food/drinks. Assume that the mass of a passenger/crew is approximately $60kg$. In addition suppose that luggage/food/cargo has a mass $m_{\text{cargo}} = 3.5\times 10^{7}kg$. How much further would the ship sink into the water if all passengers, crew, and cargo were on board the ship?

The forces that act on the ship are given by:

$\sum F_y :\; F_B - m_{ship}g - Nm_{person}g - m_{cargo}g = m_{ship}a_y = 0$. Thus the buoyant force is:

$$F_B = \rho_{saltwater}V_{saltwater}g = m_{ship}g + Nm_{person}g + m_{\text{cargo}}g \rightarrow \rho_{saltwater}V_{saltwater} = \left(m_{ship} + Nm_{person} + m_{\text{cargo}}\right)$$

$$\rho_{saltwater}A_{ship}h_{new} = \left(m_{ship} + Nm_{person} + m_{\text{cargo}}\right)$$

$$\therefore h_{new} = \frac{\left(m_{ship} + Nm_{person} + m_{\text{cargo}}\right)}{\rho_{saltwater}A_{ship}} = \frac{1\times 10^{8}kg + \left(9080\times 60kg\right) + 3.5\times 10^{7}kg}{1025\frac{kg}{m^3}\left(366m\times 47m\right)} = 7.7m \sim 25\,feet$$

Therefore the since the ship was already $5.6m$ the ship will sink an additional $2.1m$.

Useful formulas:

Motion in the r = x, y or z-directions

$$r_f = r_0 + v_{0r}t + \tfrac{1}{2}a_r t^2$$

$$v_{fr} = v_{0r} + a_r t$$

$$v_{fr}^{\ 2} = v_{0r}^{\ 2} + 2a_r \Delta r$$

Uniform Circular Motion

$$a_r = \frac{v^2}{r}$$

$$F_r = ma_r = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$F_G = G\frac{m_1 m_2}{r^2}$$

Geometry /Algebra

Circles *Triangles* *Spheres*

$C = 2\pi r$ $A = \tfrac{1}{2}bh$ $A = 4\pi r^2$

$A = \pi r^2$ $V = \tfrac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0,$

whose solutions are given by : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

$$magnitude\ of\ a\,vector = \sqrt{v_x^{\ 2} + v_y^{\ 2}}$$

$$direction\ of\ a\,vector \rightarrow \phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Useful Constants

$g = 9.8\ {}^{m}\!/_{s^2}$ $G = 6.67\times10^{-11}\ {}^{Nm^2}\!/_{kg^2}$

$N_A = 6.02\times10^{23}\ {}^{atoms}\!/_{mole}$ $k_B = 1.38\times10^{-23}\ {}^{J}\!/_{K}$

$\sigma = 5.67\times10^{-8}\ {}^{W}\!/_{m^2K^4}$ $v_{sound} = 343\ {}^{m}\!/_{s}$

Linear Momentum/Forces

$$\vec{p} = m\vec{v}$$

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$$

$$\vec{F} = m\vec{a}$$

$$\vec{F}_s = -k\vec{x}$$

$$F_f = \mu F_N$$

Work/Energy

$$K_t = \tfrac{1}{2}mv^2$$

$$K_r = \tfrac{1}{2}I\omega^2$$

$$U_g = mgh$$

$$U_S = \tfrac{1}{2}kx^2$$

$$W_T = FdCos\theta = \Delta E_T$$

$$W_R = \tau\theta = \Delta E_R$$

$$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$$

$$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$$

Heat

$$T_C = \tfrac{5}{9}[T_F - 32]$$

$$T_F = \tfrac{9}{5}T_C + 32$$

$$L_{new} = L_{old}(1 + \alpha\Delta T)$$

$$A_{new} = A_{old}(1 + 2\alpha\Delta T)$$

$$V_{new} = V_{old}(1 + \beta\Delta T):\ \beta = 3\alpha$$

$$PV = Nk_BT$$

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}mv^2$$

$$\Delta Q = mc\Delta T$$

$$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$$

$$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$$

$$\Delta U = \Delta Q - \Delta W$$

Rotational Motion

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

$$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$$

$$\tau = I\alpha = rF$$

$$L = I\omega$$

$$L_f = L_i + \tau\Delta t$$

$$\Delta s = r\Delta\theta:\ v = r\omega:\ a_t = r\alpha$$

$$a_r = r\omega^2$$

Sound

$$v = f\lambda = (331 + 0.6T)\tfrac{m}{s}$$

$$\beta = 10\log\frac{I}{I_0};\quad I_o = 1\times10^{-12}\ \tfrac{W}{m^2}$$

$$f_n = nf_1 = n\frac{v}{2L};\ f_n = nf_1 = n\frac{v}{4L}$$

Fluids

$$\rho = \frac{M}{V}$$

$$P = \frac{F}{A}$$

$$P_d = P_0 + \rho gd$$

$$F_B = \rho gV$$

$$A_1v_1 = A_2v_2$$

$$\rho_1A_1v_1 = \rho_2A_2v_2$$

$$P_1 + \tfrac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \tfrac{1}{2}\rho v^2{}_2 + \rho gh_2$$

Simple Harmonic Motion/Waves

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_S = 2\pi\sqrt{\frac{m}{k}}$$

$$T_P = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$$

$$x(t) = A\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\tfrac{2\pi t}{T}\right)$$

$$a(t) = -A\frac{k}{m}\sin\left(\tfrac{2\pi t}{T}\right)$$

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$$f_n = nf_1 = n\frac{v}{2L}$$

$$I = 2\pi^2 f^2\rho vA^2$$