Name____

Physics 110 Quiz #6, May 15, 2020

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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A box of mass $m_b = 2kg$ is at rest on a ramp inclined at an angle of $\theta = 37^0$measured with respect to the horizontal. The box is connected by a light string to a pulley (located at the top of the ramp) of mass $m_p = 6kg$ and radius $r_p = 0.2m$. The box is released from rest and slides down the incline a distance $d = 0.7m$.

a. When the box is released from rest the acceleration of the box down the incline is measred to be $a = 2\frac{m}{s^2}$. What is the moment of inertia of the pulley if we assume the incline is frictionless. Hint: You don't know the shape of the pulley. It could be a hoop, a cylinder, a ball, or something else. You don't know, so don't assume a formula.

Taking down the ramp as the positive x-direction and using a tilted coordinate system we have:

$$mg\sin\theta - F_T = ma \rightarrow F_T = mg\sin\theta - ma = 2kg\left(9.8\frac{m}{s^2}\sin 37 - 2\frac{m}{s^2}\right) = 7.8N$$

Taking the torque about the axis of rotation of the pulley (assuming counterclockwise is the positive direction for the torque) we have:

$$\tau_p = r_pF_T\sin\theta = r_pF_T\sin 90 = r_pF_T = I\alpha = I\left(\frac{a}{r_p}\right) \rightarrow I = \frac{r_p^2F_T}{a} = \frac{(0.2m)^2\times 7.8N}{2\frac{m}{s^2}} = 0.156kgm^2$$

b. Using a rotational equation of motion, if the block slides distance $d = 0.7m$ down the ramp, what is the rotational speed of the pulley about its axis of rotation?

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \rightarrow \omega_f = \sqrt{2\alpha\Delta\theta} = \sqrt{2\left(\frac{a}{r}\right)\left(\frac{x}{r}\right)} = \sqrt{\frac{2ax}{r_p^2}} = \sqrt{\frac{2\times 2\frac{m}{s^2}\times 0.7m}{(0.2m)^2}} = 8.37\frac{rad}{s}$$

c. Suppose friction existed between the box and the ramp with coefficient of friction $\mu = 0.2$. What is the constant angular acceleration of the pulley about its axis of rotation if the box is released from rest and slides down the ramp a distance $d = 0.7m$?

Taking down the ramp as the positive x-direction and using a tilted coordinate system we have:
$mg \sin\theta - F_T - F_{fr} = ma \rightarrow F_T = mg \sin\theta - \mu mg \cos\theta - mr_p\alpha$

Taking the torque about the axis of rotation of the pulley (assuming counterclockwise is the positive direction for the torque) we have:
$\tau_p = r_p F_T \sin\theta = r_p F_T \sin 90 = r_p F_T = I\alpha \rightarrow F_T = \frac{I\alpha}{r_p}$

$$F_T = \frac{I\alpha}{r_p} = mg \sin\theta - \mu mg \cos\theta - mr_p\alpha \rightarrow \alpha = \frac{g(\sin\theta - \mu\cos\theta)}{r_p\left(1+\frac{I}{mr_p^2}\right)} = \frac{9.8\frac{m}{s^2}(\sin 37 - 0.2\cos 37)}{0.2m\left(1+\frac{0.156kgm^2}{2kg(0.2m)^2}\right)} = 7.3\frac{rad}{s^2}$$

d. Suppose that you connect a motor to the pulley to pull the block back up the incline a distance of $d = 0.7m$. What torque would the motor (τ_{motor}) need to produce to pull the block back up the incline at a constant speed if the box started from rest? Assume friction still exists between the box and the ramp with coefficient of friction $\mu = 0.2$.

At a constant speed means that $a = 0$ (& $\alpha = 0$).

The net torque is given as the torque due to the motor and the torque due to the tension in the string. (Assume counterclockwise is positive for the torques.)
$-\tau_{motor} + \tau_{F_T} = -I\alpha \rightarrow \tau_{motor} = \tau_{F_T} = r_p F_T$

The tension force is determined from the block on the incline. Taking up the incline as the positive x-direction we have:
$F_T - F_{Wx} - F_{fr} = F_T - mg\sin\theta - \mu mg\cos\theta = ma = 0$
$F_T = mg\sin\theta + \mu mg\cos\theta$

$\rightarrow \tau_{motor} = r_p F_T = r_p(mg\sin\theta + \mu mg\cos\theta)$

$\rightarrow \tau_{motor} = 0.2m \times 2kg \times 9.8\frac{m}{s^2}\sin 37 + 0.2m \times 0.2 \times 2kg \times 9.8\frac{m}{s^2}\cos 37$

$\rightarrow \tau_{motor} = 2.36Nm + 0.63Nm = 2.99Nm$

Physics 110 Formulas

Motion

$\Delta x = x_f - x_i$ $v_{avg} = \frac{\Delta x}{\Delta t}$ $a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Uniform Circular Motion

$F_r = ma_r = m\frac{v^2}{r}$; $a_r = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$

$F_G = G\frac{m_1 m_2}{r^2}$

Geometry /Algebra

Circles *Triangles* *Spheres*

$C = 2\pi r$ $A = \frac{1}{2}bh$ $A = 4\pi r^2$

$A = \pi r^2$ $V = \frac{4}{3}\pi r^3$

Quadratic equation : $ax^2 + bx + c = 0$,

whose solutions are given by : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vectors

magnitude of a vector: $v = |\vec{v}| = \sqrt{v_x{}^2 + v^2{}_y}$

direction of a vector: $\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Useful Constants

$g = 9.8 \, m/s^2$ $G = 6.67\times10^{-11} \, Nm^2/kg^2$

$N_A = 6.02\times10^{23} \, atoms/mole$ $k_B = 1.38\times10^{-23} \, J/K$

$\sigma = 5.67\times10^{-8} \, W/m^2K^4$ $v_{sound} = 343 \, m/s$

Linear Momentum/Forces

$\vec{p} = m\vec{v}$

$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = m\vec{a}$

$\vec{F}_s = -k\vec{x}$

$F_f = \mu F_N$

Work/Energy

$K_t = \frac{1}{2}mv^2$

$K_r = \frac{1}{2}I\omega^2$

$U_g = mgh$

$U_S = \frac{1}{2}kx^2$

$W_T = FdCos\theta = \Delta E_T$

$W_R = \tau\theta = \Delta E_R$

$W_{net} = W_R + W_T = \Delta E_R + \Delta E_T$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = 0$

$\Delta E_R + \Delta E_T + \Delta U_g + \Delta U_S = -\Delta E_{diss}$

Heat

$T_C = \frac{5}{9}[T_F - 32]$

$T_F = \frac{9}{5}T_C + 32$

$L_{new} = L_{old}(1 + \alpha\Delta T)$

$A_{new} = A_{old}(1 + 2\alpha\Delta T)$

$V_{new} = V_{old}(1 + \beta\Delta T): \beta = 3\alpha$

$PV = Nk_BT$

$\frac{3}{2}k_BT = \frac{1}{2}mv^2$

$\Delta Q = mc\Delta T$

$P_C = \frac{\Delta Q}{\Delta t} = \frac{kA}{L}\Delta T$

$P_R = \frac{\Delta Q}{\Delta T} = \varepsilon\sigma A\Delta T^4$

$\Delta U = \Delta Q - \Delta W$

Rotational Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega^2{}_f = \omega^2{}_i + 2\alpha\Delta\theta$

$\tau = I\alpha = rF$

$L = I\omega$

$L_f = L_i + \tau\Delta t$

$\Delta s = r\Delta\theta : v = r\omega : a_t = r\alpha$

$a_r = r\omega^2$

Fluids

$\rho = \frac{M}{V}$

$P = \frac{F}{A}$

$P_d = P_0 + \rho gd$

$F_B = \rho gV$

$A_1v_1 = A_2v_2$

$\rho_1A_1v_1 = \rho_2A_2v_2$

$P_1 + \frac{1}{2}\rho v^2{}_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v^2{}_2 + \rho gh_2$

Simple Harmonic Motion/Waves

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_S = 2\pi\sqrt{\frac{m}{k}}$

$T_P = 2\pi\sqrt{\frac{l}{g}}$

$v = \pm\sqrt{\frac{k}{m}}A\left(1 - \frac{x^2}{A^2}\right)^{\frac{1}{2}}$

$x(t) = A\sin(\frac{2\pi t}{T})$

$v(t) = A\sqrt{\frac{k}{m}}\cos(\frac{2\pi t}{T})$

$a(t) = -A\frac{k}{m}\sin(\frac{2\pi t}{T})$

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = nf_1 = n\frac{v}{2L}$

$I = 2\pi^2 f^2 \rho vA^2$

Sound

$v = f\lambda = (331 + 0.6T)\frac{m}{s}$

$\beta = 10\log\frac{I}{I_0}$; $I_o = 1\times10^{-12}\frac{W}{m^2}$

$f_n = nf_1 = n\frac{v}{2L}$; $f_n = nf_1 = n\frac{v}{4L}$