Name $\qquad$
Physics 110 Quiz \#5, May 7, 2021
Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

## I affirm that I have carried out my academic endeavors with full academic honesty.

Consider the situation below in which a block of mass $m_{1}=3 \mathrm{~kg}$ connected to an initially horizontal string of negligible mass and length $L=0.75 \mathrm{~m}$ held at rest while a block of mass $m_{2}=1 \mathrm{~kg}$ sits at rest on a horizontal table.


1. Using energy ideas, what is the speed of block $m_{1}$ just before it collides with block $m_{2}$ ?

$$
\begin{aligned}
& \Delta E=\Delta K+\Delta U_{g}+\Delta U_{s} \rightarrow 0=\left(\frac{1}{2} m_{1} v_{m_{1}}^{2}-0\right)+\left(0-m_{1} g L\right) \\
& v_{3 M, i}=\sqrt{2 g L}=\sqrt{2 \times 9.8 \frac{m}{s^{2}} \times 0.75 \mathrm{~m}}=3.8 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

2. After the collision block $m_{2}$ is launched horizontally off the table and travels a horizontal distance of $4 L$ and a vertical distance of $2 L$ before landing on the ground. Using this information, what was the speed of block $m_{2}$ after the collision?

$$
\begin{aligned}
& x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2} \rightarrow 4 L=v_{M} t \rightarrow t=\frac{4 L}{v_{m_{2}}} \\
& y_{f}=y_{i}+v_{i y} t+\frac{1}{2} a_{y} t^{2} \rightarrow-2 L=-\frac{1}{2} g\left(\frac{4 L}{v_{m_{2}}}\right)^{2} \\
& \rightarrow v_{m_{2}}=\sqrt{4 g L}=\sqrt{4 \times 9.8 \frac{\mathrm{~m}}{s^{2}} \times 0.75 \mathrm{~m}}=5.4 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

3. What was the speed of block $m_{1}$ after the collision?

$$
\begin{aligned}
& p_{i, \text { system }, x}=p_{f, \text { system }, x} \rightarrow 3 M v_{3 M, i}=M v_{M}+3 M v_{3 M, f} \\
& v_{m_{1}, f}=\frac{m_{1} v_{m_{1}}-m_{2} v_{2}}{m_{1}}=\frac{3 \mathrm{~kg} \times 3.8 \frac{\mathrm{~m}}{\mathrm{~s}}-1 \mathrm{~kg} \times 5.4 \frac{\mathrm{~m}}{\mathrm{~s}}}{3 \mathrm{~kg}}=2.0 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

4. After the collision $m_{2}$ continues moving and swings through an angle $\theta_{\max }$ What is the value of $\theta_{\max }$ ? You can calculate a number for $\theta_{\max }$.
$\Delta E=\Delta K+\Delta U_{g}+\Delta U_{S} \rightarrow 0=\left(0-\frac{1}{2} m_{1} v_{m_{1, f}}^{2}\right)+\left(m_{1} g L(1-\cos \theta)-0\right)$
$\cos \theta_{\text {max }}=1-\frac{v_{m 1, f}^{2}}{2 g L}=1-\frac{\left(2 \frac{m}{s}\right)^{2}}{2 \times 9.8 \frac{m}{s^{2}} \times 0.75}=0.7280 \rightarrow \theta_{\max }=43.3^{0}$
where, $L=y_{f}+L \cos \theta_{\text {max }}$
5. What fraction of the initial energy is lost in the collision and based on this value, is the collision between blocks and elastic or inelastic?
$f=\frac{\Delta K}{K_{i}}=\frac{K_{f}-K_{i}}{K_{i}}=\frac{\left(\frac{1}{2} m_{1} v_{m_{1, f}}^{2}+\frac{1}{2} m_{2} v_{m_{2, f}}^{2}\right)-\frac{1}{2} m_{1} v_{m_{1, f}}^{2}}{\frac{1}{2} m_{1} v_{m_{1, f}}^{2}}=\frac{\left(\frac{1}{2} \times 3 \mathrm{~kg}\left(2 \frac{m}{s}\right)^{2}+\frac{1}{2} \times 1 \mathrm{~kg}\left(5.4 \frac{m}{s}\right)^{2}\right)-\left(\frac{1}{2} \times 3 \mathrm{~kg}\left(3.8 \frac{m}{s}\right)^{2}\right)}{\left(\frac{1}{2} \times 3 \mathrm{~kg}\left(3.8 \frac{m}{s}\right)^{2}\right)}$
$f=\frac{20.58-21.66}{21.66}=-0.05 \rightarrow-5 \%$
Since this is not zero, the collision is inelastic.

Vectors
$v=\sqrt{v_{x}^{2}+v_{y}^{2}}$
$\phi=\tan ^{-1}\left(\frac{v_{y}}{v_{x}}\right)$

## Motion Definitions

Displacement: $\Delta x=x_{f}-x_{i}$
Average velocity: $v_{\text {avg }}=\frac{\Delta x}{\Delta t}$
Average acceleration: $a_{a v g}=\frac{\Delta v}{\Delta t}$

## Equations of Motion

displacement: $\left\{\begin{array}{l}x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2} \\ y_{f}=y_{i}+v_{i y} t+\frac{1}{2} a_{y} t^{2}\end{array}\right.$
velocity: $\left\{\begin{array}{l}v_{f x}=v_{i x}+a_{x} t \\ v_{f y}=v_{i y}+a_{y} t\end{array}\right.$
time-independent: $\left\{\begin{array}{l}v_{f x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x \\ v_{f y}^{2}=v_{i y}^{2}+2 a_{y} \Delta y\end{array}\right.$

Rotational Motion Definitions
Angular displacement: $\Delta s=R \Delta \theta$
Angular velocity: $\omega=\frac{\Delta \theta}{\Delta t} \rightarrow v=R \omega$
Angular acceleration: $\alpha=\frac{\Delta \omega}{\Delta t} \rightarrow\left\{\begin{array}{c}a_{t}=r \alpha \\ a_{c}=r \omega^{2}\end{array}\right.$
Rotational Equations of Motion

$$
\begin{aligned}
& \theta_{f}=\theta_{i}+\omega_{i} t+\frac{1}{2} \alpha t^{2} \\
& \omega_{f}=\omega_{i}+\alpha t \\
& \omega_{f}^{2}=\omega_{i}^{2}+2 \alpha \Delta \theta
\end{aligned}
$$

Momentum \& Force
$\vec{p}=m \vec{v} \rightarrow p_{x}=m v_{x} ; p_{y}=m v_{y}$
$\Delta \vec{p}=\vec{F} \Delta t \rightarrow \vec{p}_{f}=\vec{p}_{i}+\vec{F} \Delta t$
$\vec{F}=\frac{d \vec{p}}{d t}=m \vec{a} \rightarrow F_{x}=m a_{x} ; F_{y}=m a_{y}$
$F_{f r}=\mu F_{N}$
$F_{w}=m g$
$F_{S}=-k x$
$F_{G}=G \frac{M_{1} M_{2}}{r^{2}}$
$F_{c}=m a_{c}=m \frac{v^{2}}{R}$
Work \& Energy
$\left\{\begin{array}{c}W_{T}=\int \vec{F} \cdot d \vec{r}=F d r \cos \theta=\Delta K_{T} \\ W_{R}=\int \vec{\tau} \cdot d \vec{\theta}=\tau d \theta=\Delta K_{R}\end{array}\right.$
$W_{n e t}=W_{T}+W_{R}=\Delta K_{T}+\Delta K_{R}=-\Delta U$
$K_{T}=\frac{1}{2} m v^{2}$
$K_{R}=\frac{1}{2} I \omega^{2}$
$U_{g}=m g y$
$U_{s}=\frac{1}{2} k x^{2}$
$\Delta E=\Delta E_{R}+\Delta E_{T}$
$\Delta E=\Delta K_{R}+\Delta K_{T}+\Delta U_{g}+\Delta U_{s}=\left\{\begin{array}{c}0 \\ W_{f r}\end{array}\right.$
Rotational Momentum \& Force

$$
\begin{aligned}
& \vec{\tau}=\vec{r} \times \vec{F} ; \tau=r_{\perp} F=r F_{\perp}=r F \sin \theta \\
& \tau=\frac{\Delta L}{\Delta t}=I \alpha \\
& L=I \omega \\
& \Delta \vec{L}=\vec{\tau} \Delta t \rightarrow \vec{L}_{f}=\vec{L}_{i}+\vec{\tau} \Delta t
\end{aligned}
$$

Fluids
$\rho=\frac{m}{V}$
$P=\frac{F}{A}$
$P_{y}=P_{\text {air }}+\rho g y$
$F_{B}=\rho g V$
$\rho_{1} A_{1} v_{1}=\rho_{2} A_{2} v_{2}$; compressible $A_{1} v_{1}=A_{2} v_{2}$; incompressible
$P_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g y_{1}=P_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho g y_{2}$

Simple Harmonic Motion
$\omega=2 \pi f=\frac{2 \pi}{T}$
$T_{S}=2 \pi \sqrt{\frac{m}{k}} ; \quad \omega=\sqrt{\frac{k}{m}}$
$T_{p}=2 \pi \sqrt{\frac{l}{g}} ; \quad \omega=\sqrt{\frac{g}{l}}$

Geometry/Algebra
Circles:

$$
A=\pi r^{2} \quad C=2 \pi r=\pi D
$$

Spheres: $\quad A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$
Triangles: $\quad A=\frac{1}{2} b h$
Quadratics: $\quad a x^{2}+b x+c=0 \rightarrow x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \quad v= \pm \omega x_{\max } \sqrt{1-\left(\frac{x}{x_{\max }}\right)^{2}}$

Sound
$v_{s}=f \lambda=(331+0.6 T) \frac{m}{s}$
$\beta=10 \log \frac{I}{I_{o}}$
$f_{n}=n f_{1}=n \frac{v}{2 L} ; n=1,2,3, \ldots$ open pipes
$f_{n}=n f_{1}=n \frac{v}{4 L} ; n=1,3,5, \ldots$ closed pipes
Waves
$v=f \lambda=\sqrt{\frac{F_{T}}{\mu}}$
$f_{n}=n f_{1}=n \frac{v}{2 L} ; n=1,2,3, \ldots$
$I=2 \pi^{2} f^{2} \rho v A^{2}$

Equations of Motion for SHM
$x(t)=\left\{\begin{array}{l}x_{\text {max }} \sin \left(\frac{2 \pi}{T} t\right) \\ x_{\text {max }} \cos \left(\frac{2 \pi}{T} t\right)\end{array}\right.$
$v(t)=\left\{\begin{array}{c}v_{\text {max }} \cos \left(\frac{2 \pi}{T} t\right) \\ -v_{\text {max }} \sin \left(\frac{2 \pi}{T} t\right)\end{array}\right.$
$a(t)=\left\{\begin{array}{l}-a_{\text {max }} \sin \left(\frac{2 \pi}{T} t\right) \\ -a_{\text {max }} \cos \left(\frac{2 \pi}{T} t\right)\end{array}\right.$
$v= \pm v_{\max } \sqrt{1-\left(\frac{x}{x_{\max }}\right)^{2}}$

Periodic Table of the Elements


