Name____

Physics 110 Quiz #6, May 12, 2022

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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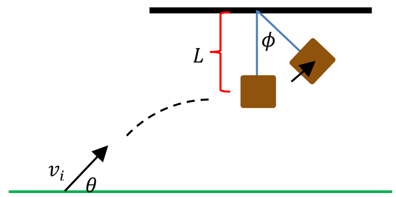
1. A dart of mass $m = 0.2kg$ is launched from the ground at an angle $\theta = 38^0$ measured with respect to the ground at a speed of $v_i = 10\frac{m}{s}$. When the dart reaches its highest point above the ground it strikes a soft wooden block of mass $M = 1.3kg$ initially at rest. How high above the ground was the wooden block placed?



$$\Delta E = \Delta K + \Delta U_g + \Delta U_S = 0$$
$$0 = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(mgy_f - mgy_i\right)$$
$$0 = \tfrac{1}{2}m(v_i\cos\theta)^2 - \tfrac{1}{2}mv_i^2 + mgh$$
$$h = \frac{v_i^2(1-\cos^2\theta)}{2g} = \frac{\left(10\frac{m}{s}\right)^2(1-\cos^2 38)}{2\times 9.8\frac{m}{s^2}} = 1.93m$$

2. Immediately after the dart strikes the wooden block it becomes stuck in the block. With what speed does the dart and block move after the collision?

$$p_{ix} = p_{fx} \to mv_{ix} = (m+M)V \to V = \left(\frac{m}{m+M}\right)v_{ix} = \left(\frac{0.2kg}{0.2kg+1.3kg}\right)10\frac{m}{s}\cos 38$$
$$V = 1.1\frac{m}{s}$$

3. Through what angle ϕ measured with respect to the vertical do the dart and block swing through if the rope has a length of $L = 1.0m$.

$$\Delta E = \Delta K + \Delta U_g + \Delta U_S = 0$$
$$0 = \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right) + \left(mgy_f - mgy_i\right) = -\tfrac{1}{2}mv_i^2 + mgL(1-\cos\phi)$$
$$\cos\phi = 1 - \frac{v_i^2}{2gL} = 1 - \frac{\left(1.1\frac{m}{s}\right)^2}{2\times 9.8\frac{m}{s^2}\times 1m} = 0.9437 \to \phi = 19.3^0$$

4. What fraction of the initial energy in the dart is lost in the collision with the wooden block?

$$f = \frac{\Delta K}{K_i} = \frac{\frac{1}{2}(m+M)V^2 - \frac{1}{2}mv_i^2}{\frac{1}{2}mv_i^2} = \left(\frac{m+M}{m}\right)\frac{V^2}{v_i^2} - 1 = \left(\frac{0.2kg + 1.3kg}{0.2kg}\right)\left(\frac{1.1\frac{m}{s}}{10\frac{m}{s}\cos 38}\right)^2 - 1$$

$$f = -0.85$$

5. Suppose that instead of the soft wooden block, a metal block of the same mass $(M = 1.3kg)$ is suspended from the rope at rest. The same dart $(m = 0.2kg)$ is launched at the same velocity as in part 1 and in this case the dart bounces off the metal block with a velocity $v_{fx} = -\frac{3}{4}v_{ix}$, where v_{ix} is the impact velocity of the dart with the metal block. Through what angle ϕ measured with respect to the vertical, did the metal block swing?

$$p_{ix} = p_{fx} \rightarrow mv_{ix,m} = mv_{fx,m} + Mv_{fx,M} = -\tfrac{3}{4}mv_{ix,m} + Mv_{fx,M} \rightarrow v_{fx,M} = \frac{7}{4}\left(\frac{m}{M}\right)v_{ix,m}$$

$$v_{fx,M} = \frac{7}{4}\left(\frac{m}{M}\right)v_{ix,m} = \frac{7 \times 0.2kg}{4 \times 1.3kg} \times 10\frac{m}{s}\cos 38 = 2.12\frac{m}{s}$$

$$\Delta E = \Delta K + \Delta U_g + \Delta U_S = 0$$

$$0 = \left(\tfrac{1}{2}Mv_f^2 - \tfrac{1}{2}Mv_i^2\right) + \left(Mgy_f - Mgy_i\right) = -\tfrac{1}{2}Mv_i^2 + MgL(1 - \cos\phi)$$

$$\cos\phi = 1 - \frac{v_i^2}{2gL} = 1 - \frac{\left(2.12\frac{m}{s}\right)^2}{2 \times 9.8\frac{m}{s^2} \times 1m} = 0.7704 \rightarrow \phi = 39.6^0$$

Physics 110 Formula sheet

Vectors

$v = \sqrt{v_x^2 + v_y^2}$

$\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Motion Definitions

Displacement: $\Delta x = x_f - x_i$

Average velocity: $v_{avg} = \frac{\Delta x}{\Delta t}$

Average acceleration: $a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y \end{cases}$

Rotational Motion Definitions

Angular displacement: $\Delta s = r\Delta\theta$

Angular velocity: $\omega = \frac{\Delta\theta}{\Delta t} \rightarrow v = r\omega$

Angular acceleration: $\alpha = \frac{\Delta\omega}{\Delta t} \rightarrow \begin{cases} a_t = r\alpha \\ a_c = r\omega^2 \end{cases}$

Rotational Equations of Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

Momentum & Force

$\vec{p} = m\vec{v} \rightarrow p_x = mv_x;\ p_y = mv_y$

$\Delta\vec{p} = \vec{F}\Delta t \rightarrow \vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a} \rightarrow F_x = ma_x; F_y = ma_y$

$F_{fr} = \mu F_N$

$F_w = mg$

$F_s = -kx$

$F_G = G\frac{M_1 M_2}{r^2}$

$F_c = ma_c = m\frac{v^2}{R}$

Work & Energy

$\begin{cases} W_T = \int \vec{F}\cdot d\vec{r} = Fdr\cos\theta = \Delta K_T \\ W_R = \int \vec{\tau}\cdot d\vec{\theta} = \tau d\theta = \Delta K_R \end{cases}$

$W_{net} = W_T + W_R = \Delta K_T + \Delta K_R = -\Delta U$

$K_T = \frac{1}{2}mv^2$

$K_R = \frac{1}{2}I\omega^2$

$U_g = mgy$

$U_s = \frac{1}{2}kx^2$

$\Delta E = \Delta E_R + \Delta E_T$

$\Delta E = \Delta K_R + \Delta K_T + \Delta U_g + \Delta U_s = \begin{cases} 0 \\ W_{fr} \end{cases}$

Rotational Momentum & Force

$\vec{\tau} = \vec{r}\times\vec{F};\ \tau = r_\perp F = rF_\perp = rF\sin\theta$

$\tau = \frac{\Delta L}{\Delta t} = I\alpha$

$L = I\omega$

$\Delta\vec{L} = \vec{\tau}\Delta t \rightarrow \vec{L}_f = \vec{L}_i + \vec{\tau}\Delta t$

Fluids

$\rho = \frac{m}{V}$

$P = \frac{F}{A}$

$P_y = P_{air} + \rho g y$

$F_B = \rho g V$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$; compressible

$A_1 v_1 = A_2 v_2$; incompressible

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

Simple Harmonic Motion

$\omega = 2\pi f = \frac{2\pi}{T}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$; $\quad \omega = \sqrt{\frac{k}{m}}$

$T_p = 2\pi\sqrt{\frac{l}{g}}$; $\quad \omega = \sqrt{\frac{g}{l}}$

Geometry/Algebra

Circles: $A = \pi r^2$ $\quad C = 2\pi r = \pi D$

Spheres: $A = 4\pi r^2$ $\quad V = \frac{4}{3}\pi r^3$

Triangles: $A = \frac{1}{2}bh$

Quadratics: $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Common Metric Prefixes

$nano = 1 \times 10^{-9}$

$micro = 1 \times 10^{-6}$

$milli = 1 \times 10^{-3}$

$centi = 1 \times 10^{-2}$

$kilo = 1 \times 10^{3}$

$mega = 1 \times 10^{6}$

Sound

$v_s = f\lambda = (331 + 0.6T)\frac{m}{s}$

$\beta = 10 \log \frac{I}{I_o}$

$f_n = n f_1 = n\frac{v}{2L}; n = 1,2,3, \ldots$ open pipes

$f_n = n f_1 = n\frac{v}{4L}; n = 1,3,5, \ldots$ closed pipes

Waves

$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$

$f_n = n f_1 = n\frac{v}{2L};\ n = 1,2,3, \ldots$

$I = 2\pi^2 f^2 \rho v A^2$

Equations of Motion for SHM

$$x(t) = \begin{cases} x_{max}\sin\left(\frac{2\pi}{T}t\right) \\ x_{max}\cos\left(\frac{2\pi}{T}t\right) \end{cases}$$

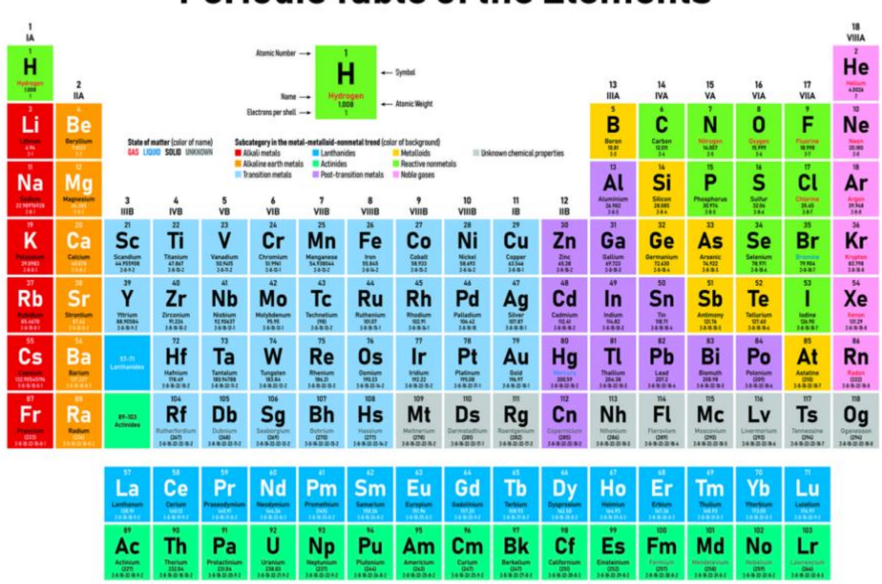
$$v(t) = \begin{cases} v_{max}\cos\left(\frac{2\pi}{T}t\right) \\ -v_{max}\sin\left(\frac{2\pi}{T}t\right) \end{cases}$$

$$a(t) = \begin{cases} -a_{max}\sin\left(\frac{2\pi}{T}t\right) \\ -a_{max}\cos\left(\frac{2\pi}{T}t\right) \end{cases}$$

$v = \pm v_{max}\sqrt{1 - \left(\frac{x}{x_{max}}\right)^2}$

$v = \pm \omega x_{max}\sqrt{1 - \left(\frac{x}{x_{max}}\right)^2}$

Periodic Table of the Elements



https://www.wuwm.com/post/periodic-table-elements-turns-150#stream/0