Name_______________________________________

Physics 110 Quiz #7, May 27, 2022

Please show all work, thoughts and/or reasoning in order to receive partial credit. The quiz is worth 10 points total.

I affirm that I have carried out my academic endeavors with full academic honesty.

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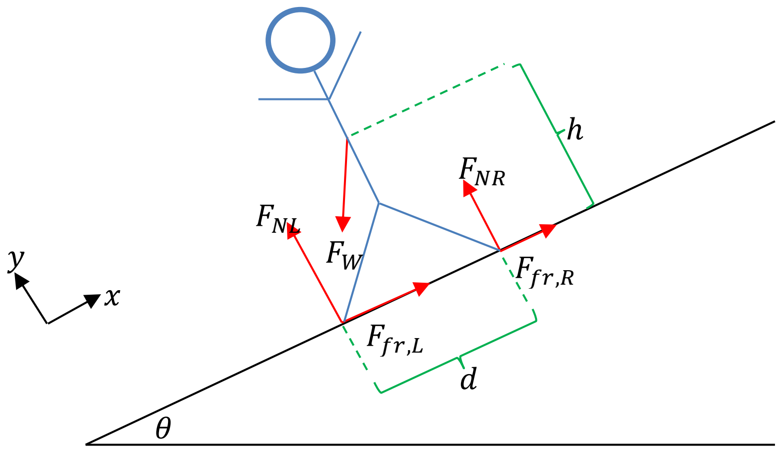
Suppose that you are standing on a hill inclined at angle θ with respect to the ground. Your legs are separated by a distance d with one foot uphill and one foot downhill. The center of your body (where your weight $F_W = mg$ acts) is at a heigh h measured perpendicular to the hill as shown below. Assume that static friction is large enough under each foot that you don't slide down the hill.



1. Assuming the tilted coordinate system shown, what is the expression for the forces parallel to the hill.

$$F_{fr,L} + F_{fr,R} - F_{wx} = F_{fr,L} + F_{fr,R} - mg\sin\theta = ma_x = 0$$
$$\rightarrow F_{fr,L} + F_{fr,R} = mg\sin\theta$$

2. What is the expression for the forces perpendicular to the hill?

$$F_{NL} + F_{NR} - F_{wy} = F_{NL} + F_{NR} - mg\cos\theta = ma_y = 0$$
$$\rightarrow F_{NL} + F_{NR} = mg\cos\theta$$

3. Taking the pivot to be at the center of your body, what is the expression for the torques about your center if you are just on the verge of tipping and falling down the hill? Assume counterclockwise rotations are positive and clockwise rotations negative.

$$-r_L F_{NL} \sin\beta + r_L F_{fr,L} \sin(90-\beta) + r_R F_{NR} \sin\beta + r_R F_{fr,R} \sin(90-\beta) = I\alpha = 0$$
$$0 = -\tfrac{d}{2} F_{NL} + hF_{fr,L} + \tfrac{d}{2} F_{NR} + hF_{fr,R}$$
$$\rightarrow (F_{NR} - F_{NL})\tfrac{d}{2} + \left(F_{fr,L} + F_{fr,R}\right)h = 0$$

4. Suppose that $m = 60kg$, $d = 0.85m$, $h = 0.7m$, and $\theta = 30^0$, what is the magnitude of the normal force on your left foot, F_{NL}?

$$0 = [(mg\cos\theta - F_{NL}) - F_{NL}]\tfrac{d}{2} + (mg\sin\theta)h$$
$$F_{NL} = \frac{mg\cos\theta}{2} + mg\frac{h}{d}\sin\theta = 60kg \times 9.8\frac{m}{s^2}\left[\frac{\cos 30}{2} + \frac{0.7m}{0.85m}\sin 30\right]$$
$$F_{NL} = 497N$$

5. What is the magnitude of the normal force on your right foot, F_{NR}? Hint, F_{NL} and F_{NR} are not equal to each other in this case.

$$F_{NR} = mg\cos\theta - F_{NL} = mg\cos\theta - \frac{mg\cos\theta}{2} - mg\frac{h}{d}\sin\theta$$
$$F_{NR} = \frac{mg\cos\theta}{2} - mg\frac{h}{d}\sin\theta = 60kg \times 9.8\frac{m}{s^2}\left[\frac{\cos 30}{2} - \frac{0.7m}{0.85m}\sin 30\right]$$
$$F_{NR} = 12N$$

Physics 110 Formula sheet

Vectors

$v = \sqrt{v_x^2 + v_y^2}$

$\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Motion Definitions

Displacement: $\Delta x = x_f - x_i$

Average velocity: $v_{avg} = \frac{\Delta x}{\Delta t}$

Average acceleration: $a_{avg} = \frac{\Delta v}{\Delta t}$

Equations of Motion

displacement: $\begin{cases} x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2 \\ y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \end{cases}$

velocity: $\begin{cases} v_{fx} = v_{ix} + a_x t \\ v_{fy} = v_{iy} + a_y t \end{cases}$

time-independent: $\begin{cases} v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y \end{cases}$

Rotational Motion Definitions

Angular displacement: $\Delta s = r\Delta\theta$

Angular velocity: $\omega = \frac{\Delta\theta}{\Delta t} \to v = r\omega$

Angular acceleration: $\alpha = \frac{\Delta\omega}{\Delta t} \to \begin{cases} a_t = r\alpha \\ a_c = r\omega^2 \end{cases}$

Rotational Equations of Motion

$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

$\omega_f = \omega_i + \alpha t$

$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

Fluids

Momentum & Force

$\vec{p} = m\vec{v} \to p_x = mv_x;\ p_y = mv_y$

$\Delta\vec{p} = \vec{F}\Delta t \to \vec{p}_f = \vec{p}_i + \vec{F}\Delta t$

$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a} \to F_x = ma_x; F_y = ma_y$

$F_{fr} = \mu F_N$

$F_w = mg$

$F_s = -kx$

$F_G = G\frac{M_1 M_2}{r^2}$

$F_c = ma_c = m\frac{v^2}{R}$

Work & Energy

$\begin{cases} W_T = \int \vec{F}\cdot d\vec{r} = Fdr\cos\theta = \Delta K_T \\ W_R = \int \vec{\tau}\cdot d\vec{\theta} = \tau d\theta = \Delta K_R \end{cases}$

$W_{net} = W_T + W_R = \Delta K_T + \Delta K_R = -\Delta U$

$K_T = \frac{1}{2}mv^2$

$K_R = \frac{1}{2}I\omega^2$

$U_g = mgy$

$U_s = \frac{1}{2}kx^2$

$\Delta E = \Delta E_R + \Delta E_T$

$\Delta E = \Delta K_R + \Delta K_T + \Delta U_g + \Delta U_s = \begin{cases} 0 \\ W_{fr} \end{cases}$

Rotational Momentum & Force

$\vec{\tau} = \vec{r}\times\vec{F};\ \tau = r_\perp F = rF_\perp = rF\sin\theta$

$\tau = \frac{\Delta L}{\Delta t} = I\alpha$

$L = I\omega$

$\Delta\vec{L} = \vec{\tau}\Delta t \to \vec{L}_f = \vec{L}_i + \vec{\tau}\Delta t$

Sound

$\rho = \frac{m}{V}$
$P = \frac{F}{A}$
$P_y = P_{air} + \rho g y$
$F_B = \rho g V$
$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$; compressible
$A_1 v_1 = A_2 v_2$; incompressible

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

Simple Harmonic Motion

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}; \quad \omega = \sqrt{\frac{k}{m}}$$

$$T_p = 2\pi\sqrt{\frac{l}{g}}; \quad \omega = \sqrt{\frac{g}{l}}$$

Geometry/Algebra

Circles: $A = \pi r^2$ $C = 2\pi r = \pi D$

Spheres: $A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

Triangles: $A = \frac{1}{2}bh$

Quadratics: $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Common Metric Prefixes

$nano = 1 \times 10^{-9}$
$micro = 1 \times 10^{-6}$
$milli = 1 \times 10^{-3}$
$centi = 1 \times 10^{-2}$
$kilo = 1 \times 10^{3}$
$mega = 1 \times 10^{6}$

$v_s = f\lambda = (331 + 0.6T)\frac{m}{s}$
$\beta = 10 \log \frac{I}{I_o}$
$f_n = n f_1 = n\frac{v}{2L}; n = 1,2,3, \ldots$ open pipes
$f_n = n f_1 = n\frac{v}{4L}; n = 1,3,5, \ldots$ closed pipes

Waves

$$v = f\lambda = \sqrt{\frac{F_T}{\mu}}$$

$f_n = n f_1 = n\frac{v}{2L};\ n = 1,2,3, \ldots$
$I = 2\pi^2 f^2 \rho v A^2$

Equations of Motion for SHM

$$x(t) = \begin{cases} x_{max} \sin\left(\frac{2\pi}{T}t\right) \\ x_{max} \cos\left(\frac{2\pi}{T}t\right) \end{cases}$$

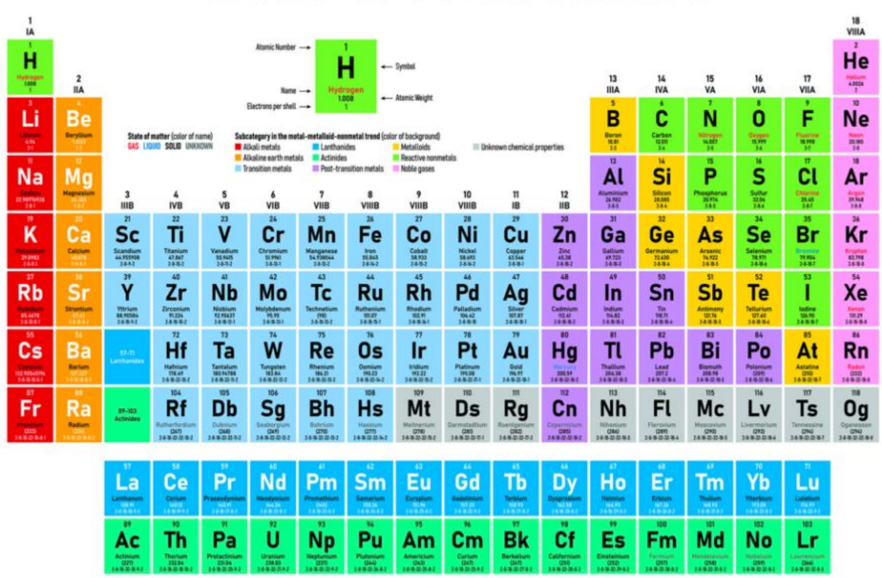
$$v(t) = \begin{cases} v_{max} \cos\left(\frac{2\pi}{T}t\right) \\ -v_{max} \sin\left(\frac{2\pi}{T}t\right) \end{cases}$$

$$a(t) = \begin{cases} -a_{max} \sin\left(\frac{2\pi}{T}t\right) \\ -a_{max} \cos\left(\frac{2\pi}{T}t\right) \end{cases}$$

$$v = \pm v_{max}\sqrt{1 - \left(\frac{x}{x_{max}}\right)^2}$$

$$v = \pm \omega x_{max}\sqrt{1 - \left(\frac{x}{x_{max}}\right)^2}$$

Periodic Table of the Elements



https://www.wuwm.com/post/periodic-table-elements-turns-150#stream/0