Physics 111

Exam #1

October 3, 2014

Name_____

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}| = (5kg) \times (2\frac{m}{s}) = 10\frac{kg \cdot m}{s}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 3 points and each free-response part is worth 8 points

Problem #1	/27
Problem #2	/22
Problem #3	/22
Total	/71

I affirm that I have carried out my academic endeavors with full academic honesty.

- 1. Suppose that you are given a system of point charges, where $q_1 = +1\mu C$ is located at the point (x,y) = (0,0), $q_2 = -2\mu C$ is located at the point (x,y) = (0.5,0), $q_3 = +1\mu C$ is located at the point (x,y) = (0.5,0.5), where all distances are in meters.
 - a. What is the electric field at a point *P* with coordinates (x,y) = (0,0.5)?

The net x-component of the field is:

$$E_{net,x} = E_2 \cos\theta - E_3 = \frac{kq_2}{2L^2} \cos\theta - \frac{kq_3}{L^2}$$
$$= 9 \times 10^9 \frac{Nm^2}{C^2} \left[\frac{2 \times 10^{-6} C}{2(0.5m)^2} (0.707) - \frac{1 \times 10^{-6} C}{(0.5m)^2} \right] = -1.06 \times 10^4 \frac{N}{C}$$

The net y-component of the field is:

$$E_{net,y} = E_1 - E_2 \sin\theta = \frac{kq_1}{L^2} - \frac{kq_2}{2L^2} \sin\theta$$
$$= 9 \times 10^9 \frac{Nm^2}{C^2} \left[\frac{1 \times 10^{-6}C}{(0.5m)^2} - \frac{2 \times 10^{-6}C}{2(0.5m)^2} (0.707) \right] = 1.06 \times 10^4 \frac{N}{C}$$

The net electric field is therefore,

$$E_{net} = \sqrt{E_{net,x}^2 + E_{net,y}^2} @\phi = \tan^{-1} \left(\frac{E_{net,y}}{E_{net,x}}\right)$$
$$= \sqrt{2(1.06 \times 10^4 \frac{N}{C})^2} @\phi = \tan^{-1}(1) = 1.5 \times 10^4 \frac{N}{C} @\phi = 45^0 \text{ above the negative x-axis}$$

b. What is the net force and how much work is required to bring in a fourth charge $q_4 = -2\mu C$ and place it at point *P*?

The net force is given by $\vec{F} = q\vec{E} = 2 \times 10^{-6} C \times 1.5 \times 10^4 \frac{N}{C} = 0.03N @\phi = 45^{\circ}$ below the positive x-axis.

The work done brining $q_4 = -2\mu C$ in from infinitely far away is

$$W = -q\Delta V = -q_4 \left[\left(V_{f,1} + V_{f,2} + V_{f,3} \right) - 0 \right] = -q_4 \left[\frac{kq_1}{r_{1,p}} + \frac{kq_2}{r_{2,p}} + \frac{kq_3}{r_{3,p}} \right]$$
$$= -\left(-2 \times 10^{-6} C \right) \left(9 \times 10^9 \frac{Nm^2}{C^2} \right) \left[\frac{1 \times 10^{-6} C}{0.5m} - \frac{2 \times 10^{-6} C}{2(0.5m)} + \frac{1 \times 10^{-6} C}{0.5m} \right].$$
$$= 2.1 \times 10^{-2} J = 21 m J$$

c. How much work went into assembling the initial collection of 3-point charges?

The work done to place all three charges is the sum of the work done to bring each charge in from infinity and place them at their specific locations.

The work done to place $q_1 = +1\mu C$ located at the point (x,y) = (0,0) is $W_1 = -q\Delta V = -q_1 \left[V_f - V_i \right] = 0J.$

The work done to place $q_2 = -2\mu C$ located at the point (x,y) = (0.5,0) is

$$W_{2} = -q\Delta V = -q_{2} \left[V_{2,1} - V_{i} \right] = -q_{2} \left[\frac{kq_{1}}{r_{1,2}} \right]$$
$$= -\left(-2 \times 10^{-6} C\right) \left(\frac{9 \times 10^{9} \frac{Nm^{2}}{C^{2}} \times 1 \times 10^{-6} C}{0.5m} \right) = 0.036 J = 36 m J$$

The work done to place $q_3 = +1\mu C$ located at the point (x,y) = (0.5,0.5) is

$$W_{3} = -q\Delta V = -q_{3} \left[V_{3,1} - V_{i} \right] - q_{3} \left[V_{3,2} - V_{i} \right] = -q_{3} \left[\frac{kq_{1}}{r_{1,3}} + \frac{kq_{2}}{r_{2,3}} \right]$$
$$= -\left(1 \times 10^{-6} C\right) \left(\frac{9 \times 10^{9} \frac{Nm^{2}}{C^{2}} \times 1 \times 10^{-6} C}{2(0.5m)} - \frac{9 \times 10^{9} \frac{Nm^{2}}{C^{2}} \times 2 \times 10^{-6} C}{(0.5m)} \right) = 0.023 J = 23 m J$$

Thus the net work done is $W_{net} = W_1 + W_2 + W_3 = 0J + 36mJ + 23mJ = 59mJ = 0.059J$.

- d. If q_4 were released from rest at point P, it would
 - 1. accelerate in the direction of the net electric field at point *P*.
 - accelerate in the direction opposite to the net electric field at point *P*.
 feel no net force and thus remain at rest at point *P*.
 - 4. feel no net force and continue moving at a constant velocity along the charges original direction of motion.

2. Tractor trailers and tank trucks are used to transport goods and products all over the country. Suppose that you have a tractor-trailer that needs to be filled with some product (say a liquid or a powder.) As the product moves through the transfer system from the source to the truck, the product interacts with pumps, valve, filters and pipe walls. This interaction strips electrons off of the pumps, valves, pipe walls, etc. and the product will be building up some amount of electrostatic charge. When the product is transferred into the tank truck, the tank truck, will in turn, become electrified. If a person outside of the truck attempts to touch the truck before it is fully discharged, a painful spark can occur. Or worse yet, if the product is say gasoline, fuel vapors can be ignited and an explosion could occur. An example of filling a tank truck is shown below. In what follows, suppose that the truck has an effective capacitance of $C = 1000 \times 10^{-12} F$ and 18 tires each with a resistance of $R = 1000 M\Omega$.



a. If the tank of the truck is completely full and if is left alone, the accumulated charge will dissipate through the tires. What would be the time constant for this discharge process?

There are 18 resistors in parallel, so the effective resistance is $\frac{1}{R_{eq}} = \frac{18}{R} = \frac{18}{1000 \times 10^6 \Omega} \rightarrow R_{eq} = 5.6 \times 10^7 \Omega$. Therefore the time constant for this circuit is $\tau = RC = 5.6 \times 10^7 \Omega \times 1000 \times 10^{-12} F = 0.06s = 60 ms$.

b. Most hydrocarbon vapors (e.g. gasoline) will ignite with a minimum input of energy less than 1mJ. How long would you have to wait for say the truck to discharge the accumulated charge through the tires, so that you could reach this minimum 1mJ of energy? Suppose that during the filling process, the truck acquires a potential difference $\Delta V = -50kV$ across it, relative to ground.

The initial energy stored is $E_i = \frac{1}{2}CV^2 = \frac{1}{2} \times 1000 \times 10^{-12} F (50000V)^2 = 1.25J$. The energy that's dissipated is given by

$$E_{f} = \frac{1}{2}CV^{2} = \frac{1}{2}C\left(V_{\max}e^{-\frac{t}{RC}}\right)^{2} = \frac{1}{2}CV_{\max}^{2}e^{-\frac{2t}{RC}} = E_{i}e^{-\frac{2t}{RC}}.$$
 The time to discharge is
$$t = -\frac{RC}{2}\ln\left(\frac{E_{f}}{E_{i}}\right) = -\frac{0.06s}{2}\ln\left(\frac{1\times10^{-3}J}{1.25J}\right) = 0.21s.$$

- c. If you wanted to discharge the truck to an energy of 1mJ or less, given the time in part b, an effective solution to discharging the truck (thereby hopefully not igniting the fuel vapors) while at the same time being able to carry the same amount of product, is to
 - 1. make the tires of the truck have a larger resistance.
 - 2. make the capacitance of the truck larger.
 - 3. make the capacitance of the truck smaller.
 - 4.) connect the truck to the earth with a wire so that any accumulated charge can flow off of the truck to ground.
 - 5. connect the truck to the earth with a wire so charge can flow from the earth onto the truck.

- d. This problem is not directly related to the previous parts. A 10V battery is connected to a parallel plate capacitor and a 100 Ω resistor. The plates have a surface area $0.01m^2$ and are separated by a distance of 3mm. Consider the wires that connect the circuit elements as having negligible resistance. When the capacitor is fully charged, a proton can be directed between the plates with a speed of $1000 \frac{m}{s}$. If the charged particle is given a speed twice its current speed, which will NOT be true:
 - 1. The acceleration of the proton will remain the same.
 - 2. The force on the proton will remain the same.
 - 3.) The force on the proton will increase.
 - 4. The path that the proton takes between the plates will change.

3. Suppose that you have the circuit shown below in which you have several resistors, each with resistance 100Ω connected to a 15V battery.



a. What is the total current that is produced by the battery?

$$R_3 \& R_4 \text{ are in parallel, so } \frac{1}{R_{34}} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{2}{100\Omega} \rightarrow R_{3,4} = 50\Omega$$

 $R_2, R_{3,4} \& R_5$ are in series, so $R_{2345} = R_2 + R_{34} + R_5 = 250\Omega$.

 $R_6, R_7 \& R_8$ are in series, so $R_{678} = R_6 + R_7 + R_8 = 300\Omega$.

 $\frac{R_{2345} \& R_{678} \text{ are in parallel, so}}{\frac{1}{R_{2345678}} = \frac{1}{R_{2345}} + \frac{1}{R_{678}} = \frac{1}{250\Omega} + \frac{1}{300\Omega} \to R_{2345678} = 136.4\Omega$

 $R_1, R_{2345678}$ & R_9 are in series, so $R_{eq} = R_{123456789} = R_1 + R_{2345678} + R_9 = 336.4 \Omega$.

Therefore the total current produced by the battery is $V = I_{total} R_{eq} \rightarrow I_{total} = \frac{V}{R_{eq}} = \frac{15V}{336.4\Omega} = 0.045 A = 45 mA$. b. What is the potential drop across resistor 6, V_6 and the current through resistor 2, I_2 ?

The potential difference across resistors R_1 and R_1 is given by $V_1 = V_9 = I_{total} R_1 = 0.045 A \times 100 \Omega = 4.5 V$. Therefore, $V_{2345678} = 15V - (2 \times 4.5V) = 6V$.

The current in the upper branch of the circuit is

$$V = IR \to I_{upper} = \frac{V_{upper}}{R_{upper}} = \frac{V_{upper}}{R_{678}} = \frac{6V}{300\Omega} = 0.02A = 20mA.$$

The potential drop across R_6 is given by $V_6 = I_{upper}R_6 = 0.020A \times 100\Omega = 2V$.

The current in the lower branch is given by

$$I_{total} = I_{upper} + I_{lower} \rightarrow I_{lower} = I_{total} - I_{upper} = 45 mA - 20 mA = 25 mA = 0.025 A \text{ or}$$

 $V = IR \rightarrow I_{lower} = \frac{V_{lower}}{R_{lower}} = \frac{V_{lower}}{R_{2345}} = \frac{6V}{250\Omega} = 0.024 A = 24 mA$.

- c. The power developed by the battery is dissipated across all of the resistors in the circuit. As a function of time, the power dissipated across all of the resistors is given by
 - 1. $P = I^2 R = I_{max}^2 R_{eq} e^{-\frac{2t}{RC}}$ 2. $P = I^2 R = I_{\max}^2 R_{eq} \left(1 - e^{-\frac{2t}{RC}} \right)$ $(3.) P = I_{\max}^2 R_{eq}$ 4. $P = I_{max}^2 R_{eq} e^{-t}$
- d. Suppose that you have a circuit (shown below) with a constant current flowing such that a magnetic field is produced (by the current) directed into the page with strength B. If a proton were shot directly up the plane of the page, the proton will feel a magnetic force given by
 - 1. F = qvB directed into the page.
 - 2. F = qvB directed out of the page.

 - 3. F = qvB directed toward the circuit. 4. F = qvB directed away from the circuit.
 - 4. F = 0



Physics 111 Equation Sheet

Electric Forces, Fields and Potentials $\vec{F} = k \frac{Q_1 Q_2}{r^2} \hat{r}$ $\vec{E} = \frac{\vec{F}}{a}$ $\vec{E}_Q = k \frac{Q}{r^2} \hat{r}$ $PE = k \frac{Q_1 Q_2}{r}$ $V(r) = k \frac{Q}{r}$ $E_x = -\frac{\Delta V}{\Delta x}$ $W = -q\Delta V$

Magnetic Forces and Fields

 $F = qvB\sin\theta$ $F = IlB\sin\theta$ $\tau = NIAB\sin\theta = \mu B\sin\theta$ $PE = -\mu B\cos\theta$ $B = \frac{\mu_0 I}{2\pi r}$ $\varepsilon_{induced} = -N \frac{\Delta \phi_B}{\Delta t} = -N \frac{\Delta (BA \cos \theta)}{\Delta t}$

Constants

 $g = 9.8 \frac{m}{r^2}$ $1e = 1.6 \times 10^{-19}C$ $k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 \frac{Nm^2}{C^2}$ $\varepsilon_o = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$ $1eV = 1.6 \times 10^{-19} J$ $\mu_{o} = 4\pi \times 10^{-7} \frac{Tm}{A}$ $c = 3 \times 10^8 \frac{m}{c}$ $h = 6.63 \times 10^{-34} Js$ $m_e = 9.11 \times 10^{-31} kg = \frac{0.511 MeV}{c^2}$ $m_p = 1.67 \times 10^{-27} kg = \frac{937.1 MeV}{c^2}$ $m_n = 1.69 \times 10^{-27} kg = \frac{948.3 MeV}{c^2}$ $1amu = 1.66 \times 10^{-27} kg = \frac{931.5 MeV}{c^2}$ $N_{4} = 6.02 \times 10^{23}$ $Ar^2 \pm Br \pm C = 0 \implies r = \frac{-B \pm \sqrt{B^2} - 4AC}{2}$

$$\begin{aligned} \mathbf{Electric Circuits} & \mathbf{Light as a Wave} \\ I &= \frac{\Delta Q}{\Delta t} & c &= f\lambda = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \\ V &= IR = I \left(\frac{\rho L}{A}\right) & S(t) = \frac{energy}{time \times area} = c\varepsilon_o \\ R_{series} &= \sum_{i=1}^{N} R_i & I = S_{avg} = \frac{1}{2} c\varepsilon_o E_{max}^2 = c \\ \frac{1}{R_{parallel}} &= \sum_{i=1}^{N} \frac{1}{R_i} & P = \frac{S}{c} = \frac{Force}{Area} \\ P &= IV = I^2 R = \frac{V^2}{R} & S = S_o \cos^2 \theta \\ Q &= CV = \left(\frac{\kappa\varepsilon_0 A}{d}\right) V = (\kappa C_0) V & v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n} \\ PE &= \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{Q^2}{2C} & \theta_{inc} = \theta_{refl} \\ n_1 \sin \theta_1 = n_2 \sin \theta_2 \\ Q_{charge}(t) &= Q_{max} \left(1 - e^{-\frac{r}{RC}}\right) & \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \\ Q_{discharge}(t) &= Q_{max} e^{-\frac{r}{RC}} & M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \\ C_{parallel} &= \sum_{i=1}^{N} \frac{1}{C_i} & d \sin \theta = m\lambda \text{ or } (m + \frac{1}{2}) \\ a \sin \theta &= m'\lambda \end{aligned}$$

Light as a Particle & Relativity

$$E = hf = \frac{hc}{\lambda} = pc$$

$$KE_{max} = hf - \phi = eV_{stop}$$

$$\Delta \lambda = \frac{h}{m_e c} (1 - \cos \phi)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \gamma mv$$

$$E_{total} = KE + E_{rest} = \gamma mc^2$$

$$E_{total}^2 = p^2 c^2 + m^2 c^4$$

$$E_{rest} = mc^2$$

$$KE = (\gamma - 1)mc^2$$

Geometry

Circles: $C = 2\pi r = \pi D$ $A = \pi r^2$ Triangles: $A = \frac{1}{2}bh$ Spheres: $A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

$$c = f\lambda = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$$

$$S(t) = \frac{energy}{time \times area} = c\varepsilon_o E^2(t) = c \frac{B^2(t)}{\mu_0}$$

$$I = S_{avg} = \frac{1}{2}c\varepsilon_o E_{max}^2 = c \frac{B_{max}^2}{2\mu_0}$$

$$P = \frac{S}{c} = \frac{Force}{Area}$$

$$S = S_o \cos^2 \theta$$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n}$$

$$\theta_{inc} = \theta_{refl}$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

$$M_{total} = \prod_{i=1}^N M_i$$

$$d \sin \theta = m\lambda \text{ or } (m + \frac{1}{2})\lambda$$

$$a \sin \phi = m'\lambda$$

Nuclear Physics $E_{binding} = \left(Zm_p + Nm_n - m_{rest} \right) c^2$ $\frac{\Delta N}{\Delta t} = -\lambda N_o \rightarrow N(t) = N_o e^{-\lambda t}$ $A(t) = A_o e^{-\lambda t}$ $m(t) = m_0 e^{-\lambda t}$ $t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

Misc. Physics 110 Formulae

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\Delta (mv)}{\Delta t} = m\vec{a}$$

$$\vec{F} = -k\vec{y}$$

$$\vec{F}_{c} = m\frac{v^{2}}{R}\hat{r}$$

$$W = \Delta KE = \frac{1}{2}m(v_{f}^{2} - v_{i}^{2}) = -\Delta PE$$

$$PE_{gravity} = mgy$$

$$PE_{spring} = \frac{1}{2}ky^{2}$$

$$x_{f} = x_{i} + v_{ix}t + \frac{1}{2}a_{x}t^{2}$$

$$v_{fx} = v_{ix} + a_{x}t$$

$$v_{fx}^{2} = v_{ix}^{2} + 2a_{x}\Delta x$$