## Physics 111

Exam \#1

September 25, 2015

Name $\qquad$

Please read and follow these instructions carefully:

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization clear.
- You must show all work, including correct vector notation.
- You will not receive full credit for correct answers without adequate explanations.
- You will not receive full credit if incorrect work or explanations are mixed in with correct work. So erase or cross out anything you don't want graded.
- Make explanations complete but brief. Do not write a lot of prose.
- Include diagrams.
- Show what goes into a calculation, not just the final number. For example $|\vec{p}| \approx m|\vec{v}|=(5 \mathrm{~kg}) \times\left(2 \frac{\mathrm{~m}}{\mathrm{~s}}\right)=10 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}}$
- Give standard SI units with your results unless specifically asked for a certain unit.
- Unless specifically asked to derive a result, you may start with the formulas given on the formula sheet including equations corresponding to the fundamental concepts.
- Go for partial credit. If you cannot do some portion of a problem, invent a symbol and/or value for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.
- All multiple choice questions are worth 4 points and each free-response part is worth 10 points

| Problem \#1 | $/ 38$ |
| :---: | :---: |
| Problem \#2 | $/ 38$ |
| Total | $/ 76$ |

I affirm that I have carried out my academic endeavors with full academic honesty.

1. Suppose that want to place a collection of three point charges $(q=1 \mu C)$, one at a time, at the following locations; $q_{1}=q$ located at the point $(x, y)=(0,0), q_{2}=q$ located at the point $(x, y)=(a, 0)$, and $q_{3}=q$ located at the point $(x, y)=(2 a, 0)$, where the distance $a=10 \mathrm{~cm}$. Assume all charges are brought in from very far away and placed at each location.
a. How much work (in $J$ and $e V$ ) would be done to assemble this collection of point charges?

$$
\begin{aligned}
& W_{1}=0 \\
& W_{2}=-q_{2} \Delta V_{1}=-q\left[\frac{k q}{a}-0\right]=-\frac{k q^{2}}{a} \\
& W_{3}=-q_{3} \Delta V_{2,1}=-q\left[\left(\frac{k q}{a}+\frac{k q}{2 a}\right)-0\right]=-\frac{3 k q^{2}}{2 a} \\
& W_{\text {net }}=\sum_{i=1}^{3} W_{i}=0-\frac{k q^{2}}{a}-\frac{3 k q^{2}}{2 a}=-\frac{5 k q^{2}}{2 a}=-\frac{5 \times 9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}} \times\left(1 \times 10^{-6} \mathrm{C}\right)^{2}}{2 \times 0.1 \mathrm{~m}}=-0.225 \mathrm{~J} \\
& W_{\text {net }}=-0.225 \mathrm{~J}=-225 \mathrm{~mJ}=-1.4 \times 10^{18} \mathrm{eV}
\end{aligned}
$$

b. What is the net electric field at a point 10 cm located directly above $q_{2}$ ?

$$
\left.\begin{array}{l}
\vec{E}_{p}=\vec{E}_{1}+\vec{E}_{2}+\vec{E}_{3} \\
E_{p, x}=0 \\
E_{p, y}=E_{1} \sin \theta+E_{2}+E_{3} \sin \theta=\frac{k q}{2 a^{2}}\left(\frac{\sqrt{2}}{2}\right)+\frac{k q}{a^{2}}+\frac{k q}{2 a^{2}}\left(\frac{\sqrt{2}}{2}\right) \\
\quad=\frac{k q}{a^{2}}\left(\frac{2 \sqrt{2}}{4}+1\right)=1.707 \frac{\mathrm{kq}}{a^{2}}=1.707 \times\left(\frac{9 \times 10^{9} \frac{\mathrm{Nm}}{} \mathrm{C}^{2}}{} \times 1 \times 10^{-6} C\right. \\
(0.1 \mathrm{~m})^{2}
\end{array}\right)=1.54 \times 10^{6} \frac{\mathrm{~N}}{\mathrm{C}} .
$$

c. Suppose that a charge $q_{4}=\frac{1}{2} \mu C$ were placed at the point 10 cm located directly above $q_{2}$, what net force would $q_{4}$ feel?
$\vec{F}_{q_{4}}=q_{4} \vec{E}=1 \times 10^{-6} \mathrm{C} \times 1.54 \times 10^{6} \frac{\mathrm{~N}}{\mathrm{C}}=0.77 \mathrm{~N}$ in the positive y-direction.
d. Suppose that charge $q_{4}=\frac{1}{2} \mu C$ were placed at the point 10 cm directly above $q_{2}$ and that $q_{4}$ were released from rest. When $q_{4}$ is very far away from the 3 -charge system

1. its acceleration is zero and its speed is zero.
(2.) its acceleration is zero and its speed is constant.
2. its acceleration is not constant and therefore its speed is not constant.
3. its acceleration is not constant and its speed is zero.
4. its acceleration cannot be determined exactly and therefore its speed cannot be determined.
e. Suppose that now you have only the 3-charge system (with no $q_{4}$ present). If charge $q_{2}$ were displaced slightly to the right (closer to charge $q_{3}$ than to charge $q_{1}$ ) and released from rest, the resulting motion of $q_{2}$ would most likely be
5. to accelerate toward $q_{1}$ and subsequently collide with $q_{1}$.
6. to accelerate toward $q_{3}$ and subsequently collide with $q_{3}$.
7. to return to the midpoint between the two charges and return to rest.
8. to oscillate back and forth about the midpoint between the charges $q_{1}$ and $q_{3}$.
9. unable to be determined since the actual forces that act on charge $q_{2}$ are not know.
10. Cardiac dysrhythmia, or irregular heartbeat, is a condition that can have a number of causes and manifestations. One example of this is fibrillation, a condition in which the heart erratically contracting due to spontaneous electrical impulses from the cardiac nodes. A defibrillator is a device that can deliver a large amount of electrical energy in a short amount of time to someone suffering from cardiac dysrhythmia. This burst of electrical energy has the potential to briefly stop a heart that is beating irregularly. The heart will then immediately restart, and will often revert to a steady, normal beating pace again, as the spontaneous impulses have now been eliminated. One of the main components in a defibrillator is a capacitor and it applies a strong electric shock to the chest over a time interval of a few milliseconds. The device contains a capacitor of several microfarads, charged to several thousand volts. Electrodes called paddles, about 8 cm across and coated with conducting paste, and are held against the chest on both sides of the heart as shown below. Their handles are insulated to prevent injury to the operator, who calls "Clear!" and pushes a button on one paddle to discharge the capacitor through the patient's chest.

a. Suppose that someone was suffering from cardiac dysrhythmia and that when they arrived at the hospital you're working at, you need to use the defibrillator. Suppose that you wanted to charge the defibrillator so that it was ready to use in a time $t=1.3 \mathrm{~s}$ to a value of $98 \%$ of the maximum charge. What is the value of the capacitance that you would need so to achieve this result and the value of the capacitive time constant that you produce for the charging circuit of the defibrillator? Assume that the resistance of the charging circuit is $R=10 \mathrm{k} \Omega$.

$$
\begin{aligned}
& Q(t)=0.98 Q_{\max }=Q_{\max }\left(1-e^{-\frac{1}{R C}}\right) \rightarrow e^{-\frac{t}{R C}}=0.02 \rightarrow-\frac{t}{R C}=\ln (0.02)=-3.91 \\
& \therefore C=\frac{1.3 s}{10000 \Omega \times 3.91}=3.3 \times 10^{-5} F=33 \mu F
\end{aligned}
$$

The time constant: $\tau=R C=10000 \Omega \times 3.3 \times 10^{-5} F=0.33 \mathrm{~s}$
b. When the defibrillator is charging, energy is stored in the electric fields. When discharged through the patient, the energy stored in the electric fields is delivered to the patient as a shock, which momentarily stops the heart and allows the heart to "reset." Assuming that the capacitor is fully charged by a 4000 V battery, after one time constant, how much energy remains to be delivered to the patient?
Assume that a paddle has an effective area of $8 \times 10^{-3} \mathrm{~m}^{2}$, the separation between the paddles when in contact with the chest is 15 cm , and that the human body acts as an insulator with a dielectric constant of $\kappa=50$.

$$
\begin{aligned}
& \begin{aligned}
E_{i} & =\frac{1}{2} C V_{i}^{2}=\frac{1}{2}\left(\frac{\kappa \varepsilon_{0} A}{d}\right) V_{i}^{2}=\frac{1}{2} \times\left(\frac{50 \times 8.85 \times 10^{-12} \frac{C^{2}}{N m^{2}} \times 8 \times 10^{-3} m^{2}}{0.15 m}\right) \times(4000 V)^{2} \\
& =1.89 \times 10^{-4} J=189 \mu J
\end{aligned} \\
& E_{f}(t)=\frac{1}{2} C V^{2}(t)=\frac{1}{2} C\left(V_{\max } e^{-\frac{1}{R C}}\right)^{2}=\frac{1}{2} C V_{\max }^{2} e^{-\frac{2 t}{R C}}=E_{i} e^{-\frac{2 t}{R C}} \\
& \therefore E_{f}=E_{i} e^{-\frac{2 R C}{R C}}=1.89 \times 10^{-4} J \times 0.135=2.56 \times 10^{-5} J=25.6 \mu J
\end{aligned}
$$

c The patient eventually recovers but requires an artificial pacemaker (shown in the picture below). An artificial pacemaker is an electronic device that is inserted into the body and is electrically connected to the heart. Each cycle of the human heart begins with a natural pacemaker pulse from a group of nerve fibers located in the sinoatrial (SA) node. The artificial pacemaker is used to pulse the heart if the person's natural pacemaker fails to cause the heart to
 beat. Suppose that the resistor and capacitor used in the pacemaker circuit are given by $R_{p m}$ and $C_{p m}$. To design a pacemaker that is capable of doubling the heart rate when the patient exercises, which statement below is true? The capacitor $C_{p m}$
(1.) needs to discharge faster, so the resistance $R_{p m}$ should be decreased.
2. needs to discharge faster, so the resistance $R_{p m}$ should be increased.
3. needs to discharge slower, so the resistance $R_{p m}$ should be decreased.
4. needs to discharge slower, so the resistance $R_{p m}$ should be increased.
5. does not affect the timing of the pacemaker circuit, regardless of the resistance.
d. Suppose that the total current that a cardiac pacemaker produces is 0.015 A . How many electrons flow during the average lifetime of a pacemaker, 9.2 yrs ?

$$
\begin{aligned}
& I=\frac{\Delta Q}{\Delta t} \rightarrow \Delta Q=I \Delta t=0.015 \frac{c}{s} \times\left(9.2 y r \times \frac{365 d}{1 y r} \times \frac{24 h}{1 d} \times \frac{3600 s}{1 h}\right)=4.35 \times 10^{6} \mathrm{C} \\
& \Delta Q=n\left|e^{-}\right| \rightarrow n=\frac{\Delta Q}{\left|e^{-}\right|}=\frac{4.35 \times 10^{6} \mathrm{C}}{1.6 \times 10^{-19} \mathrm{C}}=2.7 \times 10^{25}
\end{aligned}
$$

e. This question does not depend on any of the preceding questions. A bulb (i.e., a resistor) is connected in series to a switch, a battery, and an uncharged capacitor. At time $t=0$, the switch is closed. Which of the following graphs below best describes the brightness of the bulb as a function of time?

1. A.
2. B.
3. C.
4. D.
5. Unable to be determined.
A.

B.

C.

D.


## Physics 111 Equation Sheet

Electric Forces, Fields and Potentials

$$
\begin{aligned}
& \vec{F}=k \frac{Q_{1} Q_{2}}{r^{2}} \hat{r} \\
& \vec{E}=\frac{\vec{F}}{q} \\
& \vec{E}_{Q}=k \frac{Q}{r^{2}} \hat{r} \\
& P E=k \frac{Q_{1} Q_{2}}{r} \\
& V(r)=k \frac{Q}{r} \\
& E_{x}=-\frac{\Delta V}{\Delta x} \\
& W=-q \Delta V
\end{aligned}
$$

Magnetic Forces and Fields
$F=q v B \sin \theta$
$F=I l B \sin \theta$
$\tau=N I A B \sin \theta=\mu B \sin \theta$
$P E=-\mu B \cos \theta$
$B=\frac{\mu_{0} I}{2 \pi r}$
$\varepsilon_{\text {indued }}=-N \frac{\Delta \phi_{B}}{\Lambda t}=-N \frac{\Delta(B A \cos \theta)}{\Lambda t}$

## Constants

$g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$1 e=1.6 \times 10^{-19} \mathrm{C}$
$k=\frac{1}{4 \pi \varepsilon_{o}}=9 \times 10^{9} \frac{\mathrm{~N} \mathrm{~N}^{2}}{\mathrm{c}^{2}}$
$\varepsilon_{o}=8.85 \times 10^{-12} \frac{\mathrm{C}^{2}}{\frac{\mathrm{Nm}}{}{ }^{2}}$
$1 \mathrm{eV}=1.6 \times 10^{-19} \mathrm{~J}$
$\mu_{o}=4 \pi \times 10^{-7} \frac{\mathrm{~T}}{\mathrm{~T}} \mathrm{~A}$
$c=3 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}$
$h=6.63 \times 10^{-34} \mathrm{Js}$
$m_{e}=9.11 \times 10^{-31} \mathrm{~kg}=\frac{0.511 \mathrm{MeV}}{c^{2}}$
$m_{p}=1.67 \times 10^{-27} \mathrm{~kg}=\frac{937.1 \mathrm{MeV}}{c^{2}}$
$m_{n}=1.69 \times 10^{-27} \mathrm{~kg}=\frac{948.3 \mathrm{MeV}}{c^{2}}$
$1 \mathrm{amu}=1.66 \times 10^{-27} \mathrm{~kg}=\frac{931.5 \mathrm{MeV}}{c^{2}}$
$N_{A}=6.02 \times 10^{23}$
$A x^{2}+B x+C=0 \rightarrow x=\frac{-B \pm \sqrt{B^{2}-4 A C}}{2 A}$

## Electric Circuits

$$
\begin{aligned}
& I=\frac{\Delta Q}{\Delta t} \\
& V=I R=I\left(\frac{\rho L}{A}\right) \\
& R_{\text {series }}=\sum_{i=1}^{N} R_{i} \\
& \frac{1}{R_{\text {parallel }}}=\sum_{i=1}^{N} \frac{1}{R_{i}} \\
& P=I V=I^{2} R=\frac{V^{2}}{R} \\
& Q=C V=\left(\frac{\kappa \varepsilon_{0} A}{d}\right) V=\left(\kappa C_{0}\right) V \\
& P E=\frac{1}{2} Q V=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C}
\end{aligned}
$$

$$
Q_{\text {charge }}(t)=Q_{\max }\left(1-e^{-\frac{t}{R C}}\right)
$$

$$
Q_{\text {discharge }}(t)=Q_{\max } e^{-\frac{t}{R C}}
$$

$$
C_{\text {parallel }}=\sum_{i=1}^{N} C_{i}
$$

$$
\frac{1}{C_{\text {series }}}=\sum_{i=1}^{N} \frac{1}{C_{i}}
$$

Light as a Particle \& Relativity

$$
\begin{aligned}
& E=h f=\frac{h c}{\lambda}=p c \\
& K E_{\max }=h f-\phi=e V_{\text {stop }} \\
& \Delta \lambda=\frac{h}{m_{e} c}(1-\cos \phi) \\
& \gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \\
& p=\gamma m v \\
& E_{\text {total }}=K E+E_{\text {rest }}=\gamma m c^{2} \\
& E_{\text {total }}^{2}=p^{2} c^{2}+m^{2} c^{4} \\
& E_{\text {rest }}=m c^{2} \\
& K E=(\gamma-1) m c^{2}
\end{aligned}
$$

Geometry
Circles: $C=2 \pi r=\pi D \quad A=\pi r^{2}$
Triangles: $A=\frac{1}{2} b h$
Spheres: $A=4 \pi r^{2} \quad V=\frac{4}{3} \pi r^{3}$

Light as a Wave

$$
\begin{aligned}
& c=f \lambda=\frac{1}{\sqrt{\varepsilon_{o} \mu_{o}}} \\
& S(t)=\frac{\text { energy }}{\text { time } \times \text { area }}=c \varepsilon_{o} E^{2}(t)=c \frac{B^{2}(t)}{\mu_{0}} \\
& I=S_{\text {avg }}=\frac{1}{2} c \varepsilon_{o} E_{\max }^{2}=c \frac{B_{\max }^{2}}{2 \mu_{0}} \\
& P=\frac{S}{c}=\frac{\text { Force }}{\text { Area }} \\
& S=S_{o} \cos ^{2} \theta \\
& v=\frac{1}{\sqrt{\varepsilon \mu}}=\frac{c}{n} \\
& \theta_{\text {inc }}=\theta_{\text {refl }} \\
& n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2} \\
& \frac{1}{f}=\frac{1}{d_{o}}+\frac{1}{d_{i}} \\
& M=\frac{h_{i}}{h_{o}}=-\frac{d_{i}}{d_{o}} \\
& M_{\text {total }}=\prod_{i=1}^{N} M_{i} \\
& d \sin \theta=m \lambda \text { or }\left(m+\frac{1}{2}\right) \lambda \\
& a \sin \phi=m^{\prime} \lambda
\end{aligned}
$$

Nuclear Physics

$$
\begin{aligned}
& E_{\text {binding }}=\left(Z m_{p}+N m_{n}-m_{r ब t}\right) c^{2} \\
& \frac{\Delta N}{\Delta t}=-\lambda N_{o} \rightarrow N(t)=N_{o} e^{-\lambda t} \\
& A(t)=A_{o} e^{-\lambda t} \\
& m(t)=m_{o} e^{-\lambda t} \\
& t_{\frac{1}{2}}=\frac{\ln 2}{\lambda}
\end{aligned}
$$

Misc. Physics 110
Formulae
$\vec{F}=\frac{\Delta \vec{p}}{\Delta t}=\frac{\Delta(m v)}{\Delta t}=m \vec{a}$ $\vec{F}=-k \vec{y}$
$\vec{F}_{C}=m \frac{v^{2}}{R} \hat{r}$
$W=\Delta K E=\frac{1}{2} m\left(v_{f}^{2}-v_{i}^{2}\right)=-\Delta P E$
$P E_{g r a v i t y}=m g y$
$P E_{\text {spring }}=\frac{1}{2} k y^{2}$
$x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}$
$v_{f x}=v_{i x}+a_{x} t$
$v_{f x}^{2}=v_{i x}^{2}+2 a_{x} \Delta x$

